

Nonlinear Model Based Guidance with Deep Learning Based Target Trajectory Prediction Against Aerial Agile Attack Patterns

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Abstract—In this work, we propose a novel missile guidance algorithm that combines deep learning based trajectory prediction with nonlinear model predictive control. Although missile guidance and threat interception is a well-studied problem, existing algorithms' performance degrade significantly when the target is pulling high acceleration attack maneuvers while rapidly changing its direction. We argue that since most threats execute similar attack maneuvers, these nonlinear trajectory patterns can be processed with modern machine learning methods to build high accuracy trajectory prediction algorithms. We train a long short-term memory network (LSTM) based on a class of simulated structured agile attack patterns, then combine this predictor with quadratic programming based nonlinear model predictive control (NMPC). Our method, named nonlinear model based predictive control with target acceleration predictions (NMPC-TAP), significantly outperforms compared approaches in terms of miss distance, for the scenarios where the target/threat is executing agile maneuvers.

I. INTRODUCTION

Since the World War II, missile interception guidance has been one of the most important problems in missile studies. Several guidance laws are introduced over the years. One of the most popular and commonly used guidance law is Proportional Navigation (PN). PN can be implemented easily and can guarantee to hit targets under certain limitations such as non-maneuvering or weakly maneuvering targets [1]. Garber proposed Augmented Proportional navigation (APN) to overcome these limitations [2]. APN is developed by further optimizing the PN guidance. Thereafter, several other guidance laws are designed based on APN [3] [4], but none of these approaches are capable of explicitly predicting the future trajectory of the target. It is shown that the success of guidance algorithms can be improved with prediction of target maneuvers. Moreover, if target performs agile maneuvers and the distance between target and interceptor is small, predicting the future states of the target can be used to increase the hit performance [5]. That being said, intercepting highly agile targets is still an open problem, since the predictive capabilities of existing algorithms are usually limited to targets that execute standard (non-agile) maneuvers. The main objective of this paper is to utilize recent advances in machine learning and predictive guidance to push the state of the art performance in intercepting agile targets.

A. Previous Work

In recent years, there has been a great deal of effort in developing predictive guidance laws by employing knowledge/prediction of target trajectory and/or dynamics [6]–[8]. Best and Norton [9] proposed a predictive guidance law inspired by model predictive

control (MPC). The guidance law is based on predicting the probability density function(PDF) of target position at interception, Shima et al. [10] presented the multiple-model estimator in case the target performs evasive maneuver characterized by a random switching time. Dionne et al. [11] extends the Best and Norton [9] and presents a new terminal guidance law which uses target position that estimated a in terms of a PDF in the terminal constraint. Nobahari [12] introduced a heuristic predictive line-of-sight (LOS) guidance law based on a dynamic optimization algorithm which determines the acceleration commands. Ure [7] proposed a stochastic predictive guidance and control methodology for interception of agile targets. Akcal [8] presents a predictive guidance which uses a recursive least squares (RLS) to estimate the possible target positions and nonlinear programming is employed for selecting the optimal action. Eren et al. [13] extensively investigated potential and vast applications of MPC in aerospace guidance field in the survey paper. Li et al. [14] introduced a NMPC scheme in case of target acceleration is unknown for missile interception based on the primal–dual neural-network(PDNN) optimization. NMPC formulated as quadratic programming (QP) problem which needs to be solved at each time step and PDNN optimization implementation was shown. Integrated control and guidance of missiles based on NMPC proposed by Bachitar [15]. A multiobjective offline tuning of NMPC introduced which considered control performance and implementation cost based on required computational capacity. Kumar et al. [16] presented an integrated missile guidance and control using NMPC for missile-on-missile interception. To generate an optimal control command constrained nonlinear problem solved with a sequential quadratic programming (SQP) algorithm. Bhattacharjee [17] derived a NMPC based missile guidance algorithm for target interception.

