An Introduction to
Partial Differential Equations

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Daniel J. Arrigo
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An Introduction to Partial Differential Equations

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ABSTRACT

This book is an introduction to methods for solving partial differential equations (PDEs). After the introduction of the main four PDEs that could be considered the cornerstone of Applied Mathematics, the reader is introduced to a variety of PDEs that come from a variety of fields in the Natural Sciences and Engineering and is a springboard into this wonderful subject. The chapters include the following topics: First-order PDEs, Second-order PDEs, Fourier Series, Separation of Variables, and the Fourier Transform. The reader is guided through these chapters where techniques for solving first- and second-order PDEs are introduced. Each chapter ends with a series of exercises illustrating the material presented in each chapter.

The book can be used as a textbook for any introductory course in PDEs typically found in both science and engineering programs and has been used at the University of Central Arkansas for over ten years.

KEYWORDS

advection equation, heat equation, wave equation and Laplace's equation, method of characteristics, separation of variables, Fourier series, and the Fourier transform

Contents

Preface

This is an introductory book about obtaining exact solutions to partial differential equations (PDEs). It is based on my lecture notes from a course I have taught almost every year since 2001 at the University of Central Arkansas (UCA).

When I began teaching the course, I tried several textbooks. There are many fine textbooks on the market but they just seemed to miss the mark for students at UCA. Even though the average ACT scores of incoming freshmen at UCA are among the highest in the state, the textbooks that were available were too sophisticated for my students. I also felt that the way most books taught the subject matter could be improved upon. For example, a lot of the books start with the separation of variables, a technique used for solving second-order linear PDEs. As most books on ordinary differential equations start with solving first-order ODEs before considering second-order ODEs, I felt the same order would be beneficial in solving PDEs. This naturally led to the presentation in this book.

In Chapter 1, I introduce four basic PDEs which some would consider the cornerstone of Applied Mathematics: the advection equation, the heat equation, Laplace's equation, and the wave equation. After this, I list 12 PDEs (systems of PDEs) that appear in science and engineering and provide a springboard into the subject matter. Most of these PDEs are nonlinear in nature since our world is inherently nonlinear. However, one must first know how to solve linear PDEs before entering the nonlinear world.

In Chapter 2, I introduce the student to first-order PDEs. Through a change of variables, we solve constant coefficient and linear PDEs and are led to the method of characteristics. We continue with solving quasilinear and higher-dimensional PDEs, and then progress to fully nonlinear first-order PDES. The chapter ends with Charpit's method, a method that seeks compatibility between two first-order PDEs.

In Chapter 3, we focus on second-order PDEs, and in particular, three standard forms: (i) parabolic standard form, (ii) hyperbolic standard form, and (iii) elliptic standard form. Students learn how to transform to each standard form. The chapter ends with a derivation of the classic d'Alembert solution.

In Chapter 4, after a brief introduction to separation of variables for the heat equation, I introduce Fourier series. I introduce both the regular Fourier series and the Fourier Sine and Cosine series. Several examples are considered showing various standard functions and their Fourier series representations. At this point, I return the students to solving PDEs.

In Chapter 5, we continue our discussion with the separation of variables where we consider the heat equation, Laplace's equation and the wave equation. We start with the heat equation and consider several types of problems. One example has fixed homogeneous boundary conditions, no flux boundary conditions and radiating boundary conditions; then we consider nonhomogeneous boundary conditions. Next, we consider nonhomogeneous equations, equations with solution dependent source terms, then solution dependent convective terms. We move on to Laplace's equation and, finally, to the wave equation.

The final chapter, Chapter 6, involves the Fourier (Sine/Cosine) transform. It is a generalization of the Fourier series, where the length of the interval approaches infinity. It is through these transforms that we are able to solve a variety of PDEs on the infinite and half infinite domain.

The book is self-contained; the only requirements are a solid foundation in calculus and elementary differential equations. Chapters 1–5 have been the basis of a one-semester course at the University of Central Arkansas for over a decade. The material in Chapter 6 could certainly be included. For the times that I have taught the course, I have omitted Chapter 6 in favor of student seminars. I ask students to pick topics, extensions or applications of the material covered in class, and present oral seminars to the class with formal write-ups on their topic being due by the end of the course. My goal is that at the end of the course the students understand why studying this subject is important.

Daniel J. Arrigo
January 2018

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