Analysis and Design of Transmitarray Antennas

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# Analysis and Design of Transmitarray Antennas

Ahmed H. Abdelrahman University of Colorado Boulder

Fan Yang Tsinghua University

Atef Z. Elsherbeni Colorado School of Mines

Payam Nayeri Colorado School of Mines

SYNTHESIS LECTURES ON ANTENNAS #12

## ABSTRACT

In recent years, transmitarray antennas have attracted growing interest with many antenna researchers. Transmitarrays combines both optical and antenna array theory, leading to a low profile design with high gain, high radiation efficiency, and versatile radiation performance for many wireless communication systems. In this book, comprehensive analysis, new methodologies, and novel designs of transmitarray antennas are presented.

- Detailed analysis for the design of planar space-fed array antennas is presented. The basics of aperture field distribution and the analysis of the array elements are described. The radiation performances (directivity and gain) are discussed using array theory approach, and the impacts of element phase errors are demonstrated.
- The performance of transmitarray design using multilayer frequency selective surfaces (M-FSS) approach is carefully studied, and the transmission phase limit which are generally independent from the selection of a specific element shape is revealed. The maximum transmission phase range is determined based on the number of layers, substrate permittivity, and the separations between layers.
- In order to reduce the transmitarray design complexity and cost, three different methods have been investigated. As a result, one design is performed using quad-layer cross-slot elements with no dielectric material and another using triple-layer spiral dipole elements. Both designs were fabricated and tested at X-Band for deep space communications. Furthermore, the radiation pattern characteristics were studied under different feed polarization conditions and oblique angles of incident field from the feed.
- New design methodologies are proposed to improve the bandwidth of transmitarray antennas through the control of the transmission phase range of the elements. These design techniques are validated through the fabrication and testing of two quad-layer transmitarray antennas at Ku-band.
- A single-feed quad-beam transmitarray antenna with 50 degrees elevation separation between the beams is investigated, designed, fabricated, and tested at Ku-band.

In summary, various challenges in the analysis and design of transmitarray antennas are addressed in this book. New methodologies to improve the bandwidth of transmitarray antennas have been demonstrated. Several prototypes have been fabricated and tested, demonstrating the desirable features and potential new applications of transmitarray antennas.

## **KEYWORDS**

transmitarray antennas, frequency selective surfaces, multilayer aperture antennas, high gain antennas, wideband transmitarray antennas, multibeam transmitarray antennas

Dedicated to my parents, my beloved wife Heba, and my children Farida and Adam.

-Ahmed H. Abdelrahman

Dedicated to my colleagues and family.

– Fan Yang

To my wife, Magda, daughters, Dalia and Donia, son, Tamer, and the memory of my parents.

-Atef Z. Elsherbeni

To my parents.

– Payam Nayeri

## Contents

	List	of Figures
	List	of Tables
	Ackr	nowledgmentsxxiii
1	Intro	oduction
	1.1	Transmitarray Antenna Concept1
	1.2	Comparison with Some Related Antenna Technologies
	1.3	Transmitarray Design Approaches
		1.3.1 Multi-layer Frequency Selective Surfaces (M-FSS)
		1.3.2 Receiver-transmitter Design
		1.3.3 Metamaterial/Transformation Approach
	1.4	Overview of Research Topics
2	Spac	re-fed Array Design Method
	2.1	Phase Distribution on Transmitarray Aperture
	2.2	Unit-cell Element Analysis
	2.3	Radiation Analysis using the Array Theory9
	2.4	Directivity Calculations
		2.4.1 Method 1: Numerical Integration
		2.4.2 Method 2: Utilization of Bessel Function
		2.4.3 Method 3: Illumination Efficiency
		2.4.4 Comparison Between the Three Methods
		2.4.5 Directivity Bandwidth
	2.5	Antenna Gain
		2.5.1 Spillover Efficiency
		2.5.2 Element Losses
	2.6	Phase Error Analysis
		2.6.1 Design Errors
		2.6.2 Approximations in Unit-cell Analysis
		2.6.3 Manufacturing Errors

