Ellipse Fitting for Computer Vision

Implementation and Applications

Synthesis Lectures on Computer Vision

Editors

Gérard Medioni, *University of Southern California* Sven Dickinson, *University of Toronto*

Synthesis Lectures on Computer Vision is edited by Gérard Medioni of the University of Southern California and Sven Dickinson of the University of Toronto. The series publishes 50- to 150 page publications on topics pertaining to computer vision and pattern recognition. The scope will largely follow the purview of premier computer science conferences, such as ICCV, CVPR, and ECCV. Potential topics include, but not are limited to:

- Applications and Case Studies for Computer Vision
- Color, Illumination, and Texture
- Computational Photography and Video
- Early and Biologically-inspired Vision
- Face and Gesture Analysis
- Illumination and Reflectance Modeling
- Image-Based Modeling
- Image and Video Retrieval
- Medical Image Analysis
- Motion and Tracking
- Object Detection, Recognition, and Categorization
- Segmentation and Grouping
- Sensors
- Shape-from-X
- Stereo and Structure from Motion
- · Shape Representation and Matching

- Statistical Methods and Learning
- Performance Evaluation
- Video Analysis and Event Recognition

Ellipse Fitting for Computer Vision: Implementation and Applications

Kenichi Kanatani, Yasuyuki Sugaya, and Yasushi Kanazawa 2016

Background Subtraction: Theory and Practice

Ahmed Elgammal 2014

Vision-Based Interaction

Matthew Turk and Gang Hua 2013

Camera Networks: The Acquisition and Analysis of Videos over Wide Areas

Amit K. Roy-Chowdhury and Bi Song 2012

Deformable Surface 3D Reconstruction from Monocular Images

Mathieu Salzmann and Pascal Fua 2010

Boosting-Based Face Detection and Adaptation

Cha Zhang and Zhengyou Zhang 2010

Image-Based Modeling of Plants and Trees

Sing Bing Kang and Long Quan 2009

© Springer Nature Switzerland AG 2022 Reprint of original edition © Morgan & Claypool 2016

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means—electronic, mechanical, photocopy, recording, or any other except for brief quotations in printed reviews, without the prior permission of the publisher.

Ellipse Fitting for Computer Vision: Implementation and Applications Kenichi Kanatani, Yasuyuki Sugaya, and Yasushi Kanazawa

ISBN: 978-3-031-00687-6 paperback ISBN: 978-3-031-01815-2 ebook

DOI 10.1007/978-3-031-01815-2

A Publication in the Springer Nature series SYNTHESIS LECTURES ON COMPUTER VISION

Lecture #8

Series Editors: Gérard Medioni, *University of Southern California* Sven Dickinson, *University of Toronto*

Series ISSN

Print 2153-1056 Electronic 2153-1064

Ellipse Fitting for Computer Vision

Implementation and Applications

Kenichi Kanatani

Okayama University, Okayama, Japan

Yasuyuki Sugaya

Toyohashi University of Technology, Toyohashi, Aichi, Japan

Yasushi Kanazawa

Toyohashi University of Technology, Toyohashi, Aichi, Japan

SYNTHESIS LECTURES ON COMPUTER VISION #8

ABSTRACT

Because circular objects are projected to ellipses in images, ellipse fitting is a first step for 3-D analysis of circular objects in computer vision applications. For this reason, the study of ellipse fitting began as soon as computers came into use for image analysis in the 1970s, but it is only recently that optimal computation techniques based on the statistical properties of noise were established. These include *renormalization* (1993), which was then improved as *FNS* (2000) and *HEIV* (2000). Later, further improvements, called *hyperaccurate correction* (2006), *HyperLS* (2009), and *hyper-renormalization* (2012), were presented. Today, these are regarded as the most accurate fitting methods among all known techniques. This book describes these algorithms as well implementation details and applications to 3-D scene analysis.

We also present general mathematical theories of statistical optimization underlying all ellipse fitting algorithms, including rigorous covariance and bias analyses and the theoretical accuracy limit. The results can be directly applied to other computer vision tasks including computing fundamental matrices and homographies between images.

This book can serve not simply as a reference of ellipse fitting algorithms for researchers, but also as learning material for beginners who want to start computer vision research. The sample program codes are downloadable from the website: https://sites.google.com/a/morganclaypool.com/ellipse-fitting-for-computer-vision-implementation-and-applications/.

