A high-gain observer for stage-structured susceptible-infectious epidemic model with linear incidence rate

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A High-Gain Observer for Stage-Structured Susceptible-Infectious Epidemic Model with Linear Incidence Rate

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Abstract— **Epidemiological models play a vital role in understanding the spread and severity of a pandemic or epidemic caused by an infectious disease in a host population. The mathematical modeling of infectious diseases in the form of compartmental models are often employed in studying the probable outbreak growth. In this paper, we study the problem of state estimation for a stage-structured SI model with two classes of infected which can be considered as a simplified modelling approach to chronic diseases with progressive severity, as is the case with AIDS. Towards this end, a high-gain observer design based on system state augmentation approach is proposed to estimate the states of the SI model. Simulation results are reported with some comparisons to the standard high-gain observer.**

Index Terms— **Epidemiological modeling, Stage-structured models, SI model, High-gain observer.**

I. Introduction

The spatial spread of infectious diseases, following their introduction at distinct locations, has always been a major concern for human populations due to their high death mortality in both developed and developing countries. Epidemiological models play an important role in analyzing the origins, dynamics and spread of such diseases. These models provide notation, concepts, intuition and disease-related factors such as the infectious agent, mode of transmission, latent and infectious periods, susceptibility and resistance which can be used to capture features that are most influential in the spread of diseases.

Epidemiological constraints, such as delays in symptom appearance and positive test confirmation (due to limited testing and detection resources), may limit the real-time use of epidemiological models [1], [2]. In order to overcome such constraints, mathematical modeling of infectious diseases was employed in epidemiology, as recognized by WHO [3] and proven to be effective [4], [5]. Compartmental modeling as a class of mathematical modeling are often employed in studying the probable outbreak growth [6]. These models consist of two parts: compartments and rules. The compartments divide the population into the different possible states with respect to the disease. The rules specify the proportion of individuals moving from one class to another.

Besides the various modelling assumptions underlying the derivation of these Compartmental models, their practical use rely on two other assumptions: the model parameters are known and an appropriate initial condition, i.e., the current state of the population is known [7]. The problem of estimating the model parameters is certainly important but will not be addressed in this work, one can see for example [8], [9] and [10]. Once the model parameters are known, we can identify the current population state. In most classical models the total population is conserved and this yields an estimate of one of the compartments in terms of estimates of the remaining ones.

Although the literature on the behaviors of epidemic models endowed with a treatment function is vast, there are fewer works on these models from the observer point which allows to predict and control the propagation of diseases and virus mutation [11]. Earlier work using observers in an epidemiological context dates back to the works in [12] and since 2012 there was a growing interest in the literature such as: the estimation of sequestered infected erythrocytes in Plasmodium falciparum malaria patients in [13], parameters and states estimation for a SI-SI Dengue epidemic model [14], interval observer for uncertain SIR and SIIR-SI models [11], a state observer for a continuous and discrete time SEIR model whose values are then used to implement a vaccination strategy [15].

In this work, we analyze a class of stage-structured SI models with 2 infectious stages which can be considered as a simplified modelling approach to chronic diseases with progressive severity, as is the case with AIDS. Then, we propose a high-gain observer to track the states of the SI epidemic model in the presence of measurement uncertainty. The proposed observer is based on system state augmentation approach which transforms the original system of dimension n into an augmented system of dimension $n+j_s$ resulting in a new threshold for the observer parameter θ that guarantees the exponential convergence of the estimation error and reduces the value of the observer gain [16]. The obtained results are compared with the standard high-gain observer to further show the efficiency and superiority of the proposed technique.

II. Mathematical Modeling

The SI model splits the population into two groups, the susceptible individuals who may contract the disease and the infected individuals who may spread the disease to the susceptible. Once a susceptible becomes infected, he or she moves into the infected group, increasing the size of the infected class and decreasing the size of the susceptible class as described in the flow diagram depicted in Figure 1.

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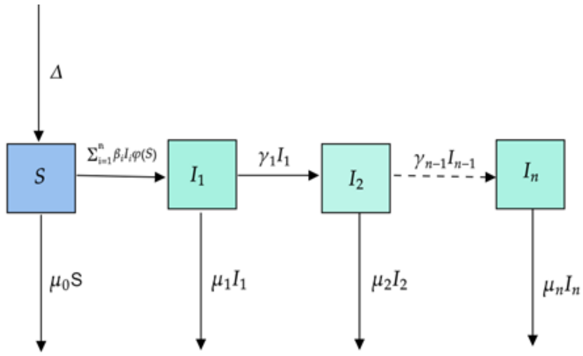

Fig. 1: The transfer diagram for SI model.

