Ellipse Fitting for Computer Vision

Implementation and Applications

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Ellipse Fitting for Computer Vision

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Kenichi Kanatani
Okayama University, Okayama, Japan

Yasuyuki Sugaya
Toyohashi University of Technology, Toyohashi, Aichi, Japan

Yasushi Kanazawa
Toyohashi University of Technology, Toyohashi, Aichi, Japan

SYNTHESIS LECTURES ON COMPUTER VISION #8

ABSTRACT

Because circular objects are projected to ellipses in images, ellipse fitting is a first step for 3-D analysis of circular objects in computer vision applications. For this reason, the study of ellipse fitting began as soon as computers came into use for image analysis in the 1970s, but it is only recently that optimal computation techniques based on the statistical properties of noise were established. These include *renormalization* (1993), which was then improved as *FNS* (2000) and *HEIV* (2000). Later, further improvements, called *hyperaccurate correction* (2006), *HyperLS* (2009), and *hyper-renormalization* (2012), were presented. Today, these are regarded as the most accurate fitting methods among all known techniques. This book describes these algorithms as well implementation details and applications to 3-D scene analysis.

We also present general mathematical theories of statistical optimization underlying all ellipse fitting algorithms, including rigorous covariance and bias analyses and the theoretical accuracy limit. The results can be directly applied to other computer vision tasks including computing fundamental matrices and homographies between images.

This book can serve not simply as a reference of ellipse fitting algorithms for researchers, but also as learning material for beginners who want to start computer vision research. The sample program codes are downloadable from the website: `https://sites.google.com/a/morganclaypool.com/ellipse-fitting-for-computer-vision-implementation-and-applications/`.

KEYWORDS

geometric distance minimization, hyperaccurate correction, HyperLS, hyper-renormalization, iterative reweight, KCR lower bound, maximum likelihood, renormalization, robust fitting, Sampson error, statistical error analysis, Taubin method

Contents

Preface

Because circular objects are projected to ellipses in images, ellipse fitting is a first step for 3-D analysis of circular objects in computer vision applications. For this reason, the study of ellipse fitting began as soon as computers came into use for image analysis in the 1970s. The basic principle was to compute the parameters so that the sum of squares of expressions that should ideally be zero is minimized, which is today called *least squares* or *algebraic distance minimization*. In the 1990s, the notion of optimal computation based on the statistical properties of noise was introduced by researchers including the authors. The first notable example was the authors' *renormalization* (1993), which was then improved as *FNS* (2000) and *HEIV* (2000) by researchers in Australia and the U.S. Later, further improvements, called *hyperaccurate correction* (2006), *HyperLS* (2009), and *hyper-renormalization* (2012), were presented by the authors. Today, these are regarded as the most accurate fitting methods among all known techniques. This book describes these algorithms as well as underlying theories, implementation details, and applications to 3-D scene analysis.

Most textbooks on computer vision begin with mathematical fundamentals followed by the resulting computational procedures. This book, in contrast, *immediately* describes computational procedures after a short statement of the purpose and the principle. The theoretical background is briefly explained as *Comments*. Thus, readers need not worry about mathematical details, which often annoy those who only want to build their vision systems. Rigorous derivations and detailed justifications are given later in separate sections, but they can be skipped if the interest is not in theories. Sample program codes of the authors are provided via the website[1] of the publisher. At the end of each chapter is given a section called *Supplemental Note*, describing historical backgrounds, related issues, and reference literature.

Chapters 1–4 specifically describe ellipse fitting algorithms. Chapter 5 discusses 3-D analysis of circular objects in the scene extracted by ellipse fitting. In Chapter 6, performance comparison experiments are conducted among the methods described in Chapters 1–4. Also, some real image applications of the 3-D analysis of Chapter 5 are shown. In Chapter 7, we point out how procedures of ellipse fitting can straightforwardly be extended to fundamental matrix and homography computation, which play a central role in 3-D analysis by computer vision. Chapters 8 and 9 give general mathematical theories of statistical optimization underlying all ellipse fitting algorithms. Finally, Chapter 10 gives a rigorous analysis of the theoretical accuracy limit. However, beginners and practice-oriented readers can skip these last three chapters.

The authors used the materials in this book as student projects for introductory computer vision research at Okayama University, Japan, and Toyohashi University of Technology, Japan. By implementing the algorithms themselves, students can learn basic programming know-hows

[1]https://sites.google.com/a/morganclaypool.com/ellipse-fitting-for-computer-vision-implementation-and-applications/

and also understand the theoretical background of vision computation as their interest deepens. We are hoping that this book can serve not simply as a reference of ellipse fitting algorithms for researchers, but also as learning material for beginners who want to start computer vision research.

The theories in this book are the fruit of the authors' collaborations and interactions with their colleagues and friends for many years. The authors thank Takayuki Okatani of Tohoku University, Japan, Mike Brooks and Wojciech Chojnacki of the University of Adelaide, Australia, Peter Meer of Rutgers University, U.S., Wolfgang Förstner, of the University of Bonn, Germany, Michael Felsberg of Linköping University, Sweden, Rudolf Mester of the University of Frankfurt, Germany, Prasanna Rangarajan of Southern Methodist University, U.S., Ali Al-Sharadqah of University of East Carolina, U.S., and Alexander Kukush of the University of Kiev, Ukraine. Special thanks are to (late) Professor Nikolai Chernov of the University of Alabama at Birmingham, U.S., without whose inspiration and assistance this work would not have been possible.

Kenichi Kanatani, Yasuyuki Sugaya, and Yasushi Kanazawa
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