Decentralized Robust Control of Coupled Multi-Agent Systems under Local Signal Temporal Logic Tasks

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Abstract—Motivated by the recent interest in formal methods-based control of multi-agent systems, we adopt a bottom-up approach. Each agent is subject to a local signal temporal logic task that may depend on other agents' behavior. These dependencies pose control challenges since some of the tasks may be opposed to each other. We first develop a local continuous feedback control law and identify conditions under which this control law guarantees satisfaction of the local tasks. If these conditions do not hold, we propose to use the developed control law in combination with an online detection & repair scheme, expressed as a local hybrid system. After detection of a critical event, a three-stage procedure is initiated to resolve the problem. The theoretical results are illustrated in simulations.

I. INTRODUCTION

Multi-agent systems under global objectives such as consensus, formation control, and connectivity maintenance have been well studied by the research community. Comprehensive overviews of these topics can be found in [1] and [2], where the derived controllers are mainly distributed control laws. The need for more complex and rich objectives in robotic applications has led to formal methods-based control strategies where temporal logics, e.g., linear temporal logic, are used to formulate high-level temporal tasks. Top-down approaches have been considered in [3], [4] by decomposing a global temporal task into local ones that need to be executed by each agent individually. Top-down approaches often require agent synchronization and are usually subject to high computational complexity and hence impractical when the problem size becomes larger. On the other hand, the works in [5], [6] favor a bottom-up approach, where local tasks are independently distributed to each agent. This leads to partially decentralized solutions that reduce the computational burden. In a bottom-up approach, feasibility of each local task does not imply feasibility of the conjunction of all local tasks [5] since some of the local tasks may be opposed to each other. The presented works in [3]-[6] rely on automata-based verification techniques that discretize the physical environment and agent dynamics. In this paper, we instead consider continuous-time and nonlinear dynamics without the need for discretizing neither environment nor agent dynamics in space or time. To the best of our knowledge, this is the first approach not making use of such discretization in the context of formal methods-based multiagent control. This paper extends our work on single-agent systems [7] to multi-agent systems.

We adopt a bottom-up approach by considering local tasks formulated in signal temporal logic [8]. These tasks can depend on each other, i.e., also oppose each other. This makes the control of multi-agent systems under signal temporal logic tasks a challenge and the main research question in this paper. Signal temporal logic introduces the notion of space robustness [9], a robustness metric stating how robustly a signal satisfies a given task. In a first step, we identify conditions under which a continuous feedback control law, which is derived by combining space robustness and prescribed performance control [10], satisfies basic signal temporal logic tasks. If these conditions do not hold, an online detection & repair scheme is introduced by defining a local hybrid system [11] for each agent. Critical events will be detected and resolved in a three-stage procedure, gradually relaxing parameters such as robustness. One advantage of our decentralized approach is the low computational complexity due to the continuous feedback control laws. Furthermore, the team of agents is allowed to be heterogeneous with additional dynamic couplings among them. Robustness is considered with respect to disturbances and with respect to the signal temporal logic task. Multi-agent systems under signal temporal logic tasks have also been considered in [12] in a centralized approach, not investigating formula dependencies, but with a special focus on communication.

The remainder is organized as follows: in Section II, notation and preliminaries are introduced, while Section III presents the problem definition. Section IV presents our solution to the stated problem, which is verified by simulations in Section V. Conclusions are given in Section VI. This online version is an extended version of the 2018 American Control Conference version.

II. PRELIMINARIES

Scalar quantities are denoted by lowercase, non-bold letters x and column vectors are lowercase, bold letters x. True and false are denoted by \top and \bot ; \mathbb{R} are the real numbers, while \mathbb{R}^n is the *n*-dimensional real vector space. The natural, non-negative, and positive real numbers are \mathbb{N} , $\mathbb{R}_{\geq 0}$, and $\mathbb{R}_{>0}$, respectively. For convenience, we define $\begin{bmatrix} x & y \end{bmatrix} := \begin{bmatrix} x^T & y^T \end{bmatrix}^T$. For two sets \mathcal{X} and \mathcal{Y} , the set-valued map $F : \mathcal{X} \Rightarrow \mathcal{Y}$ maps each $x \in \mathcal{X}$ to a set $F(x) \subseteq \mathcal{Y}$. The inverse image by a function F of a set $\mathcal{M} \subseteq \mathcal{Y}$ is given by inv $(F(\mathcal{M})) := \{x \in \mathcal{X} | F(x) \cap \mathcal{M} \neq \emptyset\}$.

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Two basic results regarding the existence of solutions for initial-value problems (IVP) are needed in this paper. Assume $y \in \Omega_y \subseteq \mathbb{R}^{n_y}$ and consider the IVP

$$\dot{\boldsymbol{y}} = H(\boldsymbol{y}, t) \text{ with } \boldsymbol{y}_0 := \boldsymbol{y}(0) \in \Omega_{\boldsymbol{y}},$$
 (1)

where $H : \Omega_{y} \times \mathbb{R}_{\geq 0} \to \mathbb{R}^{n_{y}}$ and Ω_{y} is a non-empty and open set. A solution to this IVP is a signal $y : \mathcal{J} \to \Omega_{y}$ with $\mathcal{J} \subseteq \mathbb{R}_{\geq 0}$ obeying (1).

Lemma 1: [13, Theorem 54] Consider the IVP in (1). Assume that $H: \Omega_{\boldsymbol{y}} \times \mathbb{R}_{\geq 0} \to \mathbb{R}^{n_{\boldsymbol{y}}}$ is: 1) locally Lipschitz continuous on \boldsymbol{y} for each $t \in \mathbb{R}_{\geq 0}$, 2) piecewise continuous on t for each fixed $\boldsymbol{y} \in \Omega_{\boldsymbol{y}}$. Then, there exists a unique and maximal solution $\boldsymbol{y}: \mathcal{J} \to \Omega_{\boldsymbol{y}}$ with $\mathcal{J} := [0, \tau_{\max}) \subseteq \mathbb{R}_{\geq 0}$ and $\tau_{\max} \in \mathbb{R}_{>0} \cup \infty$.

Lemma 2: [13, Proposition C.3.6] Assume that the assumptions of Lemma 1 hold. For a maximal solution \boldsymbol{y} on $\mathcal{J} = [0, \tau_{\max})$ with $\tau_{\max} < \infty$ and for any compact set $\Omega'_{\boldsymbol{y}} \subset \Omega_{\boldsymbol{y}}$, there exists $t' \in \mathcal{J}$ such that $\boldsymbol{y}(t') \notin \Omega'_{\boldsymbol{y}}$.

A. Signal Temporal Logic (STL)

Signal temporal logic (STL) is a predicate logic based on signals [8]. STL consists of predicates μ that are obtained after the evaluation of a predicate function $h : \mathbb{R}^n \to \mathbb{R}$ as

$$\mu := \begin{cases} \top \text{ if } h(\boldsymbol{x}) \ge 0 \\ \bot \text{ if } h(\boldsymbol{x}) < 0 \end{cases}$$

For instance, it is possible to express the predicate $\mu := (|x_i + x_j| \le 1)$ with the predicate function $h(x) := 1 - |x_i + x_j|$ to specify that the *i*-th and *j*-th state should be close. The STL syntax is

$$\phi ::= \top \mid \mu \mid \neg \phi \mid \phi \land \psi \mid \phi U_{[a,b]} \psi ,$$

where μ is a predicate and ϕ and ψ are STL formulas. The temporal until-operator $U_{[a,b]}$ is time bounded with time interval [a,b] where $a,b \in \mathbb{R}_{\geq 0}$ is such that $a \leq b$. The satisfaction relation $(x,t) \models \phi$ indicates if the signal $x : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ satisfies ϕ at time t. The STL semantics are given next.

Definition 1 (STL Semantics): The STL semantics are inductively defined as [8, Definition 1]:

$$\begin{aligned} (\boldsymbol{x},t) &\models \mu & \Leftrightarrow h(\boldsymbol{x}(t)) \ge 0\\ (\boldsymbol{x},t) &\models \neg \phi & \Leftrightarrow \neg ((\boldsymbol{x},t) \models \phi)\\ (\boldsymbol{x},t) &\models \phi \land \psi & \Leftrightarrow (\boldsymbol{x},t) \models \phi \land (\boldsymbol{x},t) \models \psi\\ (\boldsymbol{x},t) &\models \phi U_{[a,b]} \psi \Leftrightarrow \exists t_1 \in [t+a,t+b] \text{ s.t. } (\boldsymbol{x},t_1) \models \psi\\ \land \forall t_2 \in [t,t_1], \ (\boldsymbol{x},t_2) \models \phi \end{aligned}$$

Disjunction-, eventually-, and always-operator are derived as $\phi \lor \psi := \neg (\neg \phi \land \neg \psi)$, $F_{[a,b]}\phi := \top U_{[a,b]}\phi$, and $G_{[a,b]}\phi := \neg F_{[a,b]}\neg \phi$, respectively. Robust semantics, called space robustness and denoted by $\rho^{\phi}(\boldsymbol{x},t)$, have been introduced in [9] and are defined in Definition 2; $\rho^{\phi}(\boldsymbol{x},t)$ determines how robustly the signal $\boldsymbol{x} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ satisfies ϕ at time t. It holds that $(\boldsymbol{x},t) \models \phi$ if $\rho^{\phi}(\boldsymbol{x},t) > 0$.

Definition 2 (Space Robustness): The semantics of space

robustness are inductively defined as [9, Definition 3]:

$$\begin{split} \rho^{\mu}(\boldsymbol{x},t) &:= h(\boldsymbol{x}(t)) \\ \rho^{\neg\phi}(\boldsymbol{x},t) &:= -\rho^{\phi}(\boldsymbol{x},t) \\ \rho^{\phi\wedge\psi}(\boldsymbol{x},t) &:= \min\left(\rho^{\phi}(\boldsymbol{x},t), \rho^{\psi}(\boldsymbol{x},t)\right) \\ \rho^{F_{[a,b]}\phi}(\boldsymbol{x},t) &:= \max_{t_1 \in [t+a,t+b]} \rho^{\phi}(\boldsymbol{x},t_1) \\ \rho^{G_{[a,b]}\phi}(\boldsymbol{x},t) &:= \min_{t_1 \in [t+a,t+b]} \rho^{\phi}(\boldsymbol{x},t_1) \end{split}$$

The definitions of $\rho^{\phi \lor \psi}(\boldsymbol{x},t)$ and $\rho^{\phi U_{[a,b]}\psi}$ are omitted since they will not be considered in the remainder. We abuse the notation as $\rho^{\phi}(\boldsymbol{x}(t)) := \rho^{\phi}(\boldsymbol{x},t)$ if t is not explicitly contained in $\rho^{\phi}(\boldsymbol{x},t)$. For instance, $\rho^{\mu}(\boldsymbol{x}(t)) := \rho^{\mu}(\boldsymbol{x},t) :=$ $h(\boldsymbol{x}(t))$ since $h(\boldsymbol{x}(t))$ does not contain t as an explicit parameter. However, t is explicitly contained in $\rho^{\phi}(\boldsymbol{x},t)$ if and only if temporal operators (eventually, always, or until) are used.

B. A Bottom-up Approach for Multi-Agent Systems

Consider a multi-agent system that consists of M agents and where each agent is possibly affecting the behavior of another agent. Therefore, communication among agents is crucial. We model the communication by using a static, i.e., time-independent, and undirected graph $\mathcal{G} := (\mathcal{V}, \mathcal{E})$ [2]. The vertex set is $\mathcal{V} := \{v_1, v_2, \ldots, v_M\}$, while the edge set is $\mathcal{E} \in$ $\mathcal{V} \times \mathcal{V}$. Two agents $v_i, v_j \in \mathcal{V}$ can communicate if and only if there exists a path between v_i and v_j . A path is a sequence $v_i, v_{k_1}, \ldots, v_{k_P}, v_j$ such that $(v_i, v_{k_1}), \ldots, (v_{k_P}, v_j) \in \mathcal{E}$. As a consequence, all agents can communicate if and only if \mathcal{G} is connected.

Let $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^{m_i}$, and $w_i \in \mathcal{W}_i$ be the state, input, and additive noise of agent v_i 's dynamics with $\mathcal{W}_i \subset \mathbb{R}^n$ being a bounded set. Let $x := \begin{bmatrix} x_1 & x_2 & \dots & x_M \end{bmatrix}$ be the stacked vector of all agents' states. Each agent v_i obeys the nonlinear and coupled dynamics

$$\dot{\boldsymbol{x}}_i = f_i(\boldsymbol{x}_i) + f_i^{\rm c}(\boldsymbol{x}) + g_i(\boldsymbol{x}_i)\boldsymbol{u}_i + \boldsymbol{w}_i, \qquad (2)$$

where $f_i^c(\boldsymbol{x})$ is a term describing preassumed dynamic couplings of the multi-agent system. Also define $\boldsymbol{x}_i^{\text{ext}} := [\boldsymbol{x}_{j_1} \dots \boldsymbol{x}_{j_{M-1}}]$ such that $v_{j_1}, \dots, v_{j_{M-1}} \in \mathcal{V} \setminus \{v_i\}$, i.e., $\boldsymbol{x}_i^{\text{ext}}$ is a stacked vector containing the states of all agents except of \boldsymbol{x}_i . Note that $\boldsymbol{x}_i^{\text{ext}}$ is contained in $f_i^c(\boldsymbol{x})$ and can be seen as an external input generated by an exo-system, i.e., other agents. The functions f_i , f_i^c , and g_i need to satisfy Assumption 1.

Assumption 1: The functions $f_i : \mathbb{R}^n \to \mathbb{R}^n$, $f_i^c : \mathbb{R}^{nM} \to \mathbb{R}^n$, and $g_i : \mathbb{R}^n \to \mathbb{R}^{n \times m_i}$ are locally Lipschitz continuous, and $g_i(\boldsymbol{x}_i)g_i(\boldsymbol{x}_i^T)$ is positive definite for all $\boldsymbol{x}_i \in \mathbb{R}^n$.

Remark 1: The term $f_i^c(x)$ represents preassumed dynamic couplings that the multi-agent system is subject to. These couplings can, for instance, express consensus, formation control, connectivity maintenance, or obstacle avoidance objectives.

We now tailor the definitions of STL and its robust semantics to multi-agent systems. In our bottom-up approach, each agent $v_i \in \mathcal{V}$ is subject to a local STL formula. As a notational rule, the local formula of agent v_i is endowed with the subscript, i.e., ϕ_i . Based on [6, Definition 3], local satisfaction of ϕ_i by the signal $x_{\phi_i} : \mathbb{R}_{\geq 0} \to \mathbb{R}^{p_i}$ is defined in Definition 3. We will be more specific regarding x_{ϕ_i} and p_i after Definition 5.