In SI modeling some assumptions are made about the population and disease. First, each person in the susceptible population is assumed to be equally likely to transmit the disease through contact with an infected individual and once a person is infected, they cannot recover; they remain in the infected class forever [17]. Second, the length of the disease outbreak is short compared with the average person's lifespan, so death is not a factor [18]. Therefore, this model can be applied to diseases for which individuals never recover and for which disease spread is relatively quick. During an epidemic, the population is divided into two compartments: the healthy individuals likely to catch the disease and the infected ones, denoted by S and I, respectively, the total population is represented by N.

Let us consider a class of stage-structured SI model with n infectious stages:

$$\begin{aligned} \dot{S} &= \Delta - \sum\nolimits_{i=1}^{n} \beta_i I_i \varphi(S) - \mu_0 S \\ \dot{I}_1 &= \sum\nolimits_{i=1}^{n} \beta_i I_i \varphi(S) - (\mu_1 + \gamma_1) I_1 \\ \dot{I}_2 &= \gamma_1 I_1 - (\mu_2 + \gamma_2) I_2 \\ &\vdots \\ \dot{I}_{n-1} &= \gamma_{n-2} I_{n-2} - (\mu_{n-1} + \gamma_{n-1}) I_{n-1} \\ \dot{I}_n &= \gamma_{n-1} I_{n-1} - \mu_n I_n \end{aligned} \tag{1}$$

where Δ is the recruitment and β_i is the per capita contact in the compartment I_i. The function φ is assumed to be continuous, positive and increasing that models the exposure of susceptible individuals to contacts with infectious ones, for instance, one can use $\varphi(S) = S^p$ or $\varphi(S) = \frac{S}{1+aS}$ (with $a > 0$) to take into account saturation effects. The parameter μ_0 is the natural death rate of the susceptible individuals and μ_i is the death rate of the infected individuals in stage i, in general $\mu_i = \mu_0 + d_i$ with d_i being the additional disease induced mortality rate and γ_i is the transition rate from stage i to $i+1$.

Let $x(t) = \begin{bmatrix} S(t) & I_1(t) & \dots & I_n(t) \end{bmatrix}^\top$ be the state vector of the system (1) and suppose that we can only measure the level of infection in the last stage, i.e., the measurable output of the system is $y(t) = I_n(t)$. The aim of this work is to derive estimates $\hat{S}(t)$ and $\hat{I}_i(t)$ satisfying $\lim_{t\to\infty} \big(\hat{S}(t) - S(t)\big) = 0$ and $\lim_{t\to\infty} \big(\hat{I}(t) - I(t)\big) = 0$ with only the knowledge of $I_n(t)$, $\forall t \geq 0$ using high-gain observer techniques.

III. High-gain observer design

A. Standard high-gain observer

Let us recall some basic concepts related to the standard high-gain observer. Given a system written under the following form:

$$\begin{cases} \dot{x} = Ax + Bf(x) \\ y = Cx \end{cases}, \tag{2}$$

where

$$B = \begin{bmatrix} 0 & \dots & 0 & 1 \end{bmatrix}^T, \quad C = \begin{bmatrix} 1 & 0 & \dots & 0 \end{bmatrix} \tag{3}$$

and the state matrix A is defined by

$$(A)_{i,j} = \begin{cases} 1 & \text{if} \quad j = i+1 \\ 0 & \text{if} \quad j \neq i+1 \end{cases}. \tag{4}$$

$f : \mathbb{R}^n \to \mathbb{R}$ is a nonlinear function satisfying the Lipschitz condition formulated under the following form:

$$\Big| f(x_1 + \Delta_1, \dots, x_n + \Delta_n) - f(x_1, \dots, x_n) \Big| \leq k_f \sum_{j=1}^{n} |\Delta_j| \,. \tag{5}$$