x		
3	Ana	lysis of Multi-layer Transmitarray Antenna 33
	3.1	Single-layer FSS Analysis
		3.1.1 Theoretical Analysis of Single-layer FSS
		3.1.2 Numerical Demonstration of Single-layer FSS
		3.1.3 Single-layer of Double Square Loop Elements
		3.1.4 Single Conductor with a Substrate Layer
	3.2	Double-layer FSS Analysis
		3.2.1 Theoretical Analysis of Double-layer FSS
		3.2.2 Numerical Demonstration of Double-layer FSS
	3.3	Multi-layer FSS Analysis
		3.3.1 Analytical Analysis of Triple-layer FSS
		3.3.2 Numerical Demonstration of Triple-layer FSS
		3.3.3 Quad-layer FSS
4	AQ	uad-layer Transmitarray Antenna Using Slot-type Elements
	4.1	Cross-slot Transmitarray Antenna Design
		4.1.1 Cross-slot Element Design
		4.1.2 Transmitarray Design and Measurements
	4.2	Discussion on Oblique Incidence and Feed Polarization Effects61
		4.2.1 Element Performance under Oblique Incidence
		4.2.2 Aperture Distribution and Radiation Pattern
5	Des	ign of Triple-layer Transmitarray Antennas
	5.1	Identical Triple-layer Transmitarray Antenna
		5.1.1 Spiral Dipole Element Design
		5.1.2 Transmitarray Design
		5.1.3 Experiment and Discussion
	5.2	Non-identical Triple-layer Transmitarray Antenna
		5.2.1 Non-identical Double-layer FSS Analysis
		5.2.2 Non-identical Triple-layer FSS Analysis
	5.3	Double-layer Unit-cells
6	Wid	leband Transmitarray Antennas
	6.1	Bandwidth Analysis of a Transmitarray using Quad-layer Double Square
		Loop Elements
		6.1.1 Unit-cell Property
		6.1.2 Bandwidth Performance of Transmitarray

	6.2	Bandwidth Performance with Different Reference Phases at the Aperture Center
	6.3	Proper Selection of Element Phase Range for Improvement of
		Transmitarray Bandwidth103
	6.4	Comparison between Different Element Shapes105
	6.5	Prototype Fabrication and Measurements
7	Sing	le-feed Multi-beam Transmitarrays 115
	7.1	Design Methodologies for Single-feed Multi-beam Transmitarray Antennas . 115
	7.2	Design of Ku-band Single-feed Quad-beam Transmitarray Antennas116
	7.3	Prototype Fabrication and Measurements
	7.4	Transmitarray Approximation and Performance Discussions
		7.4.1 Oblique Incidence Effect of the Unit-cell Element
		7.4.2 Variations in Dimensions of Neighboring Elements
		7.4.3 Phase Error and Magnitude Loss Effect on the Radiation Patterns 128
8	Con	clusions
	8.1	Contributions of this Book
	8.2	Future Work
A	S-m	atrix of Cascaded Layers
	Bibl	iography
	Autl	nors' Biographies

xi

# List of Figures

1.1	Geometry of a transmitarray antenna
1.2	Antenna technologies related to transmitarrays: (a) a planar phased array, (b) a lens antenna, and (c) a planar reflectarray
1.3	Multi-layer FSS configuration
1.4	Receiver-transmitter configuration
1.5	Content of the book
2.1	Phase compensation of a multi-layer transmittarray antenna
2.2	Example of the required phase distribution in a circular aperture
	transmittarray
2.3	A 3D model a quad-layer transmitarray unit-cell in CST Microwave Studio software [27]
2.4	Transmission coefficient of a quad-layer unit-cell at 13.5 GHz10
2.5	The coordinate system of the transmitarray antenna
2.6	Radiation pattern of a $30 \times 30$ circular apperture transmitarray antenna with a broadside beam
2.7	Directivity calculations of a circular aperture transmitarray antenna using the numerical integration method
2.8	Directivity vs. frequency of $60 \times 60$ rectangular transmitarray antenna20
2.9	Spillover efficiency analysis
2.10	Sources of phase errors
2.11	Radiation pattern of a circular aperture transmitarray antenna at different quantization phase values
2.12	Transmitarray antenna gain vs. quantization phase
2.13	Phase distribution of the transmitarray antenna for different quantization phase values: (a) ideal phase distribution, (b) 3-bit phase distribution, (c) 2-bit phase distribution, and (d) 1-bit phase distribution
2.14	Radiation pattern of a circular aperture transmitarray antenna at different transmission phase ranges