Definition 3 (Local Satisfaction): The signal $x_{\phi_i} : \mathbb{R}_{\geq 0}$

 $\rightarrow \mathbb{R}^{p_i}$ locally satisfies ϕ_i if and only if $(\boldsymbol{x}_{\phi_i}, 0) \models \phi_i$.

Local feasibility of ϕ_i is next defined in Definition 4.

Definition 4 (Local Feasibility): The formula ϕ_i is locally feasible if and only if $\exists x_{\phi_i} : \mathbb{R}_{\geq 0} \to \mathbb{R}^{p_i}$ such that x_{ϕ_i} locally satisfies ϕ_i .

Each local formula ϕ_i depends on agent v_i and may also depend on some other agents $v_j \in \mathcal{V}$. Consider $x_j : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ to be the solution to (2) associated with agent v_j .

Definition 5 (Formula-Agent Dependency): If $x_j(t)$ is not contained in $x_{\phi_i}(t)$ for all $t \in \mathbb{R}_{\geq 0}$ and local satisfaction of ϕ_i , i.e., $(x_{\phi_i}, 0) \models \phi_i$, can be evaluated, then ϕ_i does not depend on v_j . Otherwise, i.e., knowledge of $x_j(t)$ is needed and hence $x_j(t)$ is contained in $x_{\phi_i}(t)$, then ϕ_i does depend on v_j and we say that agent v_j is participating in ϕ_i .

The set of participating agents in ϕ_i is

$$\mathcal{V}_{\phi_i} := \{ v_{j_1}, v_{j_2}, \dots, v_{j_{P(\phi_i)}} \} \subseteq \mathcal{V},$$

where $P(\phi_i) := \sum_{j=1}^{|\mathcal{V}|} \chi_j(\phi_i)$ is a function evaluating the total number of participating agents in ϕ_i with

$$\chi_j(\phi_i) := \begin{cases} 1 & \text{if } \phi_i \text{ depends on } v_j \\ 0 & \text{otherwise.} \end{cases}$$

It holds that each $v_j \in \mathcal{V}_{\phi_i}$ is participating in ϕ_i and $\nexists v_k \in \mathcal{V} \setminus \mathcal{V}_{\phi_i}$ such that v_k is participating in ϕ_i . Define

$$\boldsymbol{x}_{\phi_i}(t) := \begin{bmatrix} \boldsymbol{x}_{j_1}(t) & \dots & \boldsymbol{x}_{j_{P(\phi_i)}}(t) \end{bmatrix}$$

for all $t \in \mathbb{R}_{\geq 0}$ with $v_{j_1}, \ldots, v_{j_{P(\phi_i)}} \in \mathcal{V}_{\phi_i}$, i.e., all agents participating in ϕ_i . Finally, for the signal $\boldsymbol{x}_{\phi_i} : \mathbb{R}_{\geq 0} \to \mathbb{R}^{p_i}$ we conclude that $p_i := nP(\phi_i)$ so that \boldsymbol{x}_{ϕ_i} is completely defined.

We call ϕ_i a non-collaborative formula if and only if $P(\phi_i) = 1$. In other words, the satisfaction of ϕ_i does not depend on other agents $v_j \in \mathcal{V} \setminus \{v_i\}$, and hence $\boldsymbol{x}_{\phi_i} = \boldsymbol{x}_i$. Otherwise, i.e., if $P(\phi_i) > 1$, we call ϕ_i a collaborative formula. Since ϕ_i always depends on v_i , it always holds that $P(\phi_i) \geq 1$. Global satisfaction of the set of formulas $\{\phi_1, \ldots, \phi_M\}$ by the signal $\boldsymbol{x} : \mathbb{R}_{\geq 0} \to \mathbb{R}^{nM}$ is introduced in Definition 6. Note that \boldsymbol{x}_{ϕ_i} is naturally contained in \boldsymbol{x} .

Definition 6 (Global Satisfaction): The signal $\boldsymbol{x} : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^{nM}$ globally satisfies $\{\phi_1, \ldots, \phi_M\}$ if and only if \boldsymbol{x}_{ϕ_i} locally satisfies ϕ_i for all agents $v_i \in \mathcal{V}$.

In this respect, we similarly define global feasibility.

Definition 7 (Global Feasibility): The set of formulas $\{\phi_1, \ldots, \phi_M\}$ is globally feasible if and only if $\exists x : \mathbb{R}_{\geq 0} \to \mathbb{R}^{nM}$ such that x globally satisfies $\{\phi_1, \ldots, \phi_M\}$.

Next, maximal dependency clusters are introduced in a similar vein as in [5, Definition 4].

Definition 8 (Maximal Dependency Cluster): Consider the undirected dependency graph $\mathcal{G}_d := (\mathcal{V}, \mathcal{E}_d)$ where there is an edge $(v_i, v_j) \in \mathcal{E}_d \subseteq \mathcal{V} \times \mathcal{V}$ if and only if the formula ϕ_i depends on v_j in the sense of Definition 5; $\Xi \subseteq \mathcal{V}$ is a maximal dependency cluster if and only if $\forall v_i, v_j \in \Xi$ there is a path from v_i to v_j in \mathcal{G}_d and $\nexists v_i \in \Xi, v_k \in \mathcal{V} \setminus \Xi$ such that there is a path from v_i to v_k .

Consequently, a multi-agent system under $\{\phi_1, \ldots, \phi_M\}$ induces $L \leq M$ maximal dependency clusters denoted by $\overline{\Xi} := \{\Xi_1, \ldots, \Xi_L\}$. These clusters are maximal in the sense that there are no formula-agent dependencies between clusters, i.e., $\nexists v_i \in \Xi_{l_1}, v_j \in \Xi_{l_2}$ with $l_1 \neq l_2$ such that ϕ_i depends on v_j , which is different to [5, Definition 4]. Even though maximal dependency clusters have no formula-agent dependencies, dynamic couplings between clusters induced by $f_i^c(\boldsymbol{x})$ may be present.

Example 1: Consider three agents v_1 , v_2 , and v_3 with $\phi_1 := F_{[a_1,b_1]}(||\boldsymbol{x}_1 - \boldsymbol{x}_2|| \le 1)$, $\phi_2 := F_{[a_2,b_2]}(||\boldsymbol{x}_2|| \le 1)$, and $\phi_3 := F_{[a_3,b_3]}(||\boldsymbol{x}_3|| \le 1)$. Then $\overline{\Xi} := \{\Xi_1, \Xi_2\}$ with $\Xi_1 = \{v_1, v_2\}$ and $\Xi_2 = \{v_3\}$.

C. Hybrid Systems

Hybrid systems have recently been modeled and analyzed in [11] by considering hybrid inclusions, i.e., differential and difference inclusions to account for continuous and discrete dynamics. The advantage of this framework is that clocks and logical variables can be included into the system description. Hybrid systems with external inputs as in Definition 9 have explicitly been presented in [14]. Note that the value of the state z_i after a jump is denoted by \hat{z}_i . This is not a standard convention, but will ease the reading in the upcoming sections.

Definition 9: [14] A hybrid system is a tuple $\mathcal{H}_i := (C_i, F_i, D_i, G_i)$ where C_i, D_i, F_i , and G_i are the flow and jump set and the possibly set-valued flow and jump map, respectively. The continuous and discrete dynamics are

$$\begin{cases} \dot{\boldsymbol{z}}_i \in F_i(\boldsymbol{z}_i, \boldsymbol{u}_i^{\text{int}}, \boldsymbol{u}_i^{\text{ext}}) & \text{for} \quad (\boldsymbol{z}_i, \boldsymbol{u}_i^{\text{int}}, \boldsymbol{u}_i^{\text{ext}}) \in C_i \\ \dot{\boldsymbol{z}}_i \in G_i(\boldsymbol{z}_i, \boldsymbol{u}_i^{\text{int}}, \boldsymbol{u}_i^{\text{ext}}) & \text{for} \quad (\boldsymbol{z}_i, \boldsymbol{u}_i^{\text{int}}, \boldsymbol{u}_i^{\text{ext}}) \in D_i, \end{cases}$$
(3)

where $\boldsymbol{z}_i \in \mathcal{Z}_i$ is a hybrid state with domain \mathcal{Z}_i , while $\boldsymbol{u}_i^{\text{int}} \in \mathcal{U}_i^{\text{int}}$ and $\boldsymbol{u}_i^{\text{ext}} \in \mathcal{U}_i^{\text{ext}}$ are internal and external inputs with domains $\mathcal{U}_i^{\text{int}}$ and $\mathcal{U}_i^{\text{ext}}$. Furthermore, let $\mathfrak{H}_i := \mathcal{Z}_i \times \mathcal{U}_i^{\text{int}} \times \mathcal{U}_i^{\text{ext}}$.

Solutions to (3) are parametrized by (t, j), where t indicates continuous flow according to $F_i(z_i, u_i^{\text{int}}, u_i^{\text{ext}})$ and j indicates jumps according to $G(z_i, u_i^{\text{int}}, u_i^{\text{ext}})$. Hence, a solution is a function $z_i : \mathbb{R}_{\geq 0} \times \mathbb{N} \to Z_i$ that satisfies (3) with initial condition $z_i(0, 0)$. For a detailed review of the topic, the reader is referred to [11].

III. PROBLEM STATEMENT

In this paper, the following STL fragment is considered:

$$\psi ::= \top \mid \mu \mid \neg \mu \mid \psi_{(1)} \land \psi_{(2)}$$
(4a)

$$\phi ::= \underset{K}{G_{[a,b]}\psi} \mid F_{[a,b]}\psi \tag{4b}$$

$$\theta^{\mathbf{s}_1} ::= \bigwedge_{k=1} \phi_{(k)} \text{ with } b_{(k)} \le a_{(k+1)} \tag{4c}$$

where μ is a predicate and $\psi_{(1)}, \psi_{(2)}, \ldots$ are formulas of class ψ given in (4a), whereas $\phi_{(k)}$ with $k \in \{1, \ldots, K\}$ are formulas of class ϕ given in (4b) with corresponding time intervals $[a_{(k)}, b_{(k)}]$. Note the use of brackets, e.g. $\psi_{(1)}$, to distinguish from local formulas, e.g., ψ_1 . In this paper, for conjunctions of non-temporal formulas of class ψ given in (4a), e.g., $\psi := \psi_{(1)} \wedge \psi_{(2)}$, we approximate the robust semantics, e.g., $\rho^{\psi_{(1)} \wedge \psi_{(2)}}(\boldsymbol{x})$, by a smooth function.

Assumption 2: The robust semantics for a conjunction of *q* non-temporal formulas of class ψ given in (4a), i.e., $\rho^{\psi_{(1)} \wedge \ldots \wedge \psi_{(q)}}(\mathbf{x})$, are approximated by a smooth function as

$$\rho^{\psi_{(1)}\wedge\ldots\wedge\psi_{(q)}}(\boldsymbol{x})\approx-\ln\Big(\sum_{\substack{i=1\\i\neq b}(\cdot)}^{q}\exp\big(-\rho^{\psi_{(i)}}(\boldsymbol{x})\big)\Big).$$

From now on, when writing $\rho^{\psi}(\boldsymbol{x})$, $\rho^{\phi}(\boldsymbol{x},t)$, or $\rho^{\theta}(\boldsymbol{x},t)$ for formulas of class ψ , ϕ , and θ , respectively, we mean the robust semantics including the smooth approximation in Assumption 2 unless stated otherwise. This approximation is an under-approximation and preserves the property $(\boldsymbol{x}, 0) \models$ ψ if $\rho^{\psi}(\boldsymbol{x}) > 0$ as in [7].

The objective in this paper is to consider local formulas of class ϕ given in (4b) that are independently distributed to each agent $v_i \in \mathcal{V}$. The proposed solution can then be extended to local formulas of class θ given (4e) in the same vein as in [7]. In [7], a continuous feedback control law for a single agent subject to ϕ has been derived, however not considering possible multi-agent couplings as given by $f_i^c(\boldsymbol{x})$ or formula-agent dependencies. Assume hence that each agent $v_i \in \mathcal{V}$ is subject to a local formula ϕ_i of the form (4b). Two more assumption are needed.

Assumption 3: Each formula of class ψ given in (4a) that is contained in (4b) and associated with an agent v_i is: 1) s.t. $\rho^{\psi_i}(\boldsymbol{x}_{\phi_i})$ is concave and 2) well-posed in the sense that $(\boldsymbol{x}_{\phi_i}, 0) \models \psi_i$ implies $\|\boldsymbol{x}_{\phi_i}(0)\| \leq C < \infty$ for some $C \geq 0$. *Remark 2:* Part 2) of Assumption 3 is not restrictive in

practice since $\psi_i^{\text{Ass.3}} := (||\boldsymbol{x}_{\phi_i}|| \leq C)$, where *C* is a sufficiently large positive constant, can be combined with the desired ψ_i so that $\psi_i \wedge \psi_i^{\text{Ass.3}}$ is well-posed.

Next, define the global optimum of $ho^{\psi_i}(m{x}_{\phi_i})$ as

$$\rho_i^{\text{opt}} := \sup_{\boldsymbol{x}_{\phi_i} \in \mathbb{R}^{nP(\phi_i)}} \rho^{\psi_i}(\boldsymbol{x}_{\phi_i})$$

which is straightforward to compute due to Assumption 2 and 3. Next, Assumption 4 guarantees that ϕ_i is locally feasible since $\rho_i^{\text{opt}} > 0$ implies that $\rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0) > 0$ is possible.

Assumption 4: The optimum of $\rho^{\psi_i}(\boldsymbol{x}_{\phi_i})$ is s.t. $\rho_i^{\text{opt}} > 0$.

The goal is to derive a local control law $u_i(\boldsymbol{x}_{\phi_i}, t)$ for each agent v_i such that $r_i \leq \rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0) \leq \rho_i^{\max}$, where $r_i \in \mathbb{R}$ is a robustness measure, while $\rho_i^{\max} \in \mathbb{R}$ with $r_i < \rho_i^{\max}$ is a robustness delimiter. For this purpose, we look at each dependency cluster separately and distinguish between two cases that are described in the formal problem definition.