Then, the standard high-gain observer is defined as

$$\dot{\hat{x}} = A\hat{x} + Bf(\hat{x}) + L\Big(y - C\hat{x}\Big) \tag{6}$$

where, the observer gain L is written under the following form:

$$L := \mathrm{T}(\theta)K, \;\; \theta \geq 1 \tag{7}$$

and

$$\mathrm{T}(\theta) := \mathrm{diag}\Big(\theta, \dots, \theta^n\Big) \text{ with } K \in \mathbb{R}^{n \times p}.$$

Using a transformed estimation error

$$\hat{\tilde{x}} := \mathrm{T}^{-1}(\theta)\tilde{x}, \tag{8}$$

where $\tilde{x} = x - \hat{x}$, the dynamics of the error $\hat{\tilde{x}}$ are given by:

$$\dot{\hat{\tilde{x}}} = \theta\big(A - KC\big)\hat{\tilde{x}} + \frac{1}{\theta^n} B\Delta f. \tag{9}$$

From the Lipschitz condition (5) and the fact that $\theta \geq 1$, we can show as in [19] that there always exists a positive scalar constant k_f, independent of θ, such that

$$\|\mathrm{T}^{-1}(\theta)B\Delta f\| \leq k_f \|\hat{\tilde{x}}\|. \tag{10}$$

Following the high-gain methodology, the following theorem is derived.

Theorem 1 ([20]): If there exist $P > 0$, $\lambda > 0$, Y of appropriate dimensions, such that

$$A^T P + PA - C^T Y - Y^T C + \lambda I < 0, \tag{11}$$

then the observer converges exponentially to zero for

$$\theta > \max\{1, \frac{2k_f \lambda_{\max}(P)}{\lambda}\}, \tag{12}$$

and

$$K = P^{-1}Y^T$$

where $\lambda_{\max}(P)$ is the largest eigenvalue of the matrix P.

Proof: For more details about the proof of this theorem, we refer the reader to [20], [19], [21]. ■

B. State augmentation based high-gain observer

This novel approach has been proposed in [16], indeed, as demonstrated in this work, if the nonlinear function $f(.)$ satisfies the condition

$$\frac{\partial f}{\partial x_j}(x) \equiv 0, \forall\ j > n - j_s \tag{13}$$

for a given $j_s \geq 0$, then the Lipschitz inequality (10) becomes

$$\|\mathrm{T}^{-1}(\theta)B\Delta f\| \leq \frac{k_f}{\theta^{j_s}}\|\hat{\tilde{x}}\|. \tag{14}$$

It follows that the high-gain inequality (12) becomes

$$\theta > \left(\frac{2k_f\lambda_{\max}(P)}{\lambda}\right)^{\frac{1}{1+j_s}} \triangleq \theta_0^{\frac{1}{1+j_s}}. \tag{15}$$

This new threshold on θ is significantly reduced due to the power $\frac{1}{1+j_s}$. Hence, instead of $\mathrm{T}(\theta)$ in L, we have $\mathrm{T}(\theta)^{\frac{1}{1+j_s}}$. The main idea is to transform a system of dimension n, from its original coordinates, into a new system whose dimension is $n+j_s$, where the new nonlinear function does not depend on j_s last components of the new state. The following theorem is derived.

Theorem 2 ([16]): Let us consider the uniformly observable system:

$$\begin{cases} \dot{x}=\psi(x,u) \\ y=\phi(x,u) \end{cases} \tag{16}$$

Assume that there exists a state transformation given as:

$$\begin{aligned} \Psi : \mathbb{R}^n &\to \mathbb{R}^{n+j_s} \\ x &\to z = \Psi(x) \end{aligned} \tag{17}$$

which transforms the system (16) into the following:

$$\begin{cases} \dot{z} = A_\Psi z + B_\Psi f_\Psi(z) \\ y = C_\Psi z \end{cases} \tag{18}$$

where A_Ψ, B_Ψ, and C_Ψ have the same structure than A, B, and C, respectively, but with dimension $n + j_s$.
We also have:

$$f_\Psi(z) \triangleq f_\Psi(z_1,\ldots,z_n)\ \Leftrightarrow\ \frac{\partial f_\Psi}{\partial z_j}(z) \equiv 0, \forall\ j > n. \tag{19}$$