B. Contributions

In this paper, nonlinear model predictive control with target acceleration predictions (NMPC-TAP) method is proposed by exploiting target trajectory predictions generated by a deep neural network. The main idea behind using deep neural networks for this purpose stems from the fact that a large array of past target trajectories are available from operational heuristics and domain expertise in air defense systems [18]. By using these heuristic, synthetic agile target trajectories can be fed to a time series prediction network (long short-term memory networks in our case) to predict target trajectories. We show that using such networks for agile maneuvering targets is indeed more efficient compared to alternative approaches. Proposed method needs instantaneous and future target acceleration predictions along the prediction horizon of NMPC. Then a QP is solved at each time step by employing these predictions. In PDNN [14] and QP [17] based optimization approaches, control inputs and bounded unknown target accelerations are combined in a decision variable vector. The QP problem in NMPC-TAP is obtained by leaving the control input as only decision variable, since the target accelerations are predicted. The NMPC-TAP is validated by simulations for agile maneuvers. It is shown that NMPC-TAP yields significantly superior performance

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compared to PN, APN and NMPC with unknown target acceleration in agile maneuvering target interception scenarios.

The paper is organized as follows; the Section 2 gives a brief background on the interception geometry, the Section 3 provides the details of the NMPC based predictive guidance algorithms, the Section 4 describes the structure of the LSTM and finally the Section 5 gives simulation results for various guidance laws.

II. INTERCEPTION GEOMETRY

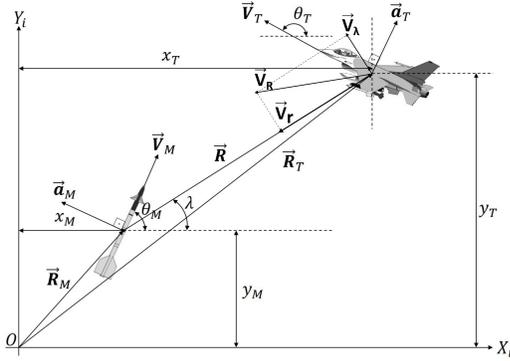


Fig. 1: The missile-target engagement geometry.

We consider a two-dimensional intercept model which is depicted in Figure 1. Assuming an inertial frame defined by X_i and Y_i axes together with corresponding unit vectors \vec{i} and \vec{j} . x_M ; y_M ; x_T ; and y_T are the Cartesian coordinates of the missile and the target while \vec{R}_M ; \vec{R}_T ; \vec{V}_M and \vec{V}_T represent the position and velocity vectors of the missile and target. In addition to the previous definitions, missile target range vector \vec{R} can be rewritten in polar coordinates as $R = (r, \lambda)$. r is the range along LOS, and λ is the LOS angle. V_R is the first time derivative of R ; V_T and a_T are target velocity and acceleration, respectively; and V_M and a_M are missile velocity and acceleration, respectively. The interception geometry is represented in polar coordinate system in [19] [14].

$$V_r = V_T \cos(\theta_T - \lambda) - V_M \cos(\theta_M - \lambda)$$

$$V_\lambda = V_T \sin(\theta_T - \lambda) - V_M \sin(\theta_M - \lambda)$$

$$a_{T_r} = a_T \sin(\theta_T - \lambda)$$

$$a_{T_\lambda} = a_T \cos(\theta_T - \lambda)$$

V_λ is a transversal component of relative velocity rotating with LOS, a_{T_r} and a_{T_λ} can be described as the projection components of the target acceleration. Let us consider the state variable vector $x = [r, V_r, \lambda, V_\lambda]^T$. Then, the following state model of missile-target interception model can be obtained in polar coordinates as follows:

$$\dot{x} = f(x) + g(x)u + d(x)w, \quad (1)$$

where $x \in \mathbb{R}^{n_x}$ is a state vector, $w = [a_{T_r}, a_{T_\lambda}]^T \in \mathbb{R}^2$ is a vector which represents target acceleration in polar coordinates, and $u = a_M \in \mathbb{R}$ is a control input. The control input is subjected to an acceleration limit, which is expressed in Eq. 2.