2.15	Transmitarray antenna gain vs. limited phase range
2.16	Surface deformation on: (a) reflectarray surface and (b) transmitarray surface 29
2.17	Feed position error of a transmitarray antenna
2.18	Standard normal distribution function
2.19	Effect of random phase error on the gain of a transmitarray antenna
3.1	Single-layer with a conducting element
3.2	Transmission coefficient of a single-layer configuration
3.3	Unit-cells of: (a) a cross dipole and (b) a square loop
3.4	Transmission coefficients of the single-layer elements: (a) $ S_{21} $ and (b) $\angle  S_{21} $
3.5	Transmission coefficient presented on polar diagrams for: (a) a cross dipole, (b) a square loop, and (c) a cross-slot elements
3.6	Unit-cell of a double square loop element
3.7	Transmission coefficient of the single-layer double square loop element: (a) $ S_{21} $ and $\angle  S_{21} $ and (b) polar plot
3.8	Transmission coefficient of the single-layer double square loop element under oblique incidence angle of $30^\circ$ : (a) $ S_{21} $ , (b) $\angle  S_{21} $ , (c) perpendicular polarization in a polar diagram, and (d) parallel polarization in a polar diagram
3.9	Single conductor thick substrate layer
3.10	Transmission coefficient of a single conductor thick substrate layer for different substrate electrical thickness $\beta L_d$ at $\varepsilon_r = 2.5$
3.11	Transmission coefficient of a single conductor thick substrate layer for different substrate permitivity $\varepsilon_r$ at $\beta L_d = 90^\circ$
3.12	Transmission cofficient of a single conductor layer when replacing the thick substrate by an air gap for different $\beta L_d$ values
3.13	Simulation and analytical results of a single conductor thick substrate layer using the double square loop shape: (a) for different substrate electrical thickness $\beta L_d$ at $\varepsilon_r = 2.5$ and (b) for different substrate permittivity $\varepsilon_r$ at $\beta L_d = 90^\circ$
3.14	Double-layer FFS configuration
3.15	Transmission coefficient of the double-layer for different dielectric permitivity by constant electrical thickness of $\beta L_d = 90^\circ45$

3.16	Transmission coefficient of the double-layer for different substrate electrical thickness using dielectric permitivity $\varepsilon_r = 1$
3.17	Simulation and analytical results of a double-layer configuration using the double square loop element for different dielectric permittivity but constant electrical thickness of $\beta L_d = 90^\circ$ : (a) transmission magnitude, (b) transmission phase, and (c) polar plot
3.18	Simulation and analytical results of a double-layer configuration using the double square loop element for different electrical thickness but constant substrate permittivity of $\varepsilon_r = 2.5$ : (a) $\beta L_d = 60^\circ$ , (b) $\beta L_d = 90^\circ$ , (c) $\beta L_d = 120^\circ$ , and (d) $\beta L_d = 270^\circ$
3.19	Simulation and analytical results of a double-layer configuration using the double square loop element for different substrate permittivity but constant substrate thickness of $L_d = \lambda 0/4$ : (a) $\varepsilon_r = 1$ ( $\beta L_d = 90^\circ$ ) and (b) $\varepsilon_r = 2.5$ ( $\beta L_d = 142.3^\circ$ )
3.20	Simulation results of a double-layer configuration using the double square loop element for both lossless and lossy materials
3.21	Triple-layer FSS configuration
3.22	Transmission coefficients of the triple layer for different substrate electrical thickness using dielectric permittivity: (a) $\varepsilon_r = 1$ and (b) $\varepsilon_r = 2$
3.23	Transmission phase range of a triple-layer FSS with electrical thickness between the conductor layers of $\beta L_d = 90^{\circ}$
3.24	Simulation and analytical results of a triple-layer configuration using the double square loop element for different dielectric permittivity but constant electrical thickness of $\beta L_d = 90^\circ$ : (a) transmission magnitude, (b) transmission phase, and (c) polar plot.
3.25	Ouad-laver FSS configuration
3.26	Transmission coefficient of the quad-layer FSS for $\beta L_d = 90^\circ$ and $\varepsilon_r = 1, \dots, 55$
3.27	Simulation and analytical results of the quad-layer configuration using the double square loop element for $\beta L_d = 90^\circ$ and $\varepsilon_r = 1$ : (a) transmission magnitude, (b) transmission phase, and (c) polar plot
4.1	Cross-slot element unit-cell: (a) top view and (b) side view
4.2	Transmission coefficient vs. the slot length $L_s$ for the four identical layers of the unit-cell shown in Fig. 4.1
4.3	A quad-layer circular aperture transmitarray antenna: (a) transmitarray mask and (b) picture of the fabricated quad-layer transmitarray