Problem 1: Assume that each agent v_i is subject to a local STL formula ϕ_i of the form (4b), hence inducing the maximal dependency clusters $\overline{\Xi} := \{\Xi_1, \ldots, \Xi_L\}$ with $L \leq M$. For each cluster Ξ_l with $l \in \{1, \ldots, L\}$, derive a control strategy as follows:

- Case A) Under the assumption that each agent v_i, v_j ∈ Ξ_l is subject to the same formula, i.e., φ_i = φ_j, design a local feedback control law u_i(x_{φ_i}, t) such that 0 < r_i ≤ ρ^{φ_i}(x_{φ_i}, 0) ≤ ρ_i^{max} for all v_i ∈ Ξ_l, which means local satisfaction of φ_i.
- Case B) Otherwise, i.e., ∃v_i, v_j ∈ Ξ_l such that φ_i ≠ φ_j, each agent v_i ∈ Ξ_l nevertheless initially applies the derived control law u_i(x_{φ_i}, t) for Case A. Design a local online detection & repair scheme for each agent v_i ∈ Ξ_l such that r_i ≤ ρ^{φ_i}(x_{φ_i}, 0) ≤ ρ_i^{max}, where r_i ∈ ℝ, possibly negative, is maximized up to a precision of δ_i > 0 with δ_i being a design parameter

If each cluster satisfies the assumption in Case A, i.e., for each $l \in \{1, ..., L\}$ it holds that $\phi_i = \phi_j$ for all agents $v_i, v_j \in \Xi_l$, the proposed solution guarantees global satisfaction of $\{\phi_1, ..., \phi_M\}$. If one or more cluster fails to satisfy this assumption, the online detection & repair scheme in Case B will apply for all agents in these clusters.

IV. PROPOSED PROBLEM SOLUTION

Two types of inter-agent dependencies have been introduced in Section II: dynamic couplings induced by $f_i^c(x)$ and formula-agent dependencies. In the proposed solution to Case A in Problem 1, presented in Section IV-B, it turns out that formula-agent dependencies do not pose any difficulties. Similarly, dynamic couplings only increase the control effort, i.e., $||u_i(t)||$. For Case B, however, both types of dependencies may lead to trajectories that do not locally satisfy the formulas. The proposed solution, introducing an online detection & repair scheme, is presented in Section IV-C.

A. A Prescribed Performance Approach

We first present the main idea of our work on singleagent systems [7], which is based on prescribed performance control [10] and now extended to multi-agent systems. For a thorough illustration, the reader is referred to [7]. Define the performance function γ_i for agent v_i in Definition 10 and the transformation function S in Definition 11.

Definition 10: The performance function $\gamma_i : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{>0}$ is continuously differentiable, bounded, positive, nonincreasing, and given by $\gamma_i(t) := (\gamma_i^0 - \gamma_i^\infty) \exp(-l_i t) + \gamma_i^\infty$ where $\gamma_i^0, \gamma_i^\infty \in \mathbb{R}_{>0}$ with $\gamma_i^0 \geq \gamma_i^\infty$ and $l_i \in \mathbb{R}_{>0}$.

Definition 11: A transformation function $S : (-1,0) \rightarrow \mathbb{R}$ is a strictly increasing function, hence injective and admitting an inverse. In particular, let $S(\xi) := \ln\left(-\frac{\xi+1}{\xi}\right)$.

The objective is to synthesize a local feedback control law $u_i(\boldsymbol{x}_{\phi_i}, t)$ for formulas ϕ_i of the form (4b) such that $r_i \leq \rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0) \leq \rho_i^{\max}$. Let ψ_i correspond to ϕ_i as in $\phi_i := G_{[a_i,b_i]}\psi_i$ or $\phi_i := F_{[a_i,b_i]}\psi_i$ and note that $\boldsymbol{x}_{\psi_i} = \boldsymbol{x}_{\phi_i}$ holds by definition. We achieve $r_i \leq \rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0) \leq \rho_i^{\max}$ by prescribing a temporal behavior to $\rho^{\psi_i}(\boldsymbol{x}_{\phi_i}(t))$ through the design parameters γ_i and ρ_i^{\max} as

$$-\gamma_i(t) + \rho_i^{\max} < \rho^{\psi_i}(\boldsymbol{x}_{\phi_i}(t)) < \rho_i^{\max}.$$
 (5)

Note the use of $\rho^{\psi_i}(\boldsymbol{x}_{\phi_i}(t))$ and not $\rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0)$ itself. When \boldsymbol{x}_{ϕ_i} is seen as a state, define the one-dimensional error, the

normalized error, and the transformed error as

$$\begin{split} e_i(\boldsymbol{x}_{\phi_i}) &:= \rho^{\psi_i}(\boldsymbol{x}_{\phi_i}) - \rho_i^{\max} \\ \xi_i(\boldsymbol{x}_{\phi_i}, t) &:= \frac{e_i(\boldsymbol{x}_{\phi_i})}{\gamma_i(t)} \\ \epsilon_i(\boldsymbol{x}_{\phi_i}, t) &:= S\big(\xi_i(\boldsymbol{x}_{\phi_i}, t)\big) = \ln\Big(-\frac{\xi_i(\boldsymbol{x}_{\phi_i}, t) + 1}{\xi_i(\boldsymbol{x}_{\phi_i}, t)}\Big), \end{split}$$

respectively. Now, (5) can be written as $-\gamma_i(t) < e_i(t) < 0$ where $e_i(t) := e_i(\boldsymbol{x}_{\phi_i}(t))$, which can be further written as $-1 < \xi_i(t) < 0$ where $\xi_i(t) := \xi_i(\boldsymbol{x}_{\phi_i}(t), t)$. Applying the transformation function S to $-1 < \xi_i(t) < 0$ gives $-\infty < \epsilon_i(t) < \infty$ with $\epsilon_i(t) := \epsilon_i(\boldsymbol{x}_{\phi_i}(t), t)$. If $\epsilon_i(t)$ is bounded for all $t \ge 0$, then inequality (5) holds. This is a consequence of the fact that S admits an inverse. The connection between $\rho^{\psi_i}(\boldsymbol{x}_{\phi_i}(t))$ and $\rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0)$ is made by the performance function γ_i , which needs to be chosen as explained in detail in [7] to obtain $0 < r_i \le \rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0) \le \rho_i^{\text{max}}$. If Assumption 4 holds, then select the parameters

$$t_i^* \in \begin{cases} a_i & \text{if } \phi_i = G_{[a_i,b_i]}\psi_i \\ [a_i,b_i] & \text{if } \phi_i = F_{[a_i,b_i]}\psi_i, \end{cases}$$
(6)

$$\rho_i^{\max} \in \left(\max\left(0, \rho^{\psi_i}(\boldsymbol{x}_{\phi_i}(0))\right), \rho_i^{\text{opt}}\right) \tag{7}$$

$$r_i \in (0, \rho_i^{\max}) \tag{8}$$

$$\gamma_i^0 \in \begin{cases} (\rho_i^{\max} - \rho^{\psi_i}(\boldsymbol{x}_{\phi_i}(0)), \infty) & \text{if } t_i^* > 0\\ (\rho_i^{\max} - \rho^{\psi_i}(\boldsymbol{x}_{\phi_i}(0)), \rho_i^{\max} - r_i] & \text{otherwise} \end{cases}$$
(9)

$$\gamma_i^{\infty} \in \left(0, \min\left(\gamma_i^0, \rho_i^{\max} - r_i\right)\right] \tag{10}$$

$$l_i \in \begin{cases} \mathbb{R}_{\geq 0} & \text{if } -\gamma_i^0 + \rho_i^{\max} \geq r_i \\ \frac{-\ln\left(\frac{r_i + \gamma_i^\infty - \rho_i^{\max}}{-(\gamma_i^0 - \gamma_i^\infty)}\right)}{t_i^*} & \text{if } -\gamma_i^0 + \rho_i^{\max} < r_i \end{cases}$$
(11)

where it has to hold that $\rho^{\psi_i}(\boldsymbol{x}_{\phi_i}(0)) > r_i$ if $t_i^* = 0$. The intuition here is that by the choice of γ_i it is ensured that $\rho^{\psi_i}(\boldsymbol{x}_{\phi_i}(t)) \geq r_i$ for all $t \geq t_i^*$. By the choice of t_i^* it consequently holds that $\rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0) \geq r_i$, i.e., $(\boldsymbol{x}_{\phi_i}, 0) \models \phi_i$.

B. Global and Local Satisfaction Guarantees

Considering the induced maximal dependency clusters $\overline{\Xi} := \{\Xi_1, \ldots, \Xi_L\}$, Theorem 1 provides a global satisfaction guarantee if all clusters satisfy the assumption of Case A in Problem 1, i.e., for each $l \in \{1, \ldots, L\}$ it holds that $\phi_i = \phi_j$ for all $v_i, v_j \in \Xi_l$.

Theorem 1: Let each agent $v_i \in \mathcal{V}$ be subject to ϕ_i as in (4b), hence inducing the maximal dependency clusters $\overline{\Xi} := \{\Xi_1, \ldots, \Xi_L\}$. Assume that for each $\Xi_l \in \overline{\Xi}$ it holds that: for all $v_i, v_j \in \Xi_l$ we have 1) v_i and v_j can communicate, 2) $\phi_i = \phi_j$, and 3) $t_i^* = t_j^*$, $\rho_i^{\text{max}} = \rho_j^{\text{max}}$, $r_i = r_j$, and $\gamma_i = \gamma_j$ are chosen as in (6)-(11). If for each agent $v_i \in \mathcal{V}$ Assumptions 1-4 hold and each agent v_i applies

$$\boldsymbol{u}_{i}(\boldsymbol{x}_{\phi_{i}},t) := -\epsilon_{i}(\boldsymbol{x}_{\phi_{i}},t)g_{i}(\boldsymbol{x}_{i})^{T}\frac{\partial\rho^{\psi_{i}}(\boldsymbol{x}_{\phi_{i}})}{\partial\boldsymbol{x}_{i}}, \qquad (12)$$

then it holds that $0 < r_i \le \rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0) \le \rho_i^{\max}$ for all agents $v_i \in \mathcal{V}$, i.e., each agent v_i locally satisfies ϕ_i , which in turn guarantees global satisfaction of $\{\phi_1, \ldots, \phi_M\}$. All closed-

loop signals are well-posed, i.e., continuous and bounded.

Proof: In a first step (Step A), we apply Lemma 1 and show that there exists a maximal solution $\xi_i(t)$ such that $\xi_i(t) := \xi_i(\boldsymbol{x}_{\phi_i}(t), t) \in \Omega_{\xi} := (-1, 0)$, which is the same as requiring that (5) holds for all $t \in \mathcal{J} := [0, \tau_{\max}) \subseteq \mathbb{R}_{\geq 0}$ and all $v_i \in \Xi_l$ with $l \in \{1, \ldots, L\}$. The second step (Step B) consists of using Lemma 2 to show that $\tau_{\max} = \infty$, which proves the main result.

Prior to Step A and B, we state the dynamics of ϵ_i as

$$\frac{d\epsilon_i}{dt} = \frac{\partial\epsilon_i}{\partial\xi_i}\frac{d\xi_i}{dt} = -\frac{1}{\gamma_i\xi_i(1+\xi_i)} \Big(\frac{\partial\rho^{\psi_i}(\boldsymbol{x}_{\phi_i})}{\partial\boldsymbol{x}}^T \dot{\boldsymbol{x}} - \xi_i \dot{\gamma}_i\Big),\tag{13}$$

which can be derived since it holds that $\frac{\partial \epsilon_i}{\partial \xi_i} = -\frac{1}{\xi_i(1+\xi_i)}$ and

$$\frac{d\xi_i}{dt} = \frac{1}{\gamma_i} \left(\frac{de_i}{dt} - \xi_i \dot{\gamma}_i \right). \tag{14}$$

Note that by $\frac{d\epsilon_i}{dt}$, $\frac{d\xi_i}{dt}$, and $\frac{de_i}{dt}$ we here mean the total derivative and that hence $\frac{de_i}{dt} = \frac{\partial e(\boldsymbol{x}_{\phi_i})}{\partial \boldsymbol{x}}^T \dot{\boldsymbol{x}}$ with $\frac{\partial e_i(\boldsymbol{x}_{\phi_i})}{\partial \boldsymbol{x}} = \frac{\partial \rho^{\psi_i}(\boldsymbol{x}_{\phi_i})}{\partial \boldsymbol{x}}$.

Step A: First, define $\boldsymbol{\xi} := (\xi_1, \xi_2, \dots, \xi_M)$ and the stacked vector $\boldsymbol{y} := (\boldsymbol{x}, \boldsymbol{\xi})$. Consider the closed-loop system $\dot{\boldsymbol{x}}_i := H_{\boldsymbol{x}_i}(\boldsymbol{x}, \xi_i)$ of agent v_i with

$$egin{aligned} H_{oldsymbol{x}_i}(oldsymbol{x},\xi_i) &:= f_i(oldsymbol{x}_i) + f_i^{ ext{c}}(oldsymbol{x}) \ &- \ln\Big(-rac{\xi_i+1}{\xi_i}\Big)g_i(oldsymbol{x}_i)g_i^T(oldsymbol{x}_i)rac{\partial
ho^{\psi_i}(oldsymbol{x}_{\phi_i})}{\partialoldsymbol{x}_i} + oldsymbol{w}_i \end{aligned}$$

that is obtained by inserting (12) into (2). The closed-loop system of all agents is then $\dot{x} =: H_x(x, \xi)$ with

$$H_{\boldsymbol{x}}(\boldsymbol{x}, \boldsymbol{\xi}) := \begin{bmatrix} H_{\boldsymbol{x}_1}(\boldsymbol{x}, \xi_1) & \dots & H_{\boldsymbol{x}_M}(\boldsymbol{x}, \xi_M) \end{bmatrix}.$$

Due to (14), we obtain $\frac{d\xi_i}{dt} =: H_{\xi_i}(\boldsymbol{x}, \xi_i, t)$ where

$$H_{\boldsymbol{\xi}_i}(\boldsymbol{x}, \xi_i, t) := rac{1}{\gamma_i(t)} \Big(rac{\partial
ho^{\psi_i}(\boldsymbol{x}_{\phi_i})}{\partial \boldsymbol{x}} H_{\boldsymbol{x}}(\boldsymbol{x}, \boldsymbol{\xi}) - \xi_i \dot{\gamma}_i(t) \Big)$$

expresses the ξ -dynamics of agent v_i . The ξ -dynamics of all agents are given by $\dot{\boldsymbol{\xi}} =: H_{\boldsymbol{\xi}}(\boldsymbol{x}, \boldsymbol{\xi}, t)$ with

$$H_{\boldsymbol{\xi}}(\boldsymbol{x},\boldsymbol{\xi},t) := \begin{bmatrix} H_{\xi_1}(\boldsymbol{x},\xi_1,t) & \dots & H_{\xi_M}(\boldsymbol{x},\xi_M,t) \end{bmatrix}.$$

Using all these definitions, the dynamics of y are finally given by $\dot{y} =: H(y, t)$ with

$$H(\boldsymbol{y},t) := \begin{bmatrix} H_{\boldsymbol{x}}(\boldsymbol{x},\boldsymbol{\xi}) & H_{\boldsymbol{\xi}}(\boldsymbol{x},\boldsymbol{\xi},t) \end{bmatrix}.$$

Note that $\boldsymbol{x}(0)$ is such that $\xi_i(\boldsymbol{x}_{\phi_i}(0), 0) \in \Omega_{\xi} := (-1, 0)$ holds for all agents $v_i \in \Xi_l$ due to the choice of γ_i^0 . Now define the time-varying and non-empty set

$$\begin{split} \Omega_{\phi_i}(t) &:= \Big\{ \boldsymbol{x}_{\phi_i} \in \mathbb{R}^{nP(\phi_i)} \big| \\ &-1 < \xi_i(\boldsymbol{x}_{\phi_i}, t) = \frac{\rho^{\psi_i}(\boldsymbol{x}_{\phi_i}) - \rho_i^{\max}}{\gamma_i(t)} < 0 \Big\}, \end{split}$$

which has the property that for $t_1 < t_2$ we have $\Omega_{\phi_i}(t_2) \subseteq \Omega_{\phi_i}(t_1)$ since γ_i is non-increasing in t. Note that $\Omega_{\phi_i}(t)$ is bounded due to Assumption 3 and since γ_i is bounded.