KEYWORDS

geometric distance minimization, hyperaccurate correction, HyperLS, hyperrenormalization, iterative reweight, KCR lower bound, maximum likelihood, renormalization, robust fitting, Sampson error, statistical error analysis, Taubin method

Contents

1	Intro	Introduction			
	1.1	Ellipse Fitting			
	1.2	Representation of Ellipses			
	1.3	Least Squares Approach			
	1.4	Noise and Covariance Matrices			
	1.5	Ellipse Fitting Approaches			
	1.6	Supplemental Note			
2	Alge	ebraic Fitting			
	2.1	Iterative Reweight and Least Squares			
	2.2	Renormalization and the Taubin Method			
	2.3	Hyper-renormalization and HyperLS			
	2.4	Summary			
	2.5	Supplemental Note			
3	Geo	Geometric Fitting			
	3.1	Geometric Distance and Sampson Error			
	3.2	FNS			
	3.3	Geometric Distance Minimization			
	3.4	Hyperaccurate Correction			
	3.5	Derivations			
	3.6	Supplemental Note			
4	Rob	Robust Fitting			
	4.1	Outlier Removal			
	4.2	Ellipse-specific Fitting			
	4.3	Supplemental Note			

	ւալ	ose-based 3-D Computation	3/
	5.1	Intersections of Ellipses	37
	5.2	Ellipse Centers, Tangents, and Perpendiculars	38
	5.3	Perspective Projection and Camera Rotation	40
	5.4	3-D Reconstruction of the Supporting Plane	43
	5.5	Projected Center of Circle	44
	5.6	Front Image of the Circle	45
	5.7	Derivations	47
	5.8	Supplemental Note	50
6	Expo	eriments and Examples	55
	6.1	Ellipse Fitting Examples	55
	6.2	Statistical Accuracy Comparison	56
	6.3	Real Image Examples 1	59
	6.4	Robust Fitting	59
	6.5	Ellipse-specific Methods	59
	6.6	Real Image Examples 2	61
	6.7	Ellipse-based 3-D Computation Examples	62
	6.8	Supplemental Note	64
7			
7		Supplemental Note	67
7	Exte	Supplemental Note Insion and Generalization Fundamental Matrix computation 7.1.1 Formulation	67 67 67
7	Exte	Supplemental Note Insion and Generalization Fundamental Matrix computation 7.1.1 Formulation 7.1.2 Rank Constraint	67 67 67 70
7	Exte 7.1	Supplemental Note Insion and Generalization Fundamental Matrix computation 7.1.1 Formulation 7.1.2 Rank Constraint 7.1.3 Outlier Removal	67 67 67 70 71
7	Exte	Supplemental Note Insion and Generalization Fundamental Matrix computation 7.1.1 Formulation 7.1.2 Rank Constraint 7.1.3 Outlier Removal Homography Computation	67 67 67 70 71 72
7	Exte 7.1	Supplemental Note Insion and Generalization Fundamental Matrix computation 7.1.1 Formulation 7.1.2 Rank Constraint 7.1.3 Outlier Removal Homography Computation 7.2.1 Formulation	67 67 70 71 72 72
7	Exte 7.1 7.2	Supplemental Note Insion and Generalization Fundamental Matrix computation 7.1.1 Formulation 7.1.2 Rank Constraint 7.1.3 Outlier Removal Homography Computation 7.2.1 Formulation 7.2.2 Outlier Removal	67 67 67 70 71 72 72 76
7	Exte 7.1	Supplemental Note Insion and Generalization Fundamental Matrix computation 7.1.1 Formulation 7.1.2 Rank Constraint 7.1.3 Outlier Removal Homography Computation 7.2.1 Formulation	67 67 67 70 71 72 72 76
7	7.1 7.2	Supplemental Note Insion and Generalization Fundamental Matrix computation 7.1.1 Formulation 7.1.2 Rank Constraint 7.1.3 Outlier Removal Homography Computation 7.2.1 Formulation 7.2.2 Outlier Removal Supplemental Note Irracy of Algebraic Fitting	67 67 67 70 71 72 72 76 77
Ť	7.1 7.2 7.3 Accus 8.1	Supplemental Note Insion and Generalization Fundamental Matrix computation 7.1.1 Formulation 7.1.2 Rank Constraint 7.1.3 Outlier Removal Homography Computation 7.2.1 Formulation 7.2.2 Outlier Removal Supplemental Note Irracy of Algebraic Fitting Error Analysis	67 67 67 70 71 72 72 76 77 79
Ť	7.1 7.2 7.3 Accuss 8.1 8.2	Supplemental Note Insion and Generalization Fundamental Matrix computation 7.1.1 Formulation 7.1.2 Rank Constraint 7.1.3 Outlier Removal Homography Computation 7.2.1 Formulation 7.2.2 Outlier Removal Supplemental Note Irracy of Algebraic Fitting Error Analysis Covariance and Bias	67 67 70 71 72 76 77 79 80
Ť	7.1 7.2 7.3 Accus 8.1 8.2 8.3	Supplemental Note Finsion and Generalization Fundamental Matrix computation 7.1.1 Formulation 7.1.2 Rank Constraint 7.1.3 Outlier Removal Homography Computation 7.2.1 Formulation 7.2.2 Outlier Removal Supplemental Note Firacy of Algebraic Fitting Error Analysis Covariance and Bias Bias Elimination and Hyper-renormalization	67 67 67 70 71 72 76 77 79 80 82
Ť	7.1 7.2 7.3 Accuss 8.1 8.2	Supplemental Note Insion and Generalization Fundamental Matrix computation 7.1.1 Formulation 7.1.2 Rank Constraint 7.1.3 Outlier Removal Homography Computation 7.2.1 Formulation 7.2.2 Outlier Removal Supplemental Note Irracy of Algebraic Fitting Error Analysis Covariance and Bias	67 67 70 71 72 76 77 79 80 82 83