xv

4.4	Transmitarray antenna setup for a near field measurement
4.5	Measured and simulated radiation pattern, considering only normal incidence plane wave in the simulation for all array elements
4.6	Transmitarray antenna measured gain vs. frequency
4.7	Transmission coefficient versus slot length at different oblique incidence angles: (a) transmission magnitude and (b) transmission phase
4.8	The transmitarray elements illumination by the feed horn
4.9	Transmission coefficient of the double square loop element at normal incidence and 30° oblique angle: (a) transmission magnitude and (b) transmission phase
4.10	Oblique incidence wave from the feed antenna on a sample array element: (a) 3D view, (b) top view ( $x-y$ plane), and (c) plane of incidence view
4.11	Transmitted vertically polarized electric field: (a) with normal incidence plane wave approximation and (b) with oblique incidence
4.12	Measured and simulated radiation pattern, considering both the oblique incidence angles and the feed polarization in the simulation of each element of the array
5.1	Triple-layer unit-cell: (a) cross-dipole shape, (b) spiral-dipole shape, and (c) triple-layer configuration of the unit-cell
5.2	Transmission coefficients of the triple-layer cross-dipole and spiral-dipole elements: (a) magnitude vs. the element dimension $L$ , (b) phase vs. the element dimension $L$ , (c) phase magnitude relation of the cross-slot element in a polar diagram, and (d) phase magnitude relation of the
	cross-slot element in a polar diagram
5.3	Transmission coefficient vs. element dimension L under different incident angles: (a) magnitudes and (b) phases
5.4	Typical geometry of a printed transmitarray antenna75
5.5	Transmitarray average element loss vs. aperture center element phase
5.6	Circular aperture transmitarray antenna: (a) top view picture of the transmitarray aperture, (b) elements transmission phase distribution, (c) elements transmission magnitude distribution, and (d) relative illumination from the feed on the transmitarray elements
5.7	Transmitarray antenna setup for a near field measurement 77
5.0	Measured $H$ plane (vz plane) and $F$ plane (vz plane) radiation patterns 77
5.8	$T_{T}$