We remark that $\boldsymbol{x}_{\phi_i}(0) \in \Omega_{\phi_i}(0)$. Due to [15, Proposition 1.4.4], the following holds: if a function is continuous, then the inverse image of an open set under this function is open. By defining $\xi_i^0(\boldsymbol{x}_{\phi_i}) := \xi_i(\boldsymbol{x}_{\phi_i}, 0)$, it holds that $\operatorname{inv}(\xi_i^0(\Omega_{\xi})) = \Omega_{\phi_i}(0)$ is open. Note therefore that $\rho^{\psi_i}(\boldsymbol{x}_{\phi_i})$ is a continuously differentiable function due to Assumption 2. Next, select $v_{i_l} \in \Xi_l$ for each $l \in \{1, \ldots, L\}$ and define

$$\Omega_{\boldsymbol{x}} := \Omega_{\phi_{i_1}}(0) \times \ldots \times \Omega_{\phi_{i_L}}(0) \subset \mathbb{R}^{nM},$$

$$\Omega_{\boldsymbol{\xi}} := \Omega_{\boldsymbol{\xi}} \times \ldots \times \Omega_{\boldsymbol{\xi}} \subset \mathbb{R}^M,$$

and the open, non-empty, and bounded set

$$\Omega_{\boldsymbol{y}} := \Omega_{\boldsymbol{x}} \times \Omega_{\boldsymbol{\xi}} \subset \mathbb{R}^{(n+1)M}$$

where it holds that $\boldsymbol{y}(0) = \begin{bmatrix} \boldsymbol{x}(0) & \boldsymbol{\xi}(0) \end{bmatrix} \in \Omega_{\boldsymbol{y}}$.

Next, we check the conditions in Lemma 1 for the initial value problem $\dot{\boldsymbol{y}} = H(\boldsymbol{y},t)$ with $\boldsymbol{y}(0) \in \Omega_{\boldsymbol{y}}$ and $H(\boldsymbol{y},t) : \Omega_{\boldsymbol{y}} \times \mathbb{R}_{\geq 0} \to \mathbb{R}^{(n+1)M}$: 1) $H(\boldsymbol{y},t)$ is locally Lipschitz continuous on \boldsymbol{y} since $f_i(\boldsymbol{x}_i), f_i^c(\boldsymbol{x}), g_i(\boldsymbol{x}_i)$, and $\epsilon_i = \ln\left(-\frac{\xi_i+1}{\xi_i}\right)$ are locally Lipschitz continuous on \boldsymbol{y} for each $t \in \mathbb{R}_{\geq 0}$. This also holds for $\frac{\partial \rho^{\psi_i}(\boldsymbol{x}_{\phi_i})}{\partial \boldsymbol{x}_i}$ due to Assumption 2. 2) $H(\boldsymbol{y},t)$ is continuous on t for each fixed $\boldsymbol{y} \in \Omega_{\boldsymbol{y}}$ due to continuity of γ_i and $\dot{\gamma}_i$. As a result of Lemma 1, there exists a maximal solution with $\boldsymbol{y}(t) \in \Omega_{\boldsymbol{y}}$ for all $t \in \mathcal{J} := [0, \tau_{\max}) \subseteq \mathbb{R}_{\geq 0}$ and $\tau_{\max} > 0$. Consequently, there exist $\boldsymbol{\xi}(t) \in \Omega_{\boldsymbol{\xi}}$ and $\boldsymbol{x}(t) \in \Omega_{\boldsymbol{x}}$ for all $t \in \mathcal{J}$.

Step B: From Step A) we have $\boldsymbol{y}(t) \in \Omega_{\boldsymbol{y}}$ for all $t \in \mathcal{J} := [0, \tau_{\max})$. Now, we show that $\tau_{\max} = \infty$ by contradiction of Lemma 2. Therefore, assume $\tau_{\max} < \infty$.

The key observation to be made is that $\xi_i(\boldsymbol{x}_{\phi_i},t) = \xi_j(\boldsymbol{x}_{\phi_j},t)$, $\epsilon_i(\boldsymbol{x}_{\phi_i},t) = \epsilon_j(\boldsymbol{x}_{\phi_j},t)$, and $\rho^{\psi_i}(\boldsymbol{x}_{\phi_i}) = \rho^{\psi_j}(\boldsymbol{x}_{\phi_i})$ for all agents $v_i, v_j \in \Xi_l$. This follows since $\boldsymbol{x}_{\phi_i} = \boldsymbol{x}_{\phi_j}$ (recall that $\phi_i = \phi_j$) and since $\rho_i^{\max} = \rho_j^{\max}$ and $\gamma_i = \gamma_j$ holds by assumption. We now show that $\epsilon_i(t)$ is bounded for all $t \in \mathbb{R}_{\geq 0}$ and then it consequently follows that $\epsilon_j(t)$ is bounded for all other agents $v_j \in \Xi_l \setminus \{v_i\}$. Since the clusters are maximal, i.e., no formula-agent dependencies between clusters exist, we can deduce the same result for the other clusters. Consider the Lyapunov function candidate $V(\epsilon_i) := \frac{1}{2}\epsilon_i\epsilon_i$ and define $\dot{V}(\epsilon_i) := \frac{\partial V}{\partial \epsilon_i}\frac{d\epsilon_i}{dt}$. We will now show that $\dot{V}(\epsilon_i) \leq 0$ if $|\epsilon_i|$ is bigger than some positive constant, which ensures that $\epsilon_i(t)$ will remain in a compact set. By using (13), it follows

$$\dot{V}(\epsilon_i) = \epsilon_i \frac{d\epsilon_i}{dt} = \epsilon_i \Big(-\frac{1}{\gamma_i \xi_i (1+\xi_i)} \Big(\frac{\partial \rho^{\psi_i}(\boldsymbol{x}_{\phi_i})}{\partial \boldsymbol{x}}^T \dot{\boldsymbol{x}} - \xi_i \dot{\gamma}_i \Big) \Big)$$

Define $\alpha_i(t) := -\frac{1}{\gamma_i \xi_i(1+\xi_i)}$ which satisfies $\alpha_i(t) \in [\frac{4}{\gamma_i^0}, \infty)$ for all $t \in \mathcal{J}$. This follows since $\frac{4}{\gamma_i^0} \leq -\frac{1}{\gamma_i^0 \xi_i(1+\xi_i)} \leq -\frac{1}{\gamma_i \xi_i(1+\xi_i)} \leq -\frac{1}{\gamma_i^\infty \xi_i(1+\xi_i)} < \infty$ for $\xi_i \in \Omega_{\xi}$. It can further be derived that

$$\dot{V}(\epsilon_i) \le \epsilon_i \alpha_i \frac{\partial \rho^{\psi_i}(\boldsymbol{x}_{\phi_i})}{\partial \boldsymbol{x}}^T H_{\boldsymbol{x}}(\boldsymbol{x}, \boldsymbol{\xi}) + |\epsilon_i| \alpha_i k_i \qquad (15)$$

where $\dot{\boldsymbol{x}} := H_{\boldsymbol{x}}(\boldsymbol{x}, \boldsymbol{\xi})$ as defined previously and $0 \le |\xi_i \dot{\gamma}_i| \le k_i < \infty$ for a positive constant k_i . This follows since $\xi_i(t) \in \Omega_{\boldsymbol{\xi}}$ for all $t \in \mathcal{J}$ and $\dot{\gamma}_i$ is bounded by definition. The

term $\frac{\partial \rho^{\psi_i}(\boldsymbol{x}_{\phi_i})}{\partial \boldsymbol{x}}^T H_{\boldsymbol{x}}(\boldsymbol{x},\boldsymbol{\xi})$ represents the couplings among the agents and can be written as

$$\frac{\partial \rho^{\psi_i}(\boldsymbol{x}_{\phi_i})}{\partial \boldsymbol{x}}^T H_{\boldsymbol{x}}(\boldsymbol{x},\boldsymbol{\xi}) = \sum_{v_j \in \Xi_l} \frac{\partial \rho^{\psi_j}(\boldsymbol{x}_{\phi_j})}{\partial \boldsymbol{x}_j}^T H_{\boldsymbol{x}_j}(\boldsymbol{x},\xi_j).$$
(16)

Plugging (16) into (15) results in

$$\dot{V}(\epsilon_{i}) \leq \epsilon_{i} \alpha_{i} \sum_{v_{j} \in \Xi_{l}} \frac{\partial \rho^{\psi_{j}}(\boldsymbol{x}_{\phi_{j}})}{\partial \boldsymbol{x}_{j}}^{T} H_{\boldsymbol{x}_{j}}(\boldsymbol{x}, \xi_{j}) + |\epsilon_{i}| \alpha_{i} k_{i}.$$
(17)

Inserting (2) and (12) into $\epsilon_i \frac{\partial \rho^{\psi_j}(\boldsymbol{x}_{\phi_j})}{\partial \boldsymbol{x}_j}^T H_{\boldsymbol{x}_j}(\boldsymbol{x},\xi_j)$ first, this term can in a second step be upper bounded as follows

$$egin{aligned} \epsilon_i rac{\partial
ho^{\psi_j}(oldsymbol{x}_{\phi_j})}{\partial oldsymbol{x}_j}^T H_{oldsymbol{x}_j}(oldsymbol{x},\xi_j) &= \epsilon_i rac{\partial
ho^{\psi_j}(oldsymbol{x}_{\phi_j})}{\partial oldsymbol{x}_j}^T \Big(f_j(oldsymbol{x}_j) + f_j^{ extsf{c}}(oldsymbol{x}) \ &- \epsilon_j g_j(oldsymbol{x}_j) g_j^T(oldsymbol{x}_j) rac{\partial
ho^{\psi_j}(oldsymbol{x}_{\phi_j})}{\partial oldsymbol{x}_j} + oldsymbol{w}_j \Big) \ &\leq |\epsilon_i| M_j - |\epsilon_i|^2 \lambda_j J_j \end{aligned}$$

where $\epsilon_i = \epsilon_j$ as remarked previously since $v_i, v_j \in \Xi_l$. Furthermore, $\lambda_j > 0$ is the positive minimum eigenvalue of $g_j(\boldsymbol{x}_j)g_j^T(\boldsymbol{x}_j)$ according to Assumption 1, and $\|\frac{\partial \rho^{\psi_j}(\boldsymbol{x}_{\phi_j})}{\partial \boldsymbol{x}_j}^T(f_j(\boldsymbol{x}_j) + f_j^c(\boldsymbol{x}) + \boldsymbol{w}_j)\| \leq M_j < \infty$ due to continuity of $\frac{\partial \rho^{\psi_j}(\boldsymbol{x}_{\phi_j})}{\partial \boldsymbol{x}_j}$, $f_j(\boldsymbol{x}_j)$, and $f_j^c(\boldsymbol{x})$, the extreme value theorem and the fact that $\Omega_{\boldsymbol{x}}$ and \mathcal{W}_i are bounded. Note therefore that the extreme value theorem guarantees that a continuous function on a compact set is bounded and that the above functions are continuous on $cl(\Omega_{\boldsymbol{x}})$, where cl denotes the closure of a set. The lower bound $J_j \in \mathbb{R}_{\geq 0}$ arises naturally due to the norm operator as $0 \leq J_j \leq \|\frac{\partial \rho^{\psi_j}(\boldsymbol{x}_{\phi_j})}{\partial \boldsymbol{x}_j})\|^2 < \infty$. Equation (17) can now be upper bounded as follows

$$\dot{V}(\epsilon_i) \le \alpha_i |\epsilon_i| \left(\hat{M}_i - |\epsilon_i| \hat{J}_i \right) \tag{18}$$

where $\hat{M}_i := \sum_{v_j \in \Xi_l} M_j + k_i$ and $\hat{J}_i := \sum_{v_j \in \Xi_l} \lambda_j J_j$. Note that $\hat{J}_i > 0$ since $\frac{\partial \rho^{\psi_i}(\boldsymbol{x}_{\phi_i})}{\partial \boldsymbol{x}_{\phi_i}} = 0$ if and only if $\rho^{\psi_i}(\boldsymbol{x}_{\phi_i}) = \rho_i^{\text{opt}}$, which is excluded since (5) holds for all $t \in \mathcal{J}$ and we selected $\rho_i^{\text{max}} < \rho_i^{\text{opt}}$. Recall that $\rho^{\psi_j}(\boldsymbol{x}_{\phi_j})$ in $\|\frac{\partial \rho^{\psi_j}(\boldsymbol{x}_{\phi_j})}{\partial \boldsymbol{x}_j}\|^2$ is concave due to Assumptions 2 and 3. In other words, at least one J_j in $v_j \in \Xi_l$ is greater than zero.