$$-u_{max} \leq u \leq u_{max}. \quad (2)$$

It is well known that MPC usually requires difference equations in discrete-time. Difference equations can be easily obtained by applying Euler discretization on Eq.(1):

$$x(k+1) = f_d(x(k)) + g_d(x(k))u(k) + d_d(x(k))w(k) \quad (3)$$

where

$$f_d(x(k)) = \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix} + \Delta(t) \begin{bmatrix} x_2(k) \\ x_4^2(k)/x_1(k) \\ x_4(k)/x_1(k) \\ -x_2(k)x_4(k)/x_1(k) \end{bmatrix}$$

$$g_d(x(k)) = \Delta(t) \begin{bmatrix} 0 \\ \sin(\theta_M - x_3(k)) \\ 0 \\ -\cos(\theta_M - x_3(k)) \end{bmatrix}, d_d(x(k)) = \Delta(t) \begin{bmatrix} 0 \\ a_{T_r} \\ 0 \\ a_{T_\lambda} \end{bmatrix}$$

III. PREDICTIVE GUIDANCE ALGORITHMS WITH CONSTRAINED OPTIMIZATION

In this section QP based formulations for predictive guidance will be derived for both predicted and unknown target accelerations.

A. NMPC with Predicted Target Accelerations

The NMPC-TAP method is described in this section. The reference predictive form [20] can be expressed by using Eq. (3) as

$$x(k+j|k) = f_d(x(k+j-1|k)) + g_d(x(k+j-1|k))u(k+j-1|k) + d_d(x(k+j-1|k))w(k+j-1|k) \quad (4)$$

where

$$u(k+j-1|k) = u(k+j-2|k) + \Delta u(k+j-1|k)$$

where $(k+j|k)$ means that the current time-step is k and the distance from the current time-step is j [20]. The standard prediction form is given by

$$X_k = F_k + G_k \Delta U_k + g_k + d_k \quad (5)$$

with

$$X_k = [x(k+1|k), x(k+2|k), \dots, x(k+N_p|k)]^T$$

$$U_k = [u(k|k), u(k+1|k), \dots, u(k+N_p-1|k)]^T$$

$$\Delta U_k = [\Delta u(k|k), \Delta u(k+1|k), \dots, \Delta u(k+N_p-1|k)]^T$$

$$F_k = \begin{bmatrix} f_d(x(k|k)) \\ f_d(x(k+1|k)) \\ \vdots \\ f_d(x(k+N_p-1|k)) \end{bmatrix}$$

$$G_k = \begin{bmatrix} g_d(x(k|k)) & \dots & 0 \\ g_d(x(k+1|k)) & \dots & 0 \\ \vdots & \ddots & \vdots \\ g_d(x(k+N_p-1|k)) & \dots & g_d(x(k+N_p-1|k)) \end{bmatrix}$$

$$g_k = \begin{bmatrix} g_d(x(k|k))u(k-1) \\ g_d(x(k+1|k))u(k-1) \\ \vdots \\ g_d(x(k+N_p-1|k-1))u(k-1) \end{bmatrix}$$

$$d_k = \begin{bmatrix} d_d(x(k|k))w(k|k) \\ \vdots \\ d_d(x(k+N_p-1|k))w(k+N_p-1|k) \end{bmatrix}$$

where $u(k-1|k)$ is the control input of the last time-step, N_p, N_c are the prediction and control horizons, respectively, $X_k \in \mathbb{R}^{n_x N_p \times 1}$, $F_k \in \mathbb{R}^{n_x N_p \times 1}$, $G_k \in \mathbb{R}^{n_x N_p \times N_c}$, $\Delta U_k \in \mathbb{R}^{N_c \times 1}$, $g_k \in \mathbb{R}^{n_x N_p \times 1}$ and $D_k \in \mathbb{R}^{n_x N_p \times 2}$. Calculating the matrices F_k , G_k , g_k is challenging, since the ΔU_k is unknown until optimization problem is solved. The problem is tackled by utilizing ΔU_k belongs to previous time step to calculate these matrices.