xvi

5.9	Transmitarray antenna gain vs. frequency
5.10	Non-identical double-layer FSS configuration
5.11	Non-identical double-layer FSS configuration: (a) $\varepsilon_r = 1$ and (b) $\varepsilon_r = 2.580$
5.12	Non-identical triple-layer FSS configuration
5.13	Non-identical triple-layer FSS configuration: (a) $\varepsilon_r = 1$ and (b) $\varepsilon_r = 2.582$
5.14	Ideal full transmission phase range of 360° of a triple-layer FSS configuration using a combination of both identical and non-identical layers with air-gap separation between layers ( $\varepsilon_r = 1$ ) and electrical thickness of $\beta L_d = 90^\circ$
5.15	Transmission coefficient of a non-identical, triple-layer FSS configuration with $\varepsilon_r = 1$ and $\beta L_d = 90^\circ$ for quantization phase of each conductor layer equal to: (a) 0.5°, (b) 2°, (c) 3°, and (d) 5°
5.16	Simulation results of a non-identical, triple-layer FSS configuration with $\varepsilon_r = 1$ and $\beta L_d = 90^\circ$ using the double square loop element at 11.3 GHz85
5.17	Sensitivity of a non-identical, triple-layer unit-cell with $\varepsilon_r = 1$ and $\beta L_d = 90^\circ$ to phase variations of $\pm 1^\circ$ in the conductor layers
5.18	Sensitivity of a high transmission coefficient, non-identical triple-layer unit-cell with $\varepsilon_r = 1$ and $\beta L_d = 90^\circ$ to phase tolerance in each layer equal to: (a) $\pm 1^\circ$ and (b) $\pm 2^\circ$
5.19	Sensitivity of a non-identical, triple-layer unit-cell with $\varepsilon_r = 1$ and $\beta L_d = 90^\circ$ and for different transmission magnitude values to phase tolerance in each layer
5.20	Practical full transmission phase range of 360° for a triple-layer FSS configuration using a combination of both identical and non-identical layers with air-gap separation between layers ( $\varepsilon_r = 1$ ) and electrical thickness of $\beta L_d = 90^\circ$
5.21	Sensitivity of a high transmission coefficient, non-identical, triple-layer unit-cell with $\varepsilon_r = 1$ and $\beta L_d = 30^\circ$ to phase tolerance in each layer equal to: (a) $\pm 1^\circ$ and (b) $\pm 2^\circ$
5.22	Full transmission phase range of 360° for a triple-layer FSS configuration using a combination of both identical and non-identical layers with air-gap separation between layers ( $\varepsilon_r = 1$ ) and electrical thickness of $\beta L_d = 30^\circ$ 91
5.23	Transmission coefficients of a double-layer unit-cell using air gap ( $\varepsilon_r = 1$ ) with electrical separation equal to $\beta L_d = 205^\circ$ : (a) the double-layer configuration and (b) transmission coefficient in a polar diagram

VV111	
VIII V	

5.24	Transmission coefficients of a double-layer unit-cell using air gap ( $\varepsilon_r = 1$ ) with electrical separation between layers equal to $\beta L_d = 155^\circ$ and taking into account the difference in electrical thickness of ( $\Delta = 50^\circ$ ) with the unit-cell of Fig. 5.23: (a) the double-layer configuration and (b) transmission coefficient in a polar diagram
5.25	Transmission coefficients of the two double-layer unit-cells of Figs. 5.23 and 5.24 in a single polar plot: (a) complete curves and (b) required transmission phase range from each unit-cell
5.26	Transmission coefficients of the two double-layer unit-cells of Figs. 5.23 and 5.24 using double square loop element: (a) transmission magnitude, (b) transmission phase, and (c) polar plot
5.27	Transmitarray antenna design using two groups of double-layer unit-cells, which have different thicknesses: (a) mask of the first layer, (b) mask of the second layer, (c) mask of the third layer, and (d) side-view cut94
6.1	The quad-layer unit-cell configuration of a double square loop element: (a) top view and (b) side view
6.2	Transmission coefficients at different frequencies: (a) magnitudes, (b) phases, (c) polar plot at 13.0 GHz, (d) polar plot at 13.5 GHz, and (e) polar plot at 14 GHz
6.3	Transmission coefficients vs. frequency for different values of $L_1$ : (a) magnitudes and (b) phases
6.4	Geometry of a printed transmitarray antenna
6.5	Effects of element phase error and element loss on the transmitarray antenna gain
6.6	Calculated gain for different phase values at the aperture center
6.7	Transmission magnitudes on the transmitarray aperture in dB with two different phase values at the aperture center $\psi_c$ and at three different frequencies.
6.8	Two different transmission phase ranges: (a) 360° at 13.5 GHz, (b) 240° at 13.5 GHz, (c) 240° at 13.0 GHz, and (d) 240° at 14.0 GHz
6.9	Transmitarray calculated gain for different transmission phase ranges 106
6.10	Aperture efficiency and 1 dB gain bandwitdth vs. transmission phase range 106
6.11	(a) DFLL element and (b) Jerusalem cross element
6.12	Bandwidth of 1 dB gain vs. transmission phase range of three different element shapes