Consider the state observer described by (20)

$$\begin{cases} \dot{\hat{z}} = A_\Psi\hat{x} + B_\Psi f_\Psi(\hat{z}) + L_\Psi\Big(y - C_\Psi\hat{z}\Big) \\ \hat{x} = \Phi(\hat{z}), \end{cases} \tag{20}$$

where Φ is a continuous left invert of the embedding Ψ satisfying $x = \Phi(z)$ and $L_\Psi \triangleq \mathrm{T}_\Psi(\theta)K_\Psi$, with $\mathrm{T}_\Psi(\theta) \triangleq \mathrm{diag}(\theta,\ldots,\theta^{n+j_s})$. If there exist $P > 0$, $\lambda > 0$, Y, and $\theta \geq 1$ such that:

$$A_\Psi^\top P + PA_\Psi - C_\Psi^\top Y - Y^\top C_\Psi + \lambda I < 0, \tag{21}$$

$$K_\Psi \triangleq P^{-1}Y^\top, \tag{22}$$

$$\theta > \sqrt[1+j_s]{\frac{2k_{f_\Psi}\lambda_{\max}(P)}{\lambda}} \triangleq \theta_\Psi^{\frac{1}{1+j_s}}, \tag{23}$$

then the estimation error $\tilde{x} = x - \hat{x}$ converges exponentially towards zero.

Proof: For the proof we refer the reader to [16]. ■

Remark 1: One way of transforming system (2) into a higher dimensional system is by adding a chain of intergrators (j_s integrators) and keeping the properties stated in Theorem 2.

IV. ISS WITH RESPECT TO MEASUREMENT NOISE

In this section we compare the properties of the standard high-gain observer (6) and the proposed observer (20) with respect to measurement noises. To this end, we consider the following system where the measurement is corrupted by a bounded disturbance:

$$\begin{aligned} \dot{x} &= Ax + Bf(x) \\ y &= Cx + \nu \end{aligned} \tag{24}$$

where ν represents the disturbance affecting the measurement y. We will show that an upper bound on the estimation error, in a ISS sens with appropriate norms, can be ensured by the observers (6) and (20), respectively. However, we will demonstrate that the use of state augmentation approach can lead to a smaller bound on the estimation error, compared to the one we get using the standard high-gain observer.

A. ISS property with standard high-gain observer

Consider system (24) and the standard high-gain observer (6), then the transformed error dynamics system is given as

$$\dot{\hat{\tilde{x}}} = \theta\underbrace{(A - KC)}_{A_K}\hat{\tilde{x}} + \frac{1}{\theta^n}B\Delta f - K\nu. \tag{25}$$

Therefore, the observer parameters designed by Theorem 1 ensure an ISS property as introduced in the following proposition.

Proposition 1: Assume that there exist a symmetric positive definite matrix P, a positive constant λ and a matrix Y of appropriate dimensions such that the inequalities (11)-(12) hold. Then with the observer gain L given in (7), there exists a positive constant α such that the estimation error $\tilde{x}(t)$ verifies the following ISS conditions:

$$\|\tilde{x}(t)\| \leq \theta^{n-1}\sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}\|\tilde{x}_0\|e^{-\frac{\beta}{2}t} + \theta^n\sqrt{\frac{\gamma(1-e^{-\beta t})}{\beta\lambda_{\min}(P)}}\sup_{s\in[0,t]}\|\nu(s)\|, \tag{26a}$$

$$\lim_{t\to+\infty}\|\tilde{x}(t)\| \leq \theta^n\sqrt{\frac{\gamma}{\beta\lambda_{\min}(P)}}\sup_{s\in[0,\ +\infty]}\|\nu(s)\|, \tag{26b}$$

where

$$\beta = \frac{\theta\lambda - 2k_f\lambda_{\max}(P) - \alpha}{\lambda_{\max}(P)},\ \gamma = \frac{\|Y\|^2}{\alpha}. \quad (27)$$