$$\begin{aligned} \Delta u(k|k) &\triangleq \Delta u((k-1)+1|k-1) \\ \Delta u(k+1|k) &\triangleq \Delta u((k-1)+2|k-1) \\ &\vdots \\ \Delta u(k+N_p-1|k) &\triangleq \Delta u((k-1)+N_p|k-1) \\ \Delta u(k+N_p|k) &\triangleq 0 \end{aligned} \quad (6)$$

We can define a quadratic cost function as:

$$J = X_k^T Q X_k + \Delta U_k^T R \Delta U_k \quad (7)$$

where $Q \in \mathbb{R}^{n_x N_p \times n_x N_p}$ and $R \in \mathbb{R}^{N_c \times N_c}$. Then, the performance objective function can be rewritten by substituting Eq.(5) into Eq. (7)

$$J = (F_k + G_k \Delta U_k + g_k + d_k)^T Q (F_k + G_k \Delta U_k + g_k + d_k) + \Delta U_k^T R \Delta U_k \quad (8)$$

The control input magnitude Eq. 2 and its rate is constrained as:

$$U_{\min} \leq U_k \leq U_{\max} = U_{\min} \leq U_{k-1} + I_{lt} \Delta U_k \leq U_{\max} \quad (9)$$

where $U_{k-1} \in \mathbb{R}^{N_c \times 1}$ and $I_{lt} \in \mathbb{R}^{N_c \times N_c}$ are given as following

$$I_{lt} = \begin{bmatrix} U_{k-1} = u(k-1|k)1_{N_c} \\ I & 0 & \dots & 0 \\ I & I & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I & I & \dots & I \end{bmatrix} \quad (10)$$

The increment on the control input constraint as follows

$$\Delta U_{\min} \leq \Delta U_k \leq \Delta U_{\max} \quad (11)$$

where $\Delta U_{\min}, \Delta U_{\max}, U_{\min}$, and U_{\max} are the lower and upper bounds of the input vectors. We can define the cost function

$$\begin{aligned} \min & \Delta U_k^T W \Delta U_k + c^T \Delta U_k \\ \text{subject to} & E \Delta U_k \leq b \end{aligned} \quad (12)$$

where

$$\begin{aligned} W &= (G_k^T Q G_k + R), \\ c &= 2(F_k + g_k + d_k)^T Q G_k \\ E &= \begin{bmatrix} -I_{lt} \\ I_{lt} \\ -I_1 \\ I_1 \end{bmatrix} \quad I_1 = I_{N_c \times N_c} \\ b &= \begin{bmatrix} -U_{\min} + u(k-1|k)1_{N_c} \\ U_{\max} + u(k-1|k)1_{N_c} \\ -\Delta U_{\min} \\ \Delta U_{\max} \end{bmatrix} \end{aligned}$$

B. NMPC with Unknown Target Accelerations

We can define the quadratic cost function for the same performance objective (5) as follows [14], [17]:

$$\begin{aligned} \min & \begin{bmatrix} \Delta U_k \\ d_k \end{bmatrix}^T W \begin{bmatrix} \Delta U_k \\ d_k \end{bmatrix} + c^T \begin{bmatrix} \Delta U_k \\ d_k \end{bmatrix} \\ \text{subject to} & E \begin{bmatrix} \Delta U_k \\ d_k \end{bmatrix} \leq b \end{aligned} \quad (13)$$

where

$$\begin{aligned} W &= \begin{bmatrix} G_k^T Q G_k + R & G^T Q \\ Q G_k & Q \end{bmatrix}, \quad c = \begin{bmatrix} 2G_k^T Q (F_k + g_k) \\ 2Q (F_k + g_k) \end{bmatrix} \\ E &= \begin{bmatrix} -I_{lt} & 0_1 \\ I_{lt} & 0_1 \\ -I_1 & 0_1 \\ I_1 & 0_1 \\ 0_2 & -I_2 \\ 0_2 & I_2 \end{bmatrix} \quad b = \begin{bmatrix} -U_{\min} + u(k-1|k)1_{N_c} \\ U_{\max} + u(k-1|k)1_{N_c} \\ -\Delta U_{\min} \\ \Delta U_{\max} \\ -d_{\min} \\ d_{\max} \end{bmatrix} \\ & 0_1 = 0_{N_p \times N_p n_x}, 0_2 = 0_{N_p n_x \times N_p}, I_2 = I_{N_p n_x} \end{aligned}$$

IV.

