

Reasoning with Probabilistic and Deterministic Graphical Models

Exact Algorithms

Second Edition

Synthesis Lectures on Artificial Intelligence and Machine Learning

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Second Edition

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University of California, Irvine

*SYNTHESIS LECTURES ON ARTIFICIAL INTELLIGENCE AND
MACHINE LEARNING #41*

ABSTRACT

Graphical models (e.g., Bayesian and constraint networks, influence diagrams, and Markov decision processes) have become a central paradigm for knowledge representation and reasoning in both artificial intelligence and computer science in general. These models are used to perform many reasoning tasks, such as scheduling, planning and learning, diagnosis and prediction, design, hardware and software verification, and bioinformatics. These problems can be stated as the formal tasks of constraint satisfaction and satisfiability, combinatorial optimization, and probabilistic inference. It is well known that the tasks are computationally hard, but research during the past three decades has yielded a variety of principles and techniques that significantly advanced the state of the art.

This book provides comprehensive coverage of the primary exact algorithms for reasoning with such models. The main feature exploited by the algorithms is the model's graph. We present inference-based, message-passing schemes (e.g., variable-elimination) and search-based, conditioning schemes (e.g., cycle-cutset conditioning and AND/OR search). Each class possesses distinguished characteristics and in particular has different time vs. space behavior. We emphasize the dependence of both schemes on few graph parameters such as the treewidth, cycle-cutset, and (the pseudo-tree) height. The new edition includes the notion of influence diagrams, which focus on sequential decision making under uncertainty. We believe the principles outlined in the book would serve well in moving forward to approximation and anytime-based schemes. The target audience of this book is researchers and students in the artificial intelligence and machine learning area, and beyond.

KEYWORDS

graphical models, Bayesian networks, constraint networks, Markov networks, influence diagrams, induced-width, treewidth, cycle-cutset, loop-cutset, pseudo-tree, bucket-elimination, variable-elimination, AND/OR search, conditioning, reasoning, inference, knowledge representation

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Preface

Graphical models, including constraint networks (hard and soft), Bayesian networks, Markov random fields, and influence diagrams, have become a central paradigm for knowledge representation and reasoning, and provide powerful tools for solving problems in a variety of application domains, including scheduling and planning, coding and information theory, signal and image processing, data mining, computational biology, and computer vision.

These models can be acquired from experts or learned from data. Once a model is available, we need to be able to make deductions and to extract various types of information. We refer to this as *reasoning* in analogy with the human process of thinking and reasoning. These reasoning problems can be stated as the formal tasks of constraint satisfaction and satisfiability, combinatorial optimization, and probabilistic inference. It is well known that these tasks are computationally hard, but research during the past three decades has yielded a variety of effective principles and led to impressive scalability of exact techniques.

In this book we provide a comprehensive coverage of the main exact algorithms for reasoning with such models. The primary feature exploited by the algorithms is the model's graph structure and they are therefore uniformly applicable across a broad range of models, where dependencies are expressed as constraints, cost functions or probabilistic relationships. We also provide a glimpse into properties of the dependencies themselves, known as context-specific independencies, when treating deterministic functions such as constraints. Clearly, exact algorithms must be complemented by approximations. Indeed, we see this book as the first phase of a broader book that would cover approximation algorithms as well. We believe, however, that in order to have effective approximations we have to start with the best exact algorithms.

The book is organized into seven chapters and a conclusion. Chapter 1 provides an introduction to the book and its contents. Chapter 2 introduces the reader to the formal definition of the general graphical model and then describes the most common models, including constraint networks and probabilistic networks, which are used throughout the book. We distinguish two classes of algorithms: inference-based, message-passing schemes (Chapters 3, 4, and 5) and search-based, conditioning schemes (Chapters 6 and 7). This division is useful because algorithms in each class possesses common and distinguished characteristics and in particular have different behavior with respect to the tradeoff between time and memory. Chapter 7 focuses on this tradeoff, introducing hybrids of search and inference schemes. We emphasize the dependence of both types on few graph parameters such as the treewidth, cycle-cutset, and (the pseudo-tree) height.

The book is based on research done in my lab over the past two decades. It is largely founded on work with my graduate and postdoctoral students including: Dan Frost, Irina

Rish, Kalev Kask, David Larkin, Robert Mateescu, Radu Marinescu, Bozhena Bidyuk, Vibhav Gogate, Lars Otten, Natasha Flerova and William Lam and my postdoctoral students Javier Larrosa, and Emma Rollon. Most heavily it relies on the work of Kalev Kask (Chapter 5) and Robert Mateescu (Chapters 6 and 7). I wish to also thank my colleagues at UCI for providing a supportive environment in our AI and machine learning labs, and especially to Alex Ihler for our recent collaboration that has been particularly inspiring and fruitful.

I owe a great deal to members of my family that took an active role in some parts of this book. First, to my son Eyal who spent several months reading and providing editing, as well as very useful suggestions regarding the book's content and exposition. Thanks also go to my husband Avi on providing editorial comments on large parts of this book and to Anat Gafni for her useful comments on Chapter 1.

The second edition of the book contains many edits and corrections that were suggested by students who took my classes over the last five years and by other readers, and I would like to thank them all. It also includes two new subsections on influence diagrams in Chapter 2 and Chapter 4.

Rina Dechter
Los Angeles, January 2019