PDE Models for Atherosclerosis Computer Implementation in R

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ISBN: 978-3-031-01286-0 paperback
ISBN: 978-3-031-02414-6 ebook
ISBN: 978-3-031-00260-1 hardcover

DOI 10.1007/978-3-031-02414-6

A Publication in the Springer series
SYNTHESIS LECTURES ON MATHEMATICS AND STATISTICS

Lecture #22
Series Editor: Steven G. Krantz, *Washington University, St. Louis*
Series ISSN
Print 1938-1743 Electronic 1938-1751

PDE Models for Atherosclerosis Computer Implementation in R

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SYNTHESIS LECTURES ON MATHEMATICS AND STATISTICS #22

ABSTRACT

Atherosclerosis is a pathological condition of the arteries in which plaque buildup and stiffening (hardening) can lead to stroke, myocardial infarction (heart attacks), and even death. Cholesterol in the blood is a key marker for atherosclerosis, with two forms: (1) LDL - low density lipoproteins and (2) HDL - high density lipoproteins. Low LDL and high HDL concentrations are generally considered essential for limited atherosclerosis and good health.

This book pertains to a mathematical model for the spatiotemporal distribution of LDL and HDL in the arterial endothelial inner layer (EIL, intima). The model consists of a system of six partial differential equations (PDEs) with the dependent variables

1. $\ell(x,t)$: concentration of modified LDL
2. $h(x,t)$: concentration of HDL
3. $p(x,t)$: concentration of chemoattractants
4. $q(x,t)$: concentration of ES cytokines
5. $m(x,t)$: density of monocytes/macrophages
6. $N(x,t)$: density of foam cells

and independent variables

1. x: distance from the inner arterial wall
2. t: time

The focus of this book is a discussion of the methodology for placing the model on modest computers for study of the numerical solutions. The foam cell density $N(x,t)$ as a function of the bloodstream LDL and HDL concentrations is of particular interest as a precursor for arterial plaque formation and stiffening.

The numerical algorithm for the solution of the model PDEs is the method of lines (MOL), a general procedure for the computer-based numerical solution of PDEs. The MOL coding (programming) is in R, a quality, open-source scientific computing system that is readily available from the Internet. The R routines for the PDE model are discussed in detail, and are available from a download link so that the reader/analyst/researcher can execute the model to duplicate the solutions reported in the book, then experiment with the model, for example, by changing the parameters (constants) and extending the model with additional equations.

KEYWORDS

atherosclerosis, cholesterol, LDL, HDL, partial differential equations, method of lines

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Preface

Atherosclerosis is a pathological condition of the arteries in which plaque buildup and stiffening (hardening) can lead to stroke, myocardial infarction (heart attacks), and even death. Cholesterol in the blood is a key marker for atherosclerosis, with two forms: (1) LDL - low density lipoproteins and (2) HDL - high density lipoproteins. Low LDL and high HDL concentrations are generally considered essential for limited atherosclerosis and good health.

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and independent variables

1. x: distance from the inner arterial wall
2. t: time

These variables are discussed in detail in [1–4]. Authoritative introductions to atherosclerosis are given with Google searches for: (1) NIH atherosclerosis, (2) MGH atherosclerosis and (3) Mayo atherosclerosis. The emphasis of this book is a discussion of the methodology for placing the model on modest computers for study of the numerical solutions, specifically, the six dependent variables $\ell(x,t)$ to $N(x,t)$ as a function of x and t. The foam cell density $N(x,t)$ as a function of the bloodstream LDL and HDL concentrations is of particular interest as a precursor for arterial plaque formation and stiffening.

The numerical algorithm for the solution of the model PDEs is the method of lines (MOL), a general procedure for the computer-based numerical solution of PDEs. The MOL coding (programming) is in R, a quality, open-source scientific computing system that is readily available from the Internet. The R routines for the PDE model are discussed in detail, and are

available from a download link so that the reader/analyst/researcher can execute the model to duplicate the solutions reported in the book, then experiment with the model, for example, by changing the parameters (constants) and extending the model with additional equations.

In summary, the intent of the book is to provide variants of a computer-based model that can be used to study the spatiotemporal dynamics of atherosclerosis. The cases discussed in the book include the possible use of LDL-lowering and HDL-raising drug therapy for altering foam cell production, and thereby, arterial plaque formation and stiffening.

I would welcome comments about the application and usefulness of the models and their computer implementation.

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October 2018

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