PREDICTION ALGORITHM AND NETWORK DESIGN

Predicting target motion is a challenging task due to highly nonlinear dynamics and large maneuver envelope of the aircraft. Furthermore, trajectories are further complicated by decisions executed by the pilot. That being said, many attack trajectories follow similar patterns [18]. Hence it is possible to either construct or collect data from radar to create maneuver library for such attacks. Then the trajectory prediction problem can be treated as a regular time series analysis problem on target trajectory. In this section, different types of neural network architectures for trajectory prediction task is discussed and evaluated.

A. Structured Attacks and Training Data

To train and demonstrate prediction and interception method, a variety of interception scenarios are simulated. The maneuver parameters for each maneuver are selected randomly under the limits. In addition, target can switch one maneuver to another with equal probability. This allows us to simulate more complex structured attack patterns. Examples of the flight data can be seen on Fig. 2. For instance, the left side of the Fig. 2 shows that target executes weave and coordinated turn maneuvers. The other sub-figures show that the target starts with level flight and executes high acceleration turn on the middle and weave maneuver at the right sub-figure.

A fixed origin frame of reference is used. The position and velocity vectors are obtained with respect to the origin. Input of the model can be written as;

$$\begin{aligned} X &= [x_{t-1}, x_{t-2}, x_{t-3}, \dots, x_{t-n}] \\ x_t &= [x^t, y^t, v_x^t, v_y^t] \end{aligned}$$

X vector contains positions and velocities from $t-n$ to t , which n is the observation history size. Min-max scaler normalization is used to fit the data into $[0,1]$ interval since the target's position, velocity, and acceleration values vary differently. Gaussian noise is added to imitate the radar measurements. A total of 500 simulations with random initial positions and 30 seconds of flight time are conducted. Next, the data are randomly sampled from flights in the form of X . The data are divided such that % 60 is used for training,

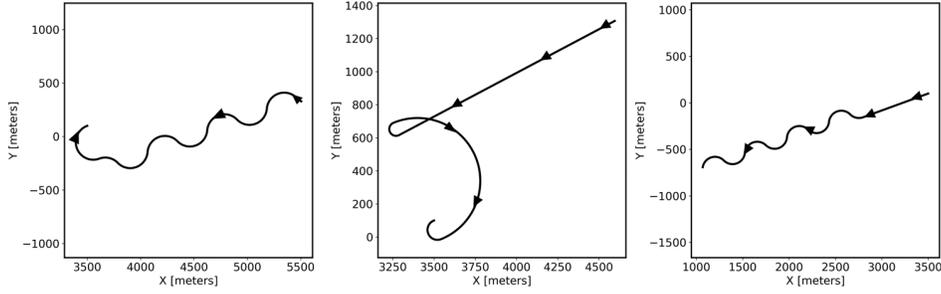


Fig. 2: Simulation Scenarios; Combination of left, right coordinated turns, level flight and weave maneuver. Arrows indicate the flow of time.

%20 for validation and %20 for test. During the simulation studies, it has been observed that at least 2 seconds of observation history is essential to obtain good predictions.

B. Network Design for Trajectory Prediction

There are plenty of deep neural network architectures that consist of Convolutional Neural Networks (CNNs), Recurrent Neural Networks (RNNs), Long-Short Term Memory Networks(LSTMs) for time-series analysis. The CNN architecture is mostly used for spatial inference from high dimensional data such as an image, and it can also be used for one-dimensional long-term dependent data [21]. It is well known that LSTM and RNN models have more suitable structures to process time series data. Since the input and output size are different, an encoder-decoder architecture is needed to deal with sequence to sequence prediction [22], [23]. Moreover, this type of architecture is capable of generalizing the solution and remove input noise.