6.13	(a) Transmitarray mask with the difference in dimensions for some elements of the two antennas and (b) elements that are different in the two antennas, as represented by the "x" symbol
6.14	Measurement setup of a transmitarray antenna using the NSI planar new-field system
6.15	Measured and simulated radiation patterns at 13.5 GHz: (a) Antenna 1 with full phase range and (b) Antenna 2 with limited phase range112
6.16	Gains vs. frequency of the two antennas: (a) theoretical and (b) measurement. 113
7.1	The quad-layer unit-cell configuration of a double square loop element: (a) top view and (b) side view
7.2	Transmission coefficient of the double square loop element with normal incidence at 13.5 GHz: (a) transmission magnitude, (b) transmission phase, and (c) polar plot
7.3	Design 1: A constant side-lobe mask: (a) ideal pattern, (b) optimized phase distribution without space delay term, (c) actual phase distribution including space delay term, (d) radiation patterns along horizontal cut, and (e) radiation patterns along vertical cut
7.4	Design 2: A tapered side-lobe mask: (a) ideal pattern, (b) optimized phase distribution without space delay term, (c) actual phase distribution including space delay term, (d) radiation patterns along horizontal cut, and (e) radiation patterns along vertical cut
7.5	Design 3: A tapered side-lobe mask and double penalty in main beam regions: (a) ideal pattern, (b) optimized phase distribution without space delay term, (c) actual phase distribution including space delay term, (d) radiation patterns along horizontal cut, and (e) radiation patterns along vertical cut
7.6	One layer of the fabricated quad-beam transmitarray prototype: (a) mask and (b) photograph
7.7	Near-field measurement setup of the single-feed quad-beam transmitarray antenna
7.8	Near-field patterns: (a) co-pol amplitude, (b) <i>x</i> -pol magnitude, (c) co-pol phase, and (d) <i>x</i> -pol phase
7.9	Far-field patterns at 13.5 GHz: (a) <i>xz</i> -plane, (b) <i>yz</i> -plane, and (c) 3-D pattern

7.10	Transmission coefficients of the double square loop element vs. element dimension $L_1$ under different incident angles: (a) magnitude of elements along x-axis, (b) magnitude of elements along y-axis, (c) phase of elements along x-axis, and (d) phase of elements along y-axis 125
7.11	Large unti-cell analysis
7.12	Transmission coefficients of the large unit-cell compared with the conventional unit-cell: (a) transmission magnitude and (b) transmission phase
7.13	Radiation patterns of 20 transmitarray trials for different standard deviations of the random phase error distribution
7.14	Average radiation patterns of 20 transmitarray trials for different standard deviations of the random phase error distribution
7.15	Radiation patterns of 20 transmitarray trials for different standard deviations of the random magnitude loss distribution
7.16	Average radiation patterns of 20 transmitarray trials for different standard deviations of the random magnitude loss distribution

xx

# **List of Tables**

2.1	Directivity calculations and the corresponding computational time of a 30 $\times$ 30 circular aperture transmitarray antenna with different $\Delta\theta$ and $\Delta\phi$ values .15
2.2	Comparison between the three methods of directivity calculations
2.3	Directivity vs. rectangular array size
3.1	Transmission phase magnitude relationship of a single conductor layer 36
3.2	Transmission phase range of a double-layer FSS
3.3	Transmission phase range of a triple-layer FSS with electrical thickness between the conductor layers of $\beta L_d = 90^\circ$
4.1	Comparison of transmitarray measured and simulated performance67
5.1	Comparison of current results with recent published work
6.1	Comparison of transmitarray antennas differ in the phase values at the center of the aperture
6.2	Comparison of four transmitarray antennas differ in the element transmission phase ranges
6.3	Design configurations of the two transmitarray prototypes110
6.4	Measurement results of the two transmitarray prototypes111
7.1	Comparison of different designs of single-feed quad-beam transmitarray antennas
7.2	Dimensions $L_1$ of the neighboring elements for the three cases of the large unit-cell

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