9	Maximum Likelihood and Geometric Fitting		
	9.1	Maximum Likelihood and Sampson Error	
	9.2	Error Analysis	
	9.3	Bias Analysis and Hyperaccurate Correction96	
	9.4	Derivations	
	9.5	Supplemental Note	
10	Theoretical Accuracy Limit		
	10.1	KCR Lower Bound	
	10.2	Derivation of the KCR Lower Bound	
	10.3	Expression of the KCR Lower Bound	
	10.4	Supplemental Note	
	Ansv	vers	
	Bibli	ography	
	Auth	ors' Biographies	
	Inde	x	

Preface

Because circular objects are projected to ellipses in images, ellipse fitting is a first step for 3-D analysis of circular objects in computer vision applications. For this reason, the study of ellipse fitting began as soon as computers came into use for image analysis in the 1970s. The basic principle was to compute the parameters so that the sum of squares of expressions that should ideally be zero is minimized, which is today called *least squares* or *algebraic distance minimization*. In the 1990s, the notion of optimal computation based on the statistical properties of noise was introduced by researchers including the authors. The first notable example was the authors' *renormalization* (1993), which was then improved as *FNS* (2000) and *HEIV* (2000) by researchers in Australia and the U.S. Later, further improvements, called *hyperaccurate correction* (2006), *HyperLS* (2009), and *hyper-renormalization* (2012), were presented by the authors. Today, these are regarded as the most accurate fitting methods among all known techniques. This book describes these algorithms as well as underlying theories, implementation details, and applications to 3-D scene analysis.

Most textbooks on computer vision begin with mathematical fundamentals followed by the resulting computational procedures. This book, in contrast, *immediately* describes computational procedures after a short statement of the purpose and the principle. The theoretical background is briefly explained as *Comments*. Thus, readers need not worry about mathematical details, which often annoy those who only want to build their vision systems. Rigorous derivations and detailed justifications are given later in separate sections, but they can be skipped if the interest is not in theories. Sample program codes of the authors are provided via the website¹ of the publisher. At the end of each chapter is given a section called *Supplemental Note*, describing historical backgrounds, related issues, and reference literature.

Chapters 1–4 specifically describe ellipse fitting algorithms. Chapter 5 discusses 3-D analysis of circular objects in the scene extracted by ellipse fitting. In Chapter 6, performance comparison experiments are conducted among the methods described in Chapters 1–4. Also, some real image applications of the 3-D analysis of Chapter 5 are shown. In Chapter 7, we point out how procedures of ellipse fitting can straightforwardly be extended to fundamental matrix and homography computation, which play a central role in 3-D analysis by computer vision. Chapters 8 and 9 give general mathematical theories of statistical optimization underlying all ellipse fitting algorithms. Finally, Chapter 10 gives a rigorous analysis of the theoretical accuracy limit. However, beginners and practice-oriented readers can skip these last three chapters.

The authors used the materials in this book as student projects for introductory computer vision research at Okayama University, Japan, and Toyohashi University of Technology, Japan. By implementing the algorithms themselves, students can learn basic programming know-hows

¹https://sites.google.com/a/morganclaypool.com/ellipse-fitting-for-computer-vision-implementation-and-applications/

xii PREFACE

and also understand the theoretical background of vision computation as their interest deepens. We are hoping that this book can serve not simply as a reference of ellipse fitting algorithms for researchers, but also as learning material for beginners who want to start computer vision research.

The theories in this book are the fruit of the authors' collaborations and interactions with their colleagues and friends for many years. The authors thank Takayuki Okatani of Tohoku University, Japan, Mike Brooks and Wojciech Chojnacki of the University of Adelaide, Australia, Peter Meer of Rutgers University, U.S., Wolfgang Förstner, of the University of Bonn, Germany, Michael Felsberg of Linköping University, Sweden, Rudolf Mester of the University of Frankfurt, Germany, Prasanna Rangarajan of Southern Methodist University, U.S., Ali Al-Sharadqah of University of East Carolina, U.S., and Alexander Kukush of the University of Kiev, Ukraine. Special thanks are to (late) Professor Nikolai Chernov of the University of Alabama at Birmingham, U.S., without whose inspiration and assistance this work would not have been possible.

Kenichi Kanatani, Yasuyuki Sugaya, and Yasushi Kanazawa March 2016