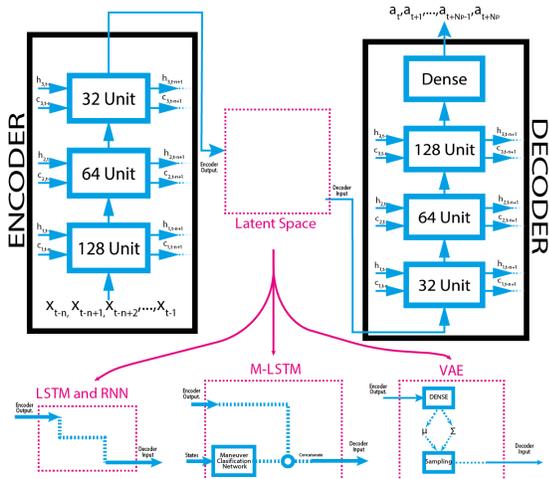


Fig. 3: Encoder-Decoder architectures for trajectory prediction.

Our preliminary studies about which kind of neural network model performs more precise acceleration predictions give rise to implement LSTM and RNN models for prediction. So, four kinds of recurrent neural network architectures such as LSTM, RNN, MLST and variational autoencoders (VAEs) [24] are compared with each other. Fig. 3 shows the studied architectures constructed over encoder-decoder. The first LSTM and RNN models are vanilla encoder-decoder which directly connect encoder outputs to decoder

inputs at the latent space. On the other hand, M-LSTM adds a maneuver classification feature into encoder outputs then sends it to decoder input. It is inspired M-LSTM model from [23] since an LSTM encoder-decoder architecture with maneuver classification improves the accuracy of position prediction. In addition to the models above, variational autoencoders derive a Gauss distribution and then take a sample from it in the latent space.

C. Implementation Details

Parameters for the prediction model are tuned according to simulation results. It is observed that adding extra layers increases computation time rather than improving the accuracy of the model. However, an increasing number of neurons in a single layer may improve the overall model accuracy. Activation functions at the last layers of the classification part in the M-LSTM are softmax. In order to prevent over-fitting, dropout method is employed. For recurrent layers 20% and for dense layers 10% dropout is implemented. Simulation studies show that Adagrad optimization method which changes learning rates according to parameter update frequency is a good fit for this problem. The number of trainable parameters varies in different architectures. The strategy was to select first encoder layer as the biggest and then decreasing and increasing the layer through the encoder decoder layers. No further improvement is obtained above 128 neurons.

D. The Prediction Results

In order to certify the effectiveness of the proposed model, SimpleRNN, LSTM, M-LSTM and VAE models are compared in simulation. The training and test accuracy are measured with Mean Square Error (MSE). The results can be seen at Table I. The acceleration limits, $[-25, 25]$, were normalized to $[0, 1]$ interval. Therefore, the MSE losses must be examined accordingly.

TABLE I: Training and Test Losses of the Models

Number of Parameters	SimpleRNN		LSTM		M-LSTM		VAE	
	Train	Test	Train	Test	Train	Test	Train	Test
Small	0.0304	0.0373	0.0121	0.0191	0.0175	0.0156	0.0653	0.0648
Medium	0.0258	0.0328	0.0169	0.0152	0.0158	0.0142	0.0651	0.0641
Large	0.0376	0.0386	0.0162	0.0096	0.0148	0.0091	0.0648	0.0631

The result show that LSTM and M-LSTM architectures are superior to SimpleRNN and VAE. In addition, M-LSTM is slightly better than LSTM architecture. Therefore, in the presence of maneuver label information, more accurate models can be used. All models also perform well on test data, which shows that models are capable of generalizing to maneuvers not used in training.