Proof: The proof is omitted. ■

B. ISS property with the proposed high-gain observer

By a straightforward analogy, we know that we can apply the results of Proposition 1 on the augmented system (18). That is the observer (20) ensures a similar ISS property than that in (26a)-(26b). However, the presence of the power $\frac{1}{1+j_s}$ in the case of augmented state based observer (20), especially in the high-gain threshold condition (23), allows reducing significantly the values of the observer gain. For instance, for an $\epsilon > 0$, if we take $\theta = \theta_0 + \epsilon$ in the standard high-gain, then according to (26b), the upper bound of the estimation error satisfies:

$$\lim_{t\to+\infty} \|\tilde{x}(t)\| \le (\theta_0+\epsilon)^n \sqrt{\frac{\gamma}{\beta\lambda_{\min}(P)}} \sup_{s\in[0,\ +\infty]} \|\nu(s)\| \quad (28)$$

while with the augmented approach for $\theta = \theta_\Psi^\epsilon \triangleq \theta_\Psi + \epsilon$, we get

$$\lim_{t\to+\infty} \|\tilde{x}(t)\| \le (\theta_\Psi^\epsilon)^{\frac{n+j_s}{1+j_s}} \sqrt{\frac{\gamma_\Psi}{\beta_\Psi\lambda_{\min}(P_\Psi)}} \sup_{s\in[0,\ +\infty]} \|\nu(s)\|. \quad (29)$$

V. Application to a two-stage-structured epidemic model

In this section we study the special case where infected batch I is made up of two compartments, namely, the infected in the first stage of the disease and the infected in the terminal phase of the infection, denoted I_1 and I_2, respectively. The corresponding SI model is given as follows,

$$\begin{cases} \dot{S} = \Delta - (\beta_1 I_1 + \beta_2 I_2)S - \mu_0 S, \\ \dot{I}_1 = (\beta_1 I_1 + \beta_2 I_2)S - (\mu_1+\gamma)I_1, \\ \dot{I}_2 = \gamma I_1 - \mu_2 I_2 \end{cases} \quad (30)$$

where

- S, I_1, I_2 represent the compartments of susceptible, first stage infected and second stage infected, respectively.
- $\Delta, \mu, \alpha, 1/\gamma$ are recruitment, mortality rate, recovery rate and time taken for an early-stage infected to become in the final phase of infection, respectively.

Let $x(t) = \begin{bmatrix} S(t) & I_1(t) & I_2(t) \end{bmatrix}^\top$ and $y = I_2 \in \mathbb{R}^3$, so the model given by (30) has the following form:

$$\Gamma : \begin{cases} \dot{x} = f(x) \\ y = h(x). \end{cases} \quad (31)$$

When system (30) is observable, the map $\Psi : x \to \Psi(x)$ is a diffeomorphism and,

$$z = \Psi(x) = \begin{bmatrix} I_2 \\ \gamma I_1 - \mu_2 I_2 \\ \gamma(\beta_1 I_1 + \beta_2 I_2)S - \gamma m I_1 + \mu_2^2 I_2 \end{bmatrix}, \quad (32)$$

where

$$m = \mu_1 + \mu_2 + \gamma.$$

With the change of variable $z = \Psi(S, I_1, I_2)$, the expression of Ψ^{-1} is

$$\Psi^{-1} : z \to \begin{cases} \dfrac{\mu_2(\mu_1+\gamma)z_1 + (\mu_1+\mu_2+\gamma)z_2 + z_3}{(\mu_2\beta_1 + \gamma\beta_2)z_1 + \beta_1 z_2}, \\ \frac{\mu_2}{\gamma} z_1 + \frac{1}{\gamma} z_2 \\ z_1 \end{cases} \quad (33)$$

Then, system given by (30) is rewritten in the following triangular form:

$$\Gamma' : \begin{cases} \dot{z} = F'(z) = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} z_2 \\ z_3 \\ \varphi(z) \end{bmatrix}, \\ y = Cz = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \end{cases} \quad (34)$$

This latter form allows us to make use of the high-gain observer techniques presented in the previous section to estimate the evolution of susceptible individuals $S(t)$ and infected individuals $I(t)$. The parameters used in these simulations are given as follows: $\Delta = 40$, $\beta_1 = 0.01$, $\beta_2 = 0.15$, $\mu_0 = \mu_1 = 0.01$, $\mu_2 = 0.025$ and $\gamma = 0.02$.