It holds that $\dot{V}(\epsilon_i) \leq 0$ if $\frac{\hat{M}_i}{\hat{J}_i} \leq |\epsilon_i|$. We can conclude that $|\epsilon_i|$ will be upper bounded due to the level sets of V as

$$|\epsilon_i(t)| \le \max\left(|\epsilon_i(0)|, \frac{M_i}{\hat{J}_i}\right)$$

which leads to the conclusion that $\epsilon_i(t)$ is upper and lower bounded by some constants ϵ_i^u and ϵ_i^l , respectively. In other words, it holds that $\epsilon_i^l \leq \epsilon_i(t) \leq \epsilon_i^u$ for all $t \in \mathcal{J}$. By using the inverse of S and defining $\xi_i^l := -\frac{1}{\exp(\epsilon_i^l + 1)}$ and $\xi_i^u := -\frac{1}{\exp(\epsilon_i^l + 1)}$, $\xi_i(t)$ is bounded by $-1 < \xi_i^l \leq \xi_i(t) \leq \xi_i^u < 0$, which translates to

$$\xi_i(t) \in \Omega'_{\xi_i} := [\xi_i^l, \xi_i^u] \subset \Omega_{\xi}$$

for all $t \in \mathcal{J}$. Recall that $\xi_i(\boldsymbol{x}_{\phi_i}, t) = \frac{\rho^{\psi_i}(\boldsymbol{x}_{\phi_i}) - \rho_i^{\max}}{\gamma_i(t)}$ and note the following: if $\xi_i(t)$ evolves in a compact set, then $\rho^{\psi_i}(\boldsymbol{x}_{\phi_i}(t))$ will evolve in a compact set $\Omega'_{\rho^{\psi_i}} := [\rho_i^l, \rho_i^u]$ for some constants ρ_i^l and ρ_i^u . Again, due to [15, Proposition 1.4.4] it holds that the inverse image

$$\Omega_{\phi_i}' := \operatorname{inv}(\rho^{\psi_i}(\Omega_{\rho^{\psi_i}}')) = \{ \boldsymbol{x}_{\phi_i} \in \Omega_{\phi_i}(0) | \rho_i^l \le \rho^{\psi_i}(\boldsymbol{x}_{\phi_i}) \le \rho_i^u \}$$

is closed and also bounded since it is a subset of Ω_{ϕ_i} . Select $v_{i_l} \in \Xi_l$ for each $l \in \{1, \ldots, L\}$. It can be concluded that $\boldsymbol{x}_{\phi_{i_l}}(t)$ evolves in a compact set, i.e., $\boldsymbol{x}_{\phi_{i_l}}(t) \in \Omega'_{\phi_{i_l}} \subset \Omega_{\phi_{i_l}}(0)$ for all $t \in \mathcal{J}$ and all v_{i_l} . Next, define

$$\Omega'_{\boldsymbol{x}} := \Omega'_{\phi_{i_1}} \times \ldots \times \Omega'_{\phi_{i_L}} \subset \mathbb{R}^{nM}$$
$$\Omega_{\boldsymbol{\xi}} := \Omega'_{\boldsymbol{\xi}_*} \times \ldots \times \Omega'_{\boldsymbol{\xi}_{*'}} \subset \mathbb{R}^M,$$

and the compact set

$$\Omega'_{\boldsymbol{y}} := \Omega'_{\boldsymbol{x}} \times \Omega'_{\boldsymbol{\xi}} \subset \mathbb{R}^{(n+1)M}$$

for which it holds that $\boldsymbol{y}(t) \in \Omega'_{\boldsymbol{y}}$ for all $t \in \mathcal{J}$. It is also true that $\Omega'_{\boldsymbol{y}} \subset \Omega_{\boldsymbol{y}}$ by which it follows that there is no $t \in \mathcal{J} := [0, \tau_{\max})$ such that $\boldsymbol{y}(t) \notin \Omega'_{\boldsymbol{y}}$. By contradiction of Lemma 2 it holds that $\tau_{\max} = \infty$, i.e., $\mathcal{J} = \mathbb{R}_{\geq 0}$. This in turn says that (5) holds for all agents $v_i \in \mathcal{V}$ and for all $t \in \mathbb{R}_{\geq 0}$. By the choice of ρ_i^{\max} , r_i , and γ_i as in (7)-(11) and [7, Theorem 2], it then holds that ϕ_i is locally satisfied for each agent $v_i \in \mathcal{V}$.

The control law $u_i(x_i, t)$ is well-posed, i.e., continuous and bounded, because $\rho^{\psi_i}(x_i)$ is approximated by a smooth function, while $\epsilon_i(x_i, t)$ and $g_i(x_i)$ are continuous. Furthermore, γ_i is continuous with $0 < \gamma(t) < \infty$. Due to the extreme value theorem, these functions are also bounded. It follows that all closed-loop signals are well-posed.

If L = M, i.e., each agent $v_i \in \mathcal{V}$ is subject to a noncollaborative formula ϕ_i , Theorem 1 trivially applies since no formula dependencies among agents exist. Recall that dynamic couplings induced by $f_i^c(\boldsymbol{x})$ may still be present.

For the next result, a stronger assumption on the dynamic couplings $f_i^c(x)$ is needed.

Assumption 5: The function $f_i^c : \mathbb{R}^{nM} \to \mathbb{R}^n$ is bounded.

Now consider a formula ϕ of the form (4b) and assume that each $v_i \in \mathcal{V}_{\phi}$ is subject to $\phi_i := \phi$. Then, Theorem 2 guarantees satisfaction of ϕ if all agents $v_i \in \mathcal{V}_{\phi}$ collaborate.

Theorem 2: Let each agent $v_i \in \mathcal{V}$ satisfy Assumption 1 and 5. Consider a formula ϕ as in (4b) and let each agent $v_i \in \mathcal{V}_{\phi}$ be subject to $\phi_i := \phi$. Assume that for all $v_i, v_j \in \mathcal{V}_{\phi}$ it holds that: 1) v_i and v_j can communicate and 2) $t_i^* = t_j^*$, $\rho_i^{\max} = \rho_j^{\max}$, $r_i = r_j$, and $\gamma_i = \gamma_j$ are chosen as in (6)-(11). Assume further that all agents $v_k \in \mathcal{V} \setminus \mathcal{V}_{\phi}$ apply a control law u'_k such that x_k remains in a compact set Ω'_k . If for each agent $v_i \in \mathcal{V}_{\phi}$ Assumptions 2-4 hold and each $v_i \in \mathcal{V}_{\phi}$ applies (12), then it holds that $0 < r := r_i \leq \rho^{\phi}(x_{\phi}, 0) \leq \rho_i^{\max} =: \rho^{\max}$, i.e., $(x_{\phi}, 0) \models \phi$. All closed-loop signals are well-posed. *Proof:* The proof is similar to the proof in Theorem 1 and is provided in the appendix.

The assumption of u'_k is not restrictive and excludes finite escape time. For instance, if Assumption 5 holds and $\dot{\boldsymbol{x}}_k := f_k(\boldsymbol{x}_k)$ is asymptotically stable, then the feedback control law $\boldsymbol{u}_k'(\boldsymbol{x}_k) := -g_k(\boldsymbol{x}_k)^T \boldsymbol{x}_k$ keeps the state \boldsymbol{x}_k in a compact set. If all agents $v_i \in \mathcal{V}_{\phi}$ apply the control law (12) under the conditions in Theorem 2 to satisfy ϕ , we refer to this as *collaborative control* in the remainder. Theorem 2 has further implications with respect to Case A in Problem 1. Consider again the induced maximal dependency clusters $\overline{\Xi} := \{\Xi_1, \ldots, \Xi_L\}$. Assume that the cluster Ξ_l with $l \in \{1, \ldots, L\}$ satisfies the assumption of Case A, while there exists another cluster Ξ_m with $m \neq l$ such that Ξ_m does not satisfy this assumption. In other words, for all $v_i, v_j \in \Xi_l$ it holds that $\phi_i = \phi_j$, while $\exists v_i, v_j \in \Xi_m$ with $m \neq l$ such that $\phi_i \neq \phi_j$. Consequently, Theorem 2 guarantees local satisfaction of ϕ_i for all $v_i \in \Xi_l$ without considering task satisfaction of agents in $\mathcal{V} \setminus \Xi_l$.

Note that Assumption 4 in Theorem 2 restricts the formula ϕ to be locally feasible. However, this assumption can be relaxed at the expense of not locally satisfying $\phi_i := \phi$ and instead finding a, possibly least violating, solution by relaxing r_i and ρ_i^{max} . Recall that $\rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0) \ge r_i$ with $r_i < 0$ does not imply local satisfaction of ϕ_i .

Corollary 1: Assume that all assumptions of Theorem 1 hold for each agent $v_i \in \mathcal{V}$ except for Assumption 4 and the choice of ρ_i^{\max} and r_i . If instead $\rho_i^{\max} \in (\rho^{\psi_i}(\boldsymbol{x}_{\phi_i}(0)), \rho_i^{\text{opt}})$ and $r_i \in (-\infty, \rho_i^{\max})$, then it holds that $r_i \leq \rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0) \leq \rho_i^{\max}$ for all agents $v_i \in \mathcal{V}$.

Proof: Follows the same line of proof as in Theorem 1 and 2. Note that it has already been stated in [7] that $r_i \leq \rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0) \leq \rho_i^{\max}$ follows from (5) by the choice of γ_i . Therefore, it consequently holds that r_i can be chosen negative as long as $r_i < \rho_i^{\max} < \rho_i^{\text{opt}}$.

C. An Online Detection & Repair Scheme

Assume now that the cluster Ξ_l with $l \in \{1, \ldots, L\}$ may not satisfy the assumption of Case A in Problem 1. We propose that each agent $v_i \in \Xi_l$ initially applies the control law (12) with parameters as in (6)-(11). The control law (12) consists of two components, one determining the control strength and one the control direction; $\epsilon_i(\boldsymbol{x}_{\phi_i}, t)$ determines the control strength. The closer $\xi_i(\boldsymbol{x}_{\phi_i}, t)$ gets to $\Omega_{\xi} := \{-1, 0\}$, i.e., the funnel boundary, the bigger gets $\epsilon_i(\boldsymbol{x}_{\phi_i}, t)$ and consequently also $\|\boldsymbol{u}(\boldsymbol{x}_{\phi_i}, t)\|$. Note that $\|\boldsymbol{u}(\boldsymbol{x}_{\phi_i},t)\| \to \infty$ as $\xi_i(\boldsymbol{x}_{\phi_i},t) \to \Omega_{\xi}$. The control direction is determined by $\frac{\partial \rho^{\psi_i}(\boldsymbol{x}_{\phi_i})}{\partial \boldsymbol{x}_i}$, i.e., in which direction control action should mainly happen. In summary, the control law always steers in the direction away from the funnel boundary, and the control effort increases close to the funnel boundary. We reason that applying the control law (12) is hence a good initial choice such that ϕ_i will be locally satisfied if the participating agents $\mathcal{V}_{\phi_i} \setminus \{v_i\}$ behave reasonably. The resulting trajectory x_{ϕ_i} may, however, hit the funnel boundary, i.e., $\xi_i(\boldsymbol{x}_{\phi_i}, t) = \{-1, 0\}$, and lead to critical events.

Example 2: Consider three agents v_1 , v_2 , and v_3 . Agent v_2 is subject to the formula $\phi_2 := F_{[5,15]}(||\mathbf{x}_2 - [90 \ 90]|| \le 5)$, while agent v_3 is subject to $\phi_3 := F_{[5,15]}(||\mathbf{x}_3 - [90 \ 10]|| \le 5)$, i.e., both agents are subject to non-collaborative formulas. Agent v_1 is subject to the collaborative formula $\phi_1 := G_{[0,15]}(||\mathbf{x}_1 - \mathbf{x}_2|| \le 10 \land ||\mathbf{x}_1 - \mathbf{x}_3|| \le 10)$. Note that the set of formulas $\{\phi_1, \phi_2, \phi_3\}$ is not globally feasible, although each formula itself is locally feasible. Under (12), agents v_2 and v_3 move to $[90 \ 90]$ and $[90 \ 10]$, respectively. Agent v_1 can consequently not satisfy ϕ_1 and only decrease the robustness such that $r_i < 0$ to achieve $r_i \le \rho^{\phi_i}(\mathbf{x}_{\phi_i}, 0) \le 0$ similar to Corollary 1.

In Example 2, the set of local formulas is globally infeasible. However, even if the set $\{\phi_1, \ldots, \phi_M\}$ is globally feasible, there are reasons why the resulting trajectory may not globally satisfy $\{\phi_1, \ldots, \phi_M\}$ as illustrated next.

Example 3: Consider two agents v_4 and v_5 with $\phi_4 := F_{[5,10]}(||\mathbf{x}_4 - \mathbf{x}_5|| \le 10 \land ||\mathbf{x}_4 - [50 \quad 70] || \le 10)$ (collaborative formula) and $\phi_5 := F_{[5,15]}(||\mathbf{x}_5 - [10 \quad 10] || \le 5)$ (non-collaborative formula). Under (12), agent v_5 moves to $[10 \quad 10]$ by at latest 15 time units. However, agent v_4 is forced to move to $[50 \quad 70]$ and be close to agent v_5 by at latest 10 time units. This may lead to critical events where (5) is violated for agent v_4 . If agent v_5 cooperates, it can first help to locally satisfy ϕ_4 , e.g., by using *collaborative control* as in Theorem 2, and locally satisfy ϕ_5 afterwards.

To overcome these potential problems, we propose an online detection & repair scheme by using a local hybrid system $\mathcal{H}_i := (C_i, F_i, D_i, G_i)$ for each agent $v_i \in \Xi_l$. We detect critical events that may lead to trajectories that do not locally satisfy ϕ_i . Then, agent v_i tries to locally repair the funnel, i.e., the design parameters t_i^* , ρ_i^{max} , r_i , and γ_i , in a first stage. If this is not successful, collaborative control as in Theorem 2 will be considered in a second stage (Example 3). If collaborative control is not applicable, r_i is successively decreased by $\delta_i > 0$ in the third stage (Example 2), where δ_i is a design parameter. The jump set D_i will detect critical events, while the jump map G_i will take repair actions.