TABLE II: Example Prediction Results on Test Data

Time Step	10 Horizon		10 Horizon		20 Horizon		20 Horizon	
	Real	Prediction	Real	Prediction	Real	Prediction	Real	Prediction
k	-16.21	-16.88	-24.67	-25	14.829	14.572	15.182	14.749
k+1	-16.21	-15.74	-24.67	-25	14.829	14.643	15.182	13.991
.	16.21	-8.67	-24.67	-25
.	16.21	12.18	-24.67	-11.08	.	.	15.182	12.97
.	16.21	19.25	24.67	20.57	.	.	15.182	9.227
.	16.21	17.84	24.67	25.11	14.829	13.624	15.182	-0.084
.	16.21	17.54	24.67	23.83	14.829	13.61	.	.
.	16.21	17.26	24.67	23.91	14.829	13.578	-15.182	-15.699
k+n	16.21	17.22	24.67	24.19	14.829	13.561	-15.182	-16.032

Several acceleration prediction results can be seen at Table II. The results show that the model can predict target acceleration with high accuracy on a large set of agile maneuvers. It should be noted that the model also captures and predicts switches between different maneuvers types and associated accelerations. This property provides our proposed method NMPC-TAP a considerable edge against the alternative approaches.

V. SIMULATION RESULTS

In this section, simulation studies are conducted by comparing proposed NMPC-TAP algorithm with aforementioned methods from literature such as PN, APN and NMPC. Trajectory predictions governed by the M-LSTM from table I. To compare these methods, 100 Monte-Carlo simulation are run for each guidance algorithm with 5% noise. Simulation parameters of NMPC-TAP are given by Table III.

TABLE III: Simulation parameters

Δt	0.02 [s]	d_{max}	$8g \times \Delta t [m/s^2]$
g	9.81 [m/s^2]	d_{min}	$-d_{max}$
u_{max}	25g	Δu_{max}	$0.025u_{max}$
u_{min}	$-u_{max}$	Δu_{min}	$-\Delta u_{max}$
q	[0,0,0,100]	R	I
$\theta_{M,0}$	0°	$\theta_{T,0}$	190°
$x_{M,0}$	0	$x_{T,0}$	1000
$y_{M,0}$	0	$y_{T,0}$	1000
V_M	150	V_T	100
$N' = 3$			

Here $Q = \text{diag}(q, \dots, q)$ and $q \in \mathbb{R}^{n_x \times 1}$, N' is navigation constant for PN and APN. The NMPC-TAP cost function Eq. (7) is chosen to penalize the V_λ and input rate. An agile maneuvering target is used for simulation purposes. The maneuver is assumed that the target begins with left coordinated turn with $4g$ and 70° then makes 100m level flight and weave maneuver with $8g$ and 4s period. Interception performance of each guidance law against aforementioned agile maneuvering target is shown in Fig. 4. It is apparently seen that each approach achieves target interception. However the interception performance can not be evaluated objectively by only analyzing X, Y trajectories. Then, relative velocity component perpendicular to the LOS, V_λ is given by Fig. 5, since the quadratic cost Eq. (7) in NMPC based approaches primarily aims to penalize V_λ . NMPC unknown and NMPC-TAP has superior V_λ responses. V_λ is being enforced to zero much more earlier than PN and APN laws with a navigation constant equal to 3. Moreover, NMPC-TAP provides smaller magnitudes along whole simulation history. Fig.6 shows the control input histories. Benefits of employing target acceleration predictions in optimization problem is revealed by control inputs. NMPC-TAP changes direction of control input few steps earlier than NMPC-unknown. In addition, control input constrains are not violated by both NMPC based approaches. In

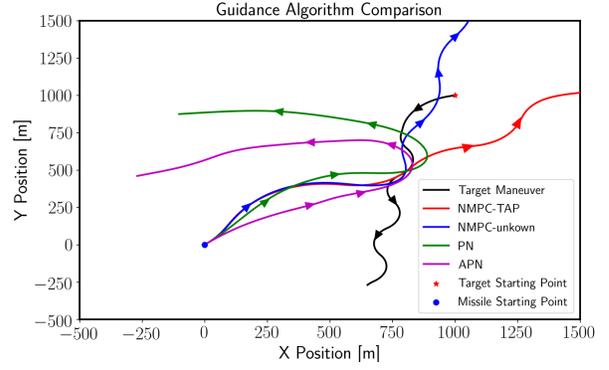


Fig. 4: Missile trajectories generated by each guidance algorithms during missile-target interception.