In order to test the efficiency of the proposed observer given in (20), we performed a batch of numerical simulations to compare with the standard high-gain observer in the presence of high-frequency measurement noise, numerically taken as Gaussian distributed noise with zero mean and standard deviation of 0.01. First, we design a standard high-gain observer for the system (34) following Theorem 1, then by using Matlab & YALMIP, we can obtain the value of the observer gain K and the tuning parameter θ. Next, by augmenting the state of the system following the transformation described in Section III-B, we obtain the values for the corresponding gain K_ψ and the new observer parameter θ_ψ. The results are summarized in Table I where we can clearly notice that the proposed observer provides lower gain with smaller value of the tuning parameter compared to the standard high-gain observer.

TABLE I: Simulation results for the two observers.

Standard high-gain observer				
θ	K			
24.2308	8.8781	15.7945	7.9158	
State augmentation approach $j_s = 1$				
θ_ψ	K_ψ			
4.7056	6.0195	12.0353	11.4500	4.5601

In Figure 2, the absolute value of the estimation error $\|\hat{x}_{i,HG} - x_i\|$ and $\|\hat{x}_{i1} - x_i\|$ are plotted for $i = 1 \ldots 3$, where $\hat{x}_{i,HG}$ denotes the state estimate using the standard high-gain observer and $\hat{x}_{i1}$ the state estimate using the proposed observer. An additional zero mean Gaussian disturbances with standard deviation of 0.01 are applied to the output measurements during time t= 5s and t=10s. It is clear that both observers converge towards the true state with roughly the same speed, however, our proposed observer has better transient response, in particular for the states x_2 and x_3. We also note that our proposed observer is less sensitive to measurement noises since the peaking phenomenon that is typical of the standard high-gain observer is prevented using our proposed structure as shown particularly by the plot of $\|\hat{x}_3 - x_3\|$.

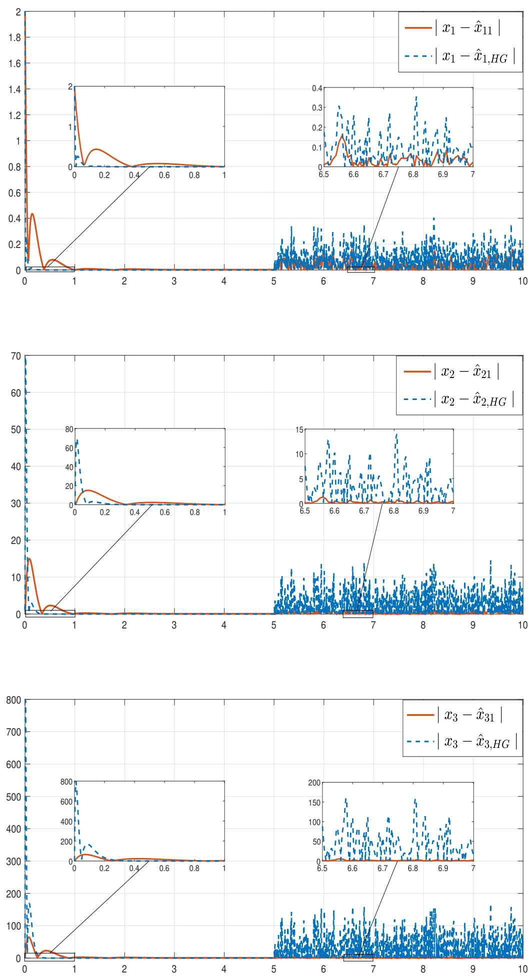

Fig. 2: Absolute values of estimation errors with the presence of measurement noise.

VI. Conclusion

In this paper, we considered a SI epidemiological model with two stages of infected individuals. For this class, we have proposed simple and easy implementable observer based on the high-gain approach. This proposed observer is characterized by two main features, namely better transient performances and less sensitivity to measurement noises with respect to the standard high-gain observer. The stability and robustness properties are established analytically using the theory of input-to-state stability. The efficiency of the obtained results is illustrated through numerical simulations.

References

[1] W. E. R. Team, “Ebola virus disease in west africa—the first 9 months of the epidemic and forward projections,” *New England Journal of Medicine*, vol. 371, no. 16, pp. 1481–1495, 2014.

[2] A. J. Kucharski, T. W. Russell, C. Diamond, Y. Liu, J. Edmunds, S. Funk, R. M. Eggo, F. Sun, M. Jit, J. D. Munday *et al.*, “Early dynamics of transmission and control of COVID-19: a mathematical modelling study,” *The lancet infectious diseases*, vol. 20, no. 5, pp. 553–558, 2020.