Let $p_i^{\gamma} := \begin{bmatrix} \gamma_i^0 & \gamma_i^{\infty} & l_i \end{bmatrix}$ and $p_i^{\text{f}} := \begin{bmatrix} t_i^* & \rho_i^{\max} & r_i & p_i^{\gamma} \end{bmatrix}$ contain the parameters that define (5), and let $p_i^{\text{r}} := \begin{bmatrix} \mathbf{n}_i & \mathbf{c}_i \end{bmatrix}$; \mathbf{n}_i indicates the number of repair attempts in the first repair stage, while \mathbf{c}_i is used in the second repair stage (**c** for collaborative). If $\mathbf{c}_i \in \{1, \ldots, M\}$, collaborative control as in Theorem 2 is used to collaboratively satisfy $\phi_{\mathbf{c}_i}$. If $\mathbf{c}_i = 0$, then agent v_i tries to locally satisfy ϕ_i by itself and if $\mathbf{c}_i = -1$, then agent v_i is free, i.e., not subject to a task. We define the hybrid state as $\mathbf{z}_i := \begin{bmatrix} \mathbf{x}_i & t_i & \mathbf{p}_i^f & \mathbf{p}_i^r \end{bmatrix} \in \mathcal{Z}_i$, where t_i is a clock, $\mathcal{Z}_i := \mathbb{R}^n \times \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0}^6 \times \mathbb{Z}^2$ and $\mathbf{z}_i(0,0) := \begin{bmatrix} \mathbf{x}_i(0) & 0 & \mathbf{p}_i^f(0) & \mathbf{0}_2 \end{bmatrix}$ with \mathbb{Z} being the set of integers. The elements in $\mathbf{p}_i^f(0)$ are as chosen according to (6)-(11). Additionally, we choose $\mathbf{p}_i^f(0) = \mathbf{p}_j^f(0)$ if Case A holds for all agents $v_i, v_j \in \Xi_l$. Next, define

$$\boldsymbol{u}_{i}^{\text{int}} = \begin{cases} \boldsymbol{0}_{m_{i}} & \text{if } \boldsymbol{\mathfrak{c}}_{i} = -1 \\ -\epsilon_{i}(\boldsymbol{x}_{\phi_{i}}, t_{i})g_{i}(\boldsymbol{x}_{i})^{T}\frac{\partial\rho^{\psi_{i}}(\boldsymbol{x}_{\phi_{i}})}{\partial\boldsymbol{x}_{i}} & \text{if } \boldsymbol{\mathfrak{c}}_{i} = 0 \\ -\epsilon_{\mathfrak{c}_{i}}(\boldsymbol{x}_{\phi_{\mathfrak{c}_{i}}}, t_{i})g_{i}(\boldsymbol{x}_{i})^{T}\frac{\partial\rho^{\psi_{\mathfrak{c}_{i}}}(\boldsymbol{x}_{\phi_{\mathfrak{c}_{i}}})}{\partial\boldsymbol{x}_{i}} & \text{if } \boldsymbol{\mathfrak{c}}_{i} > 0 \end{cases}$$

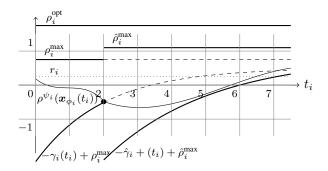


Fig. 1: Funnel repair in the first stage for $\phi_i := F_{[4,6]}\psi_i$.

so that the flow map can be written as

$$\begin{aligned} F_i(\boldsymbol{x}_i, \boldsymbol{u}_i^{\text{int}}, \boldsymbol{u}_i^{\text{ext}}) &:= \\ \begin{bmatrix} f_i(\boldsymbol{x}_i) + f_i^{\text{c}}(\boldsymbol{x}) + g_i(\boldsymbol{x}_i) \boldsymbol{u}_i^{\text{int}} + \boldsymbol{w}_i & 1 & \boldsymbol{0}_6 & \boldsymbol{0}_2 \end{bmatrix}. \end{aligned}$$

External inputs are w_i and x_i^{ext} . By assuming $v_i \in \Xi_l$, we define $\mathbf{c}_i^{\text{ext}} := [\mathbf{c}_{j_1} \dots \mathbf{c}_{j_{|\Xi_l|-1}}]$ and $p_i^{\text{f,ext}} := [p_{j_1}^f \dots p_{j_{|\Xi_l|-1}}^f]$ such that $v_{j_1}, \dots, v_{j_{|\Xi_l|-1}} \in \Xi_l \setminus \{v_i\}$. Note that $\mathbf{c}_i^{\text{ext}}$ and $p_i^{\text{f,ext}}$ contain states of all agents in the same dependency cluster Ξ_l . Ultimately, define the external input as $u_i^{\text{ext}} := [w_i \ x_i^{\text{ext}} \ \mathbf{c}_i^{\text{ext}} \ p_i^{\text{f,ext}}]$.

The set \mathcal{D}'_i is used to *detect* a critical event when the funnel in (5) is violated, i.e., when $\xi_i(t_i) \notin \Omega_{\xi} := (-1, 0)$.

$$\mathcal{D}'_i := \{ (\boldsymbol{z}_i, \boldsymbol{u}_i^{\text{int}}, \boldsymbol{u}_i^{\text{ext}}) \in \mathfrak{H}_i | \xi_i(t_i) \in \{-1, 0\}, \ \mathfrak{c}_i = 0 \}.$$

Remark 3: Note that $\xi_i(t_i) \in \{-1, 0\}$ implies $\epsilon_i(t_i) \to \infty$ and therefore $u_i(t_i) \to \infty$. In practice, the input will be saturated at some point.

Throughout the paper, we assume that agent v_i detects the critical event, while the agents with subscript j as $v_j \in \mathcal{V}_{\phi_i} \setminus \{v_i\}$ are asked to help agent v_i . Detection of a critical event by \mathcal{D}'_i does not necessarily mean that it is not possible to locally satisfy ϕ_i anymore. It rather means that the user-defined funnel boundary is touched and that repairs can help satisfying ϕ_i . We introduce the notation $\{\hat{z}_i \in \mathcal{Z}_i | \hat{z}_i = z_i ; exception\}$ denoting the set of $\hat{z}_i \in \mathcal{Z}_i$ such that $\hat{z}_i = z_i$ after the jump except for the elements in \hat{z}_i explicitly mentioned after the semicolon, here denoted by the placeholder exception.

1) Repair of Critical Events - Stage 1: The first repair stage is indicated by

$$\mathcal{D}_{i,1}' := \mathcal{D}_i' \cap \{(\boldsymbol{z}_i, \boldsymbol{u}_i^{\mathrm{int}}, \boldsymbol{u}_i^{\mathrm{ext}}) \in \mathfrak{H}_i | \mathfrak{n}_i < N_i\}$$

where $N_i \in \mathbb{N}$ is a design parameter representing the maximum number of repair attempts in the first stage. If $(\boldsymbol{z}_i, \boldsymbol{u}_i^{\text{int}}, \boldsymbol{u}_i^{\text{ext}}) \in \mathcal{D}'_{i,1}$, we first relax the parameters $t_i^*, \rho_i^{\text{max}}, r_i$, and γ_i in a way that still guarantees local satisfaction of ϕ_i . Pictorially speaking, we make the funnel in (5) bigger.

Example 4: Consider the formula $\phi_i := F_{[4,6]}\psi_i$ with $r_i := 0.25$ as the desired initial robustness, which is supposed to be achieved at $t_i^* \approx 4.8$. The original funnel is shown in Fig. 1 and given by ρ_i^{\max} and $-\gamma_i + \rho_i^{\max}$ as in (5). Without detection of a critical event, it would

hence hold that $\rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0) \geq r_i$ since $\rho^{\psi_i}(\boldsymbol{x}_{\phi_i}(t_i^*)) \geq r_i$ would be achieved. However, at $t_r := 2$, where t_r indicates the time where a critical event is detected, the trajectory $\rho^{\psi_i}(\boldsymbol{x}_{\phi_i}(t))$ touches the lower funnel boundary and repair action is needed. This is done by setting $\hat{t}_i^* := 6$ (time relaxation), $\hat{r}_i := 0.0001$ (robustness relaxation), $\hat{\rho}_i^{\max} :=$ 1.1 (upper funnel relaxation), and also adjusting $\hat{\gamma}_i$ (lower funnel relaxation). The funnel is hence relaxed to $\hat{\rho}_i^{\max}$ and $-\hat{\gamma}_i + \hat{\rho}_i^{\max}$ as depicted in Fig. 1. At the time of critical event detection t_r , the lower funnel is relaxed to $-\hat{\gamma}_i(t_r) + \hat{\rho}_i^{\max}$ where we especially denote $\gamma_i^r := \hat{\gamma}_i(t_r)$. Due to repair action, \boldsymbol{x}_{ϕ_i} locally satisfies ϕ_i as shown in Fig. 1.

With Example 4 in mind, set

$$\begin{split} \mathcal{G}_{i,1}' &:= \Big\{ \hat{\boldsymbol{z}}_i \in \mathcal{Z}_i | \hat{\boldsymbol{z}}_i = \boldsymbol{z}_i \; ; \; \hat{t}_i^* := \begin{cases} b_i & \text{if } \phi_i = F_{[a_i,b_i]} \psi_i \\ t_i^* & \text{if } \phi_i = G_{[a_i,b_i]} \psi_i, \end{cases} \\ \hat{\rho}_i^{\max} &= \rho_i^{\max} + \zeta_i^{\mathfrak{u}}, \; \hat{r}_i \in (0,r_i), \; \hat{\boldsymbol{p}}_i^{\gamma} = \boldsymbol{p}_i^{\gamma, \operatorname{new}}, \; \hat{\mathfrak{n}}_i = \mathfrak{n}_i + 1 \Big\} \end{split}$$

where the variables $\zeta_i^{\rm u}$ and $\boldsymbol{p}_i^{\gamma,{\rm new}}$ are defined in the sequel. In words, we set $\hat{t}_i^* := b_i$ if $\phi_i = F_{[a_i,b_i]}\psi_i$ (time relaxation) and keep $\hat{t}_i^* := t_i^* = a_i$ otherwise. The parameter r_i is decreased to $\hat{r}_i \in (0, r_i)$ (robustness relaxation) to ensure local satisfaction of ϕ_i . The variable $\zeta_i^{\rm u}$ relaxes the upper funnel and needs to be such that $\hat{\rho}_i^{\rm max} := \rho_i^{\rm max} + \zeta_i^{\rm u} < \rho_i^{\rm opt}$ (upper funnel relaxation) according to (7), i.e., let $\zeta_i^{\rm u} \in$ $(0, \rho_i^{\rm opt} - \rho_i^{\rm max})$. At t_r , the detection time of a critical event, we set $\gamma_i^{\rm r} := \hat{\gamma}_i(t_r) := \hat{\rho}_i^{\rm max} - \rho^{\psi_i}(\boldsymbol{x}_{\phi_i}) + \zeta_i^{\rm l}$ with

$$\zeta_i^{\rm l} \in \begin{cases} \mathbb{R}_{>0} & \text{if } \hat{t}_i^* > t_i \\ (0, \rho^{\psi_i}(\pmb{x}_{\phi_i}) - \hat{r}_i] & \text{otherwise}, \end{cases}$$

which resembles (9) (lower funnel relaxation); $\zeta_i^{\rm u}$ and $\zeta_i^{\rm l}$ determine the margin by how much the funnel is relaxed. Let $\boldsymbol{p}_i^{\gamma,{\rm new}} := \begin{bmatrix} \gamma_i^{0,{\rm new}} & \gamma_i^{\infty,{\rm new}} & l_i^{{\rm new}} \end{bmatrix}$ and select, similar to (10) and (11), $\gamma_i^{\infty,{\rm new}} \in (0,\min(\gamma_i^{\rm r},\hat{\rho}_i^{{\rm max}} - \hat{r}_i)]$ and

$$l_i^{\text{new}} := \begin{cases} 0 & \text{if } -\gamma_i^{\text{r}} + \hat{\rho}_i^{\max} \ge \hat{r}_i \\ \frac{-\ln\left(\frac{\hat{r}_i + \gamma_i^{\infty, \text{new}} - \hat{\rho}_i^{\max}}{-(\gamma_i^{\text{r}} - \gamma_i^{\infty, \text{new}})}\right)}{\hat{t}_i^{*} - t_i} & \text{if } -\gamma_i^{\text{r}} + \hat{\rho}_i^{\max} < \hat{r}_i. \end{cases}$$

Finally, set $\gamma_i^{0,\text{new}} := (\gamma_i^{r} - \gamma_i^{\infty,\text{new}}) \exp(l_i^{\text{new}} t_i) + \gamma_i^{\infty,\text{new}}$ to account for the clock t_i that is not reset $(\hat{t}_i := t_i)$.

2) Repair of Critical Events - Stage 2: Repairs of the second and third stage are detected by

$$\mathcal{D}'_{i,\{2,3\}} := \mathcal{D}'_i \cap \{(\boldsymbol{z}_i, \boldsymbol{u}_i^{\text{int}}, \boldsymbol{u}_i^{\text{ext}}) \in \mathfrak{H}_i | \mathfrak{n}_i \geq N_i\}.$$

The second stage will only be initiated if some timing constraints hold. Then, *collaborative control* as in Theorem 2 is used to satisfy ϕ_i . The second stage is detected by

$$\mathcal{D}'_{i,2} := \mathcal{D}'_{i,\{2,3\}} \cap \Big\{ (\boldsymbol{z}_i, \boldsymbol{u}_i^{\text{int}}, \boldsymbol{u}_i^{\text{ext}}) \in \mathfrak{H}_i | \forall v_j \in \mathcal{V}_{\phi_i} \setminus \{v_i\}, \\ (\mathfrak{c}_j = -1) \text{ or } \Big(\mathfrak{c}_j = 0, b_i < \begin{cases} b_j \text{ if } \phi_j = F_{[a_j, b_j]} \psi_j \\ a_j \text{ if } \phi_j = G_{[a_j, b_j]} \phi_j \end{cases} \Big) \Big\}.$$

i.e., each agent $v_j \in \mathcal{V}_{\phi_i} \setminus \{v_i\}$ is either free or postpones satisfaction of ϕ_j to collaboratively deal with ϕ_i first, while ensuring that there is enough time to deal with ϕ_i afterwards. If $(\boldsymbol{z}_i, \boldsymbol{u}_i^{\text{int}}, \boldsymbol{u}_i^{\text{ext}}) \in \mathcal{D}'_{i,2}$, all agents in \mathcal{V}_{ϕ_i} will use *collaborative control* to deal with ϕ_i . Therefore, let

$$\begin{split} \mathcal{G}_{i,2}' := & \left\{ \hat{\boldsymbol{z}}_i \in \mathcal{Z}_i | \hat{\boldsymbol{z}}_i = \boldsymbol{z}_i \ ; \ \hat{\rho}_i^{\max} = \rho_i^{\max} + \zeta_i^{\mathrm{u}} \\ \hat{r}_i \in (0, r_i), \ \hat{\boldsymbol{p}}_i^{\gamma} = \boldsymbol{p}_i^{\gamma, \mathrm{new}}, \ \hat{\boldsymbol{\mathfrak{c}}}_i = i \right\} \end{split}$$

where $\hat{c}_i := i$ initiates *collaborative control*, while again relaxing the funnel parameters as in the first repair stage. The jump set $\mathcal{D}'_{i,2}$ applies if agent v_i detects a critical event. Now changing the perspective to the participating agents $v_j \in \mathcal{V}_{\phi_i} \setminus \{v_i\}$, all agents v_j need to participate in *collaborative control*. Assume that $v_j \in \Xi_l$, then

$$\mathcal{D}_{j,2}'' := \{ (\boldsymbol{z}_j, \boldsymbol{u}_j^{\text{int}}, \boldsymbol{u}_j^{\text{ext}}) \in \mathfrak{H}_j | \boldsymbol{\mathfrak{c}}_j \in \{-1, 0\},\\ \exists v_i \in \Xi_l \setminus \{v_j\}, v_j \in \mathcal{V}_{\phi_i}, \boldsymbol{\mathfrak{c}}_i = i\},$$

is the jump set, which is activated when agent v_i asks agent v_j for collaborative control. If $(\boldsymbol{z}_i, \boldsymbol{u}_i^{\text{int}}, \boldsymbol{u}_i^{\text{ext}}) \in \mathcal{D}''_{i,2}$, set

$$\mathcal{G}_{j,2}^{\prime\prime}:=\Bigl\{\hat{oldsymbol{z}}_j\in\mathcal{Z}_j|\hat{oldsymbol{z}}_j=oldsymbol{z}_j\ ;\ \hat{oldsymbol{p}}_j^{\mathrm{f}}=oldsymbol{p}_i^{\mathrm{f}},\ \hat{\mathfrak{c}}_j=\mathfrak{c}_i\Bigr\}$$

where $\hat{\mathbf{c}}_j = \mathbf{c}_i$ and $\hat{\mathbf{p}}_j^{\text{I}} = \mathbf{p}_i^{\text{f}}$ enforce that all conditions in Theorem 2 hold such that ϕ_i will be locally satisfied.