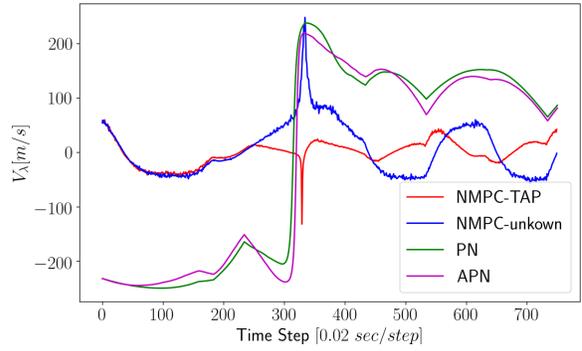
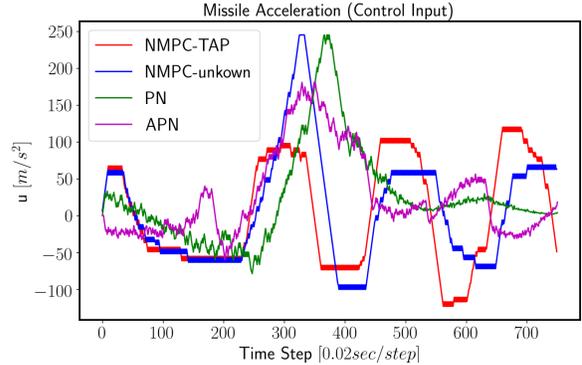

 Fig. 5: V_λ histories during missile-target interception.


Fig. 6: Missile acceleration histories during missile-target interception.

order to increase reliability of comparisons, results are averaged over 100 Monte-Carlo (MC) simulations with the values of $N_p = 10$, $N_p = 30$ and $N_p = 40$. It is also assumed that $N_p = N_c$ for each simulation. The results are evaluated in terms of miss distance, interception time and input rate. Table IV compares PN, APN, NMPC and NMPC-TAP algorithms as given above.

Here, MD_{mean} denotes mean of miss distances, MD_{STD} represents standard deviation of miss distances, IT stands for interception time and AIR presents average absolute input rate. Increasing the dimension of prediction horizon is efficiently decreasing the MD_{mean} values. Since the target is performing agile escape maneu-

TABLE IV: Results obtained by MC Simulations.

	MD_{mean} (m)	MD_{STD} (m)	IT (s)	AIR (m/s^2)
APN	18.69	22.75	6.52	5.21
PN	26.378	7.72	6.52	5.3
Results for $N_p = 10$				
NMPC-TAP	7.2	12.91	6.54	5.64
NMPC	9.2	17.49	6.67	4.76
Results for $N_p = 30$				
NMPC-TAP	6.58	8.80	6.56	6.06
NMPC	11.22	6.19	6.57	6.06
Results for $N_p = 40$				
NMPC-TAP	3.57	2.70	6.62	6.025
NMPC	14.58	6.05	6.73	5.96

vers, LSTM predictions provide invaluable future information along the prediction horizon. NMPC-TAP outperforms other guidance algorithms as long as the prediction horizon is sufficiently long. Despite the fact that NMPC-TAP presents much lower MD_{mean} values, limitations on the allowable control signal is not sacrificed.

VI. CONCLUSIONS

A novel nonlinear model predictive guidance that uses target acceleration prediction provided by an LSTM NN is proposed in this work. Target acceleration predictions are directly included in the NMPC scheme for propagating the interception problem's states. The NMPC formulates the interception problem as a QP problem in order to find the sub-optimal missile acceleration command. Simulations, which are conducted against an agile target to evaluate the performance and effectiveness of the proposed method, demonstrate that the desired guidance method is superior when compared to the others.

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