[3] M. Egger, L. Johnson, C. Althaus, A. Schöni, G. Salanti, N. Low, and S. L. Norris, “Developing who guidelines: time to formally include evidence from mathematical modelling studies,” *F1000Research*, vol. 6, 2017.

[4] N. P. Jewell, J. A. Lewnard, and B. L. Jewell, “Predictive mathematical models of the COVID-19 pandemic: underlying principles and value of projections,” *Jama*, vol. 323, no. 19, pp. 1893–1894, 2020.

[5] D. Adam, “Special report: The simulations driving the world’s response to COVID-19.” *Nature*, vol. 580, no. 7802, pp. 316–319, 2020.

[6] D. Chen, B. Moulin, and J. Wu, *Analyzing and modeling spatial and temporal dynamics of infectious diseases*. John Wiley & Sons, 2014.

[7] A. Iggidr and M. O. Souza, “State estimators for some epidemiological systems,” *Journal of mathematical biology*, vol. 78, no. 1, pp. 225–256, 2019.

[8] S. Audoly, L. D’Angio, M. P. Saccomani, and C. Cobelli, “Global identifiability of linear compartmental models−a computer algebra algorithm,” *IEEE Transactions on Biomedical Engineering*, vol. 45, no. 1, pp. 36–47, 1998.

[9] J. A. Jacquez and P. Greif, “Numerical parameter identifiability and estimability: Integrating identifiability, estimability, and optimal sampling design,” *Mathematical Biosciences*, vol. 77, no. 1-2, pp. 201–227, 1985.

[10] T. Toni, D. Welch, N. Strelkowa, A. Ipsen, and M. P. Stumpf, “Approximate bayesian computation scheme for parameter inference and model selection in dynamical systems,” *Journal of the Royal Society Interface*, vol. 6, no. 31, pp. 187–202, 2009.

[11] A. Abdelhedi, D. Boutat, L. Sbita, and R. Tami, “Extended observer to estimate the spreading of contagious disease,” in *2014 European Control Conference (ECC)*, 2014, pp. 1843–1848.

[12] J. Alvarez-Ramirez, M. Meraz, and J. X. Velasco-Hernandez, “Feedback control of the chemotherapy of HIV,” *International Journal of Bifurcation and Chaos*, vol. 10, no. 09, pp. 2207–2219, 2000.

[13] D. Bichara, N. Cozic, and A. Iggidr, “On the estimation of sequestered infected erythrocytes in plasmodium falciparum malaria patients,” *Mathematical Biosciences and engineering*, vol. 11, no. 4, pp. 741–759, 2014.

[14] R. Tami, D. Boutat, G. Zheng, and F. Kratz, “Parameters and states estimation for Dengue epidemic model,” in *2014 European Control Conference (ECC)*, 2014, pp. 528–533.

[15] A. Ibeas, M. de la Sen, S. Alonso-Quesada, and I. Zamani, “Stability analysis and observer design for discrete-time SEIR epidemic models,” *Advances in Difference Equations*, vol. 2015, no. 1, pp. 1–21, 2015.

[16] D. Bouhadjra, A. Zemouche, A. Alessandri, and P. Bagnerini, “High-gain nonlinear observer using system state augmentation,” in *Proc. of the 59th IEEE Conference on Decision and Control*, Jeju, Korea (South), 2020.

[17] L. J. Allen, “Some discrete-time SI, SIR, and SIS epidemic models,” *Mathematical Biosciences*, vol. 124, no. 1, pp. 83–105, 1994. [Online]. Available: https://www.sciencedirect.com/science/article/pii/0025556494900256

[18] R. Smith, *Modelling Disease Ecology with Mathematics*. American Institute of Mathematical Sciences, 2008.

[19] A. Alessandri and A. Rossi, “Time-varying increasing-gain observers for nonlinear systems,” *Automatica*, vol. 49, no. 9, pp. 2845–2852, 2013.

[20] J. Gauthier and I. Kupka, “Observability and observers for nonlinear systems,” *SIAM J. on Control and Optimization*, vol. 32, no. 4, pp. 975–994, 1994.

[21] A. Alessandri and A. Rossi, “Increasing-gain observers for nonlinear systems: stability and design,” *Automatica*, vol. 57, no. 7, pp. 180–188, 2015.