3) Repair of Critical Events - Stage 3: If the timing constraints in $\mathcal{D}'_{i,2}$ do not apply, repairs of the third stage are initiated by

$$\mathcal{D}'_{i,3} := \mathcal{D}'_{i,\{2,3\}} \setminus \mathcal{D}'_{i,2}$$

Agent v_i reacts in this case by reducing the robustness r_i by $\delta_i > 0$ as illustrated in Example 2 and according to

$$\begin{aligned} \mathcal{G}_{i,3}' := & \left\{ \hat{\boldsymbol{z}}_i \in \mathcal{Z}_i | \hat{\boldsymbol{z}}_i = \boldsymbol{z}_i \; ; \; \hat{\rho}_i^{\max} = \rho_i^{\max} + \zeta_i^{\mathrm{u}}, \\ \hat{r}_i = r_i - \delta_i, \; \hat{\rho}_i^{\max} = \rho_i^{\mathrm{opt}} + \sigma_i, \; \hat{\boldsymbol{p}}_i^{\gamma} = \boldsymbol{p}_i^{\gamma, \mathrm{new}} \right\}. \end{aligned}$$

where now $\gamma_i^{\rm r} := \hat{\rho}_i^{\max} - \rho^{\psi_i}(\boldsymbol{x}_{\phi_i}) + \delta_i$ is used to calculate $\boldsymbol{p}_i^{\gamma,{\rm new}}$, while $\sigma_i > 0$ will avoid Zeno behavior.

4) The Overall System: It now needs to be determined what happens when a task ϕ_i is locally satisfied. Define $\nu_i := \int \mathfrak{c}_i$ if $\mathfrak{c}_i > 0$

$$\begin{cases} i & \text{if } \mathbf{c}_i = 0 \\ \mathcal{D}_{i,\text{sat}} := \left\{ (\boldsymbol{z}_i, \boldsymbol{u}_i^{\text{int}}, \boldsymbol{u}_i^{\text{ext}}) \in \mathfrak{H}_i | r_{\nu_i} \le \rho^{\psi_{\nu_i}} (\boldsymbol{x}_{\phi_{\nu_i}}) \le \rho_{\nu_i}^{\text{max}}, \ \mathbf{c}_i \ge 0, \\ t_i \in \begin{cases} [a_{\nu_i}, b_{\nu_i}] & \text{if } \phi_{\nu_i} = F_{[a_{\nu_i}, b_{\nu_i}]} \psi_{\nu_i} \\ b_{\nu_i} & \text{if } \phi_{\nu_i} = G_{[a_{\nu_i}, b_{\nu_i}]} \psi_{\nu_i} \end{cases} \right\} \setminus (\mathcal{D}'_i \cup \mathcal{D}''_{i,2}), \end{cases}$$

where the set substraction of $\mathcal{D}'_i \cup \mathcal{D}''_{i,2}$ exludes the case where \mathcal{D}'_i or $\mathcal{D}''_{i,2}$ apply simultaneously with $\mathcal{D}_{i,\text{sat}}$. This hence prevents cases when two jump options are available, which would induce an undesirable non-determism endangering the logic behind the hybrid system. If $(\boldsymbol{z}_i, \boldsymbol{u}_i^{\text{int}}, \boldsymbol{u}_i^{\text{ext}}) \in \mathcal{D}_{i,\text{sat}}$, let

$$\begin{aligned} \mathcal{G}_{i,\text{sat}} &:= \left\{ \hat{\boldsymbol{z}}_i \in \mathcal{Z}_i | \hat{\boldsymbol{z}}_i = \boldsymbol{z}_i \; ; \; \hat{t}_i^* = \begin{cases} b_i & \text{if } \phi_i = F_{[a_i,b_i]} \psi_i \\ a_i & \text{if } \phi_i = G_{[a_i,b_i]} \psi_i, \end{cases} \\ \hat{\rho}_i^{\max} &= \tilde{\rho}_i^{\max}, \; \hat{r}_i = \tilde{r}_i, \\ \hat{\boldsymbol{p}}_i^{\gamma} &= \boldsymbol{p}_i^{\gamma,\text{new}}, \; \hat{\boldsymbol{c}}_i = \begin{cases} 0 & \text{if } \boldsymbol{c}_i > 0 \text{ and } \boldsymbol{c}_i \neq i \\ -1 & \text{if } \boldsymbol{c}_i = 0 \text{ or } \boldsymbol{c}_i = i \end{cases} \right\} \end{aligned}$$

where $\tilde{\rho}_i^{\text{max}}$ and \tilde{r}_i are chosen according to (7) and (8), but evaluated with $\boldsymbol{x}_{\phi_i}(t_i)$ instead of $\boldsymbol{x}_{\phi_i}(0)$. If $\hat{c}_i = 0$ in $\mathcal{G}_{i,\text{sat}}$, the task ϕ_i will be pursued next, while ϕ_i has already been satisfied if $\hat{c}_i = -1$ so that the agent becomes free.

Note that $\mathcal{D}'_i = \mathcal{D}'_{i,1} \cup \mathcal{D}'_{i,2} \cup \mathcal{D}'_{i,3}$ with $\mathcal{D}'_{i,1} \cap \mathcal{D}'_{i,2} \cap \mathcal{D}'_{i,3} = \emptyset$. The hybrid system \mathcal{H}_i is given by $D_i := \mathcal{D}'_i \cup \mathcal{D}''_{i,2} \cup \mathcal{D}_{i,\text{sat}}$ and $C_i := \mathcal{Z}_i \setminus D_i$. The flow map has already been defined and the jump map is

$$\begin{split} G_{i}(\boldsymbol{z}_{i},\boldsymbol{u}_{i}^{\text{int}},\boldsymbol{u}_{i}^{\text{ext}}) &:= \\ \begin{cases} \mathcal{G}'_{i,1}(\boldsymbol{z}_{i},\boldsymbol{u}_{i}^{\text{int}},\boldsymbol{u}_{i}^{\text{ext}}) & \text{for } (\boldsymbol{z}_{i},\boldsymbol{u}_{i}^{\text{int}},\boldsymbol{u}_{i}^{\text{ext}}) \in \mathcal{D}'_{i,1} \\ \mathcal{G}'_{i,2}(\boldsymbol{z}_{i},\boldsymbol{u}_{i}^{\text{int}},\boldsymbol{u}_{i}^{\text{ext}}) & \text{for } (\boldsymbol{z}_{i},\boldsymbol{u}_{i}^{\text{int}},\boldsymbol{u}_{i}^{\text{ext}}) \in \mathcal{D}'_{i,2} \\ \mathcal{G}''_{i,2}(\boldsymbol{z}_{i},\boldsymbol{u}_{i}^{\text{int}},\boldsymbol{u}_{i}^{\text{ext}}) & \text{for } (\boldsymbol{z}_{i},\boldsymbol{u}_{i}^{\text{int}},\boldsymbol{u}_{i}^{\text{ext}}) \in \mathcal{D}'_{i,2} \\ \mathcal{G}'_{i,3}(\boldsymbol{z}_{i},\boldsymbol{u}_{i}^{\text{int}},\boldsymbol{u}_{i}^{\text{ext}}) & \text{for } (\boldsymbol{z}_{i},\boldsymbol{u}_{i}^{\text{int}},\boldsymbol{u}_{i}^{\text{ext}}) \in \mathcal{D}'_{i,3} \\ \mathcal{G}_{i,\text{sat}}(\boldsymbol{z}_{i},\boldsymbol{u}_{i}^{\text{int}},\boldsymbol{u}_{i}^{\text{ext}}) & \text{for } (\boldsymbol{z}_{i},\boldsymbol{u}_{i}^{\text{int}},\boldsymbol{u}_{i}^{\text{ext}}) \in \mathcal{D}'_{i,3} \\ \end{array}$$

Note now that the sets \mathcal{D}'_i and $\mathcal{D}_{i,\text{sat}}$ as well as $\mathcal{D}''_{i,2}$ and $\mathcal{D}_{i,\text{sat}}$ are non-intersecting. However, \mathcal{D}'_i and $\mathcal{D}''_{i,2}$ are intersecting. Therefore, if $(z_i, u_i^{\text{int}}, u_i^{\text{ext}}) \in \mathcal{D}'_i \cap \mathcal{D}''_{i,2}$, which will rarely happen in practice, we only execute the jump induced by $\mathcal{D}''_{i,2}$ to not endager the logic behind the hybrid system. Thereby, we can say that the sets $\mathcal{D}'_i, \mathcal{D}''_{i,2}$, and $\mathcal{D}_{i,\text{sat}}$ are technically non-intersecting.

Theorem 3: Assume that each agent $v_i \in \mathcal{V}$ is subject to ϕ_i of the form (4b) and controlled by $\mathcal{H}_i := (C_i, F_i, D_i, G_i)$, while Assumptions 1-5 are satisfied. The induced dependency clusters $\overline{\Xi} = \{\Xi_1, \ldots, \Xi_L\}$ are such that for each $\Xi_l \in \overline{\Xi}$ it holds that v_i and v_j can communicate for all $v_i, v_j \in \Xi_l$. For $v_i \in \Xi_l$ it then holds that $\rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0) \geq r_i$, where either $r_i := r_i(0, 0)$ (initial robustness) if $\phi_i = \phi_j$ for all $v_i, v_j \in \Xi_l$ or r_i is lower bounded and maximized up to a precision of δ_i otherwise. Zeno behavior is excluded.

Proof: Note first that there will never be the option of two jumps at the same time since the jump sets $\mathcal{D}'_{i,1}$, $\mathcal{D}'_{i,2}, \mathcal{D}''_{i,2}, \mathcal{D}'_{i,3}$, and $\mathcal{D}_{i,sat}$ are technically non-intersecting. In the first repair stage, the parameters t_i^* , ρ_i^{max} , r_i , γ_i^0 , γ_i^{∞} , and l_i are repaired in a way that still guarantees local satisfaction of ϕ_i . Zeno behavior is excluded for this stage since detection of a critical event is directly followed by a jump into the interior of the funnel, i.e., into the flow set C_i and since only a finite number of jumps, i.e., N_i jumps, are permitted. For the second repair stage, collaborative control guarantees finishing the task ϕ_i by the guarantees given in Theorem 2. Afterwards, participating agents $v_i \in \mathcal{V}_{\phi_i} \setminus \{v_i\}$ have enough time to deal with their own local task ϕ_j , which is initiated by $\mathcal{D}_{j,sat}$. If the timing constraints for collaborative control do not hold, the third repair stage is initiated and r_i is successively decreased by δ_i so that eventually $\rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0) \geq r_i$ has to hold, i.e. maximizing $\rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0)$ to a precision of δ_i . Note that r_i has to be lower bounded due to Assumption 3. This assumption states the well-posedness of ψ_i and means that for local satisfaction of ϕ_i the state $oldsymbol{x}_{\phi_i}$ is bounded. Hence, all agents aim to stay within a bounded set. Consequently, successively reducing r_i will eventually lead to $\rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0) \geq r_i$. This again means that only a finite number of jumps is possible when the lower funnel is touched. Touching the upper funnel will also only lead to a finite number of jumps since $\hat{\rho}_i^{\max} = \rho_i^{\text{opt}} + \sigma_i$ in $\mathcal{G}'_{i,3}$, hence excluding Zeno behavior of \mathcal{H}_i .

D. Extension to θ -formulas

If each agent $v_i \in \mathcal{V}$ is subject to θ_i of the form (4c), the same result can be obtained by extending the hybrid system $\mathcal{H}_i = (C_i, F_i, D_i, G_i)$ as instructed in [7]. The detection & repair mechanism introduced in the previous section can be applied in exactly the same way. Due to space limitations, the illustration is omitted.

V. SIMULATIONS

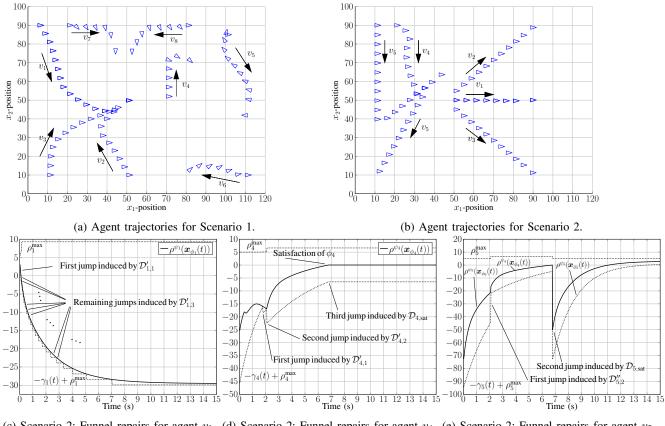
We consider omni-directional robots as in [16] with two states x_1 and x_2 indicating the robot position and one state x_3 indicating the robot orientation with respect to the x_1 axis. Let $x_{i,j}$ with $j \in \{1, 2, 3\}$ denote the *j*-th element of agent v_i 's state and let $p_i := \begin{bmatrix} x_{i,1} & x_{i,2} \end{bmatrix}$. We hence have $x_i := \begin{bmatrix} p_i & x_{i,3} \end{bmatrix} = \begin{bmatrix} x_{i,1} & x_{i,2} & x_{i,3} \end{bmatrix}$ with the dynamics

$$\dot{\boldsymbol{x}}_{i} = \begin{bmatrix} \cos(x_{i,3}) & -\sin(x_{i,3}) & 0\\ \sin(x_{i,3}) & \cos(x_{i,3}) & 0\\ 0 & 0 & 1 \end{bmatrix} \left(B_{i}^{T} \right)^{-1} R_{i} \boldsymbol{u}_{i},$$

where $R_i := 0.02$ is the wheel radius and $B_i := \begin{bmatrix} 0 & \cos(\pi/6) & -\cos(\pi/6) \\ -1 & \sin(\pi/6) & \sin(\pi/6) \\ L_i & L_i & L_i \end{bmatrix}$ describes geometrical constraints with $L_i := 0.2$ as the radius of the robot body. Each element of u_i corresponds to the angular velocity of exactly

one wheel. All simulations have been performed in real-time on a two-core 1,8 GHz CPU with 4 GB of RAM. Computational complexity is not an issue due to the computationallyefficient and easy-to-implement feedback control laws.

Scenario 1: This scenario employs eight agents in three clusters with $v_1, v_2, v_3 \in \Xi_1$, $v_4, v_5, v_6 \in \Xi_2$, and $v_7, v_8 \in$ Ξ_3 , where the agents in each cluster are subject to the same formula, consequently satisfying the conditions in Theorem 1. The first cluster Ξ_1 should eventually gather, while at the same time v_1 should approach the point $\boldsymbol{x}_A := \begin{bmatrix} 50 & 50 \end{bmatrix}$. The second cluster Ξ_2 should eventually form a triangular formation, while the robot's orientation point to each other and agent v_5 approaches $x_B := \begin{bmatrix} 110 & 40 \end{bmatrix}$. The third cluster should eventually move to some predefined points $\boldsymbol{x}_C := \begin{bmatrix} 40 & 70 \end{bmatrix}$ and $\boldsymbol{x}_D := \begin{bmatrix} 55 & 70 \end{bmatrix}$, while staying as close as possible to each other and having an orientation that is pointing into the $-x_2$ -direction. In formulas, we have $\phi_1 := \phi_2 := \phi_3 := F_{[10,15]} \psi_{l_1}$ with $\psi_{l_1} := (\| \boldsymbol{p}_1 - \boldsymbol{p}_2 \| <$ $2) \wedge (\|\boldsymbol{p}_1 - \boldsymbol{p}_3\| < 2) \wedge (\|\boldsymbol{p}_2 - \boldsymbol{p}_3\| < 2) \wedge (\|\boldsymbol{p}_1 - \boldsymbol{p}_A\| < 2).$ For the second cluster, $\phi_4 := \phi_5 := \phi_6 := F_{[10,15]} \psi_{l_2}$ is used with $\psi_{l_2} := (\| \boldsymbol{p}_5 - \boldsymbol{p}_B \| < 5) \land (27 < x_{5,1} - x_{4,1} <$ $(33) \wedge (27 < x_{5,1} - x_{6,1} < 33) \wedge (27 < x_{4,2} - x_{5,2} < 33)$ $(33) \wedge (27 < x_{5,2} - x_{6,2} < 33) \wedge (|\deg(x_{4,3}) + 45| < 33))$ $(|\deg(x_{5,3}) - 180| < 5) \land (|\deg(x_{6,3}) - 45| < 5)),$ where $deg(\cdot)$ transforms radians into degrees. Finally, the third cluster employs $\phi_7 := \phi_8 := F_{[10,15]}\psi_{l_3}$ with $\psi_{l_3} :=$ $\| \boldsymbol{p}_7 - \boldsymbol{p}_8 \| < 10 \rangle \wedge (\| \boldsymbol{p}_7 - \boldsymbol{p}_C \| < 10 \rangle \wedge (\| \boldsymbol{p}_8 - \boldsymbol{p}_D \| < 10 \rangle \wedge \| \boldsymbol{p}_8 - \boldsymbol{p}_D \| < 10 \rangle$ $10) \wedge (|\deg(x_{7,3}) + 90| < 5) \wedge (|\deg(x_{5,3}) + 90| < 5))$. The



(c) Scenario 2: Funnel repairs for agent v_1 (d) Scenario 2: Funnel repairs for agent v_4 (e) Scenario 2: Funnel repairs for agent v_5 Fig. 2: Simulation results for Scenario 1 and 2.

simulation result is shown in Fig. 2a, where each robot's initial orientation is 0, indicated by the direction of the triangle. Note that the tasks are satisfied within the time interval [10, 15].

Scenario 2: This scenario employs five agents in two clusters with $v_1, v_2, v_3 \in \Xi_1$ and $v_4, v_5 \in \Xi_2$ simulating Example 2 and 3, respectively. Recall that in these examples we had $\phi_1 := G_{[0,15]}(\|\boldsymbol{p}_1 - \boldsymbol{p}_2\| \le 10 \land \|\boldsymbol{p}_1 - \boldsymbol{p}_3\| \le$ 10), $\phi_2 := F_{[5,15]}(\|p_2 - [90\ 90]\| \le 5)$, and $\phi_3 :=$ $F_{[5,15]}(\|\boldsymbol{p}_3 - [90 \ 10]\| \le 5)$ as well as $\phi_4 := F_{[5,10]}(\|\boldsymbol{p}_4 - [90 \ 10]\| \ge 5)$ $\| \mathbf{p}_5 \| \le 10 \land \| \mathbf{p}_4 - [50] \ 70 \| \| \le 10)$ and $\phi_5 := F_{[5,15]}(\| \mathbf{p}_5 - \mathbf{p}_5 \| \| \mathbf{p}_5 - \mathbf{p}_5 \| \| \| \mathbf{p}_5 \| \| \| \mathbf{p}_5 \| \| \| \mathbf{p}_5 \| \| \| \mathbf{p}_5 \| \| \| \| \mathbf{p}_5 \| \| \mathbf{p}_5 \| \| \| \mathbf{p}_5 \| \| \| \mathbf{p}_5 \| \|$ $\begin{bmatrix} 10 & 10 \end{bmatrix} \parallel \le 5$ so that ϕ_1 and ϕ_4 are collaborative tasks. We set $\delta_i := 1.5$ and $N_i := 1$ for all agents $v_i \in \mathcal{V}$. Agent trajectories are shown in Fig. 2b, while Fig. 2c shows the funnel (5) for agent v_1 . It is visible that agent v_1 first tries to repair its parameter in Stage 1, and then initiates Stage 3 to successively reduces the robustness r_1 and consequently also the lower funnel as visible in Fig. 2c. Agent v_1 hence finds a trade-off between staying close to agent v_2 and v_3 , i.e., staying in the middle of them as visible in Fig. 2b. In other words, agent v_1 can not satisfy ϕ_1 , but a least violating solution is found. Agent v_4 first tries to repair its parameters in Stage 1, but then requests agent v_5 to use *collaborative control* to satisfy ϕ_4 as indicated in Fig. 2d and 2e. Agent v_5 collaborates with agent v_4 to satisfy ϕ_4 and satisfies ϕ_5 afterwards. We can conclude that ϕ_2 , ϕ_3 , ϕ_4 , and ϕ_5 are

locally satisfied with robustness $r_2 = r_3 = r_4 = r_5 = 0.5$, while ϕ_1 is not locally satisfied, but is forced to achieve $\rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0) > r_1 = -30$.

VI. CONCLUSION

We presented a framework for the control of multi-agent systems under signal temporal logic tasks. We adopted a bottom-up approach where each agent is subject to a local signal temporal logic task. By leveraging ideas from prescribed performance control, we developed a continuous feedback control law that achieves satisfaction of all local tasks under some given conditions. If these conditions do not hold, we proposed to combine the developed feedback control law with an online detection & repair scheme, expressed as a hybrid system. This scheme detects critical events and repairs them. Advantages of our framework are low computation times and robustness that is taken care of by the robust semantics of signal temporal logic and by the prescribed performance approach.

Possible future extensions are the improvement of the repair stages in the online detection & repair scheme. We proposed a three-stage procedure, but several other steps are possible. A next step is also to perform physical experiments.

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APPENDIX

Proof of Corollary 2: We proceed similar to the proof of Theorem 1.

Step A: First, define $\boldsymbol{\xi} := \begin{bmatrix} \xi_{i_1} & \xi_{i_2} & \cdots & \xi_{i_{|\mathcal{V}_{\phi}|}} \end{bmatrix}$ where $|\mathcal{V}_{\phi}|$ is the cardinality of \mathcal{V}_{ϕ} and $v_{i_1}, \ldots, v_{i_{|\mathcal{V}_{\phi}|}} \in \mathcal{V}_{\phi}$ are all agents participating in ϕ . Define again the stacked vector $\boldsymbol{y} := \begin{bmatrix} \boldsymbol{x} & \boldsymbol{\xi} \end{bmatrix}$. Consider the closed-loop system $\dot{\boldsymbol{x}}_i =: H_{\boldsymbol{x}_i}(\boldsymbol{x}, \xi_i)$ with $H_{\boldsymbol{x}_i}(\boldsymbol{x}, \xi_i) := f_i(\boldsymbol{x}_i) + f_i^c(\boldsymbol{x}) + g_i(\boldsymbol{x}_i)\boldsymbol{u}_i + \boldsymbol{w}_i$ where

$$oldsymbol{u}_i := egin{cases} -\ln(-rac{\xi_i+1}{\xi_i})g_i(oldsymbol{x}_i)^Trac{\partial
ho^{\psi_i}(oldsymbol{x}_{\phi_i})}{\partialoldsymbol{x}_i} & ext{if } v_i \in \mathcal{V}_\phi \ oldsymbol{u}_i' & ext{if } v_i \in \mathcal{V}\setminus\mathcal{V}_\phi. \end{cases}$$

Recall that u'_i is the control law given in the assumptions. Next, define $\dot{x} =: H_x(x, \xi)$ and $\frac{d\xi_i}{dt} =: H_{\xi_i}(x, \xi_i, t)$ with $H_{\boldsymbol{x}}(\boldsymbol{x},\boldsymbol{\xi})$ and $H_{\xi_i}(\boldsymbol{x},\xi_i,t)$ as in the proof of Theorem 1. Let

$$H_{\boldsymbol{\xi}}(\boldsymbol{x}, \boldsymbol{\xi}, t) := \begin{bmatrix} H_{\xi_{i_1}}(\boldsymbol{x}, \xi_{i_1}, t) & \dots & H_{\xi_{i_{|\mathcal{V}_{\phi}|}}}(\boldsymbol{x}, \xi_{i_{|\mathcal{V}_{\phi}|}}, t) \end{bmatrix}$$

so that the dynamics of \boldsymbol{y} can be written as $\dot{\boldsymbol{y}} =: H(\boldsymbol{y},t)$ with

$$H(\boldsymbol{y},t) := \begin{bmatrix} H_{\boldsymbol{x}}(\boldsymbol{x},\boldsymbol{\xi}) & H_{\boldsymbol{\xi}}(\boldsymbol{x},\boldsymbol{\xi},t) \end{bmatrix}.$$

It again holds that $\boldsymbol{x}(0)$ is such that $\xi_i(\boldsymbol{x}_{\phi_i}(0), 0) \in \Omega_{\xi} := (-1, 0)$ holds for all agents $v_i \in \mathcal{V}_{\phi}$ by the choice of γ_i^0 . As in the proof of Theorem 1, define the open, bounded, and non-empty set $\Omega_{\phi_i}(t)$. Next, assume that for each agent $v_k \in \mathcal{V} \setminus \mathcal{V}_{\phi}$ the corresponding state \boldsymbol{x}_k is initially contained in the open set Ω_k , i.e., $\boldsymbol{x}_k(0) \in \Omega_k$, where Ω_k exists due the assumptions, i.e., each state \boldsymbol{x}_k remains in the compact set Ω'_k . Let $v_i \in \mathcal{V}_{\phi}$ and define

$$\Omega_{\boldsymbol{\xi}} := \Omega_{\boldsymbol{\xi}} \times \ldots \times \Omega_{\boldsymbol{\xi}} \subset \mathbb{R}^{|\mathcal{V}_{\phi}|}$$
$$\Omega_{\boldsymbol{x}} := \Omega_{\phi_i}(0) \times \Omega_{k_1} \times \ldots \Omega_{k_{M-|\mathcal{V}_{\phi}|}} \subset \mathbb{R}^{nM},$$

where $v_{k_1}, \ldots, v_{k_{M-|\mathcal{V}_{\phi}|}} \in \mathcal{V} \setminus \mathcal{V}_{\phi}$ are all agents not belonging to \mathcal{V}_{ϕ} . Finally, define the open, non-empty, and bounded set

$$\Omega_{\boldsymbol{y}} := \Omega_{\boldsymbol{x}} \times \Omega_{\boldsymbol{\xi}} \subset \mathbb{R}^{nM + |\mathcal{V}_{\phi}|}.$$

It consequently holds that $\boldsymbol{y}(0) = [\boldsymbol{x}(0) \ \boldsymbol{\xi}(0)] \in \Omega_{\boldsymbol{y}}$. Next, note that the conditions in Lemma 1 for the initial value problem $\boldsymbol{\dot{y}} = H(\boldsymbol{y},t)$ with $\boldsymbol{y}(0) \in \Omega_{\boldsymbol{y}}$ and $H(\boldsymbol{y},t) : \Omega_{\boldsymbol{y}} \times \mathbb{R}_{\geq 0} \to \mathbb{R}^{nM+|\mathcal{V}_{\phi}|}$ are satisfied as in the proof of Theorem 1 since the control law \boldsymbol{u}'_k guarantees existence of nontrivial solutions. As a result, there again exists a maximal solution with $\boldsymbol{y}(t) \in \Omega_{\boldsymbol{y}}$ for all $t \in \mathcal{J} := [0, \tau_{\max}) \subseteq \mathbb{R}_{\geq 0}$ and $\tau_{\max} > 0$.

Step B: The Lyapunov analysis to show that $\tau_{\max} = \infty$ follows similar steps as in the proof of Theorem 1 that are not shown here. It can again be shown that $\xi_i(t) \in \Omega'_{\xi_i}$ and $\boldsymbol{x}_{\phi_i}(t) \in \Omega'_{\phi_i}$ for all agents $v_i \in \mathcal{V}_{\phi}$, where Ω'_{ξ_i} and Ω'_{ϕ_i} are compact subsets of Ω_{ξ_i} and Ω_{ϕ_i} , respectively. It consequently follows that $0 < r \le \rho^{\phi}(\boldsymbol{x}_{\phi}, 0) \le \rho^{\max}$, i.e., $(\boldsymbol{x}_{\phi}, 0) \models \phi$.