

APPLICATION OF NONLINEAR DISTURBANCE DECOUPLING TO ACTIVE CAR STEERING

N.P.I. Aneke*, J. Ackermann†, T. Bunte‡, H. Nijmeijer*‡

* Eindhoven University of Technology, Faculty of Mechanical Engineering,
P.O.Box 513, 5600 MB Eindhoven, The Netherlands
Fax : +31 40 246 4618 and e-mail : edo@wfw.wtb.tue.nl

† DLR Oberpfaffenhofen, Institute of Robotics and System Dynamics,
P.O.Box 1116, D-82230 Wessling, Germany

Fax: +49 8153 28 1847 and e-mail : Juergen.Ackermann@dlr.de, Tilman.Buente@dlr.de

‡ University of Twente, Faculty of Mathematical Sciences,
P.O.Box 217, 7500 AE Enschede, The Netherlands

Fax: +31 53 434 0733 and e-mail : H.Nijmeijer@math.utwente.nl

Keywords : Vehicle dynamics control, active steering, nonlinear disturbance decoupling, vehicle safety.

Abstract

A control design approach is presented for active steering of front wheel steered vehicles. The approach consists of the application of disturbance decoupling via state-derivative feedback. Simulations with the resulting controller show that the use of state-derivative information may considerably enhance vehicle safety, in particular under adverse operating conditions such as icy roads.

to the feedback loop we are indirectly providing the controller with disturbance information, and we are actually solving a variant of the disturbance decoupling problem with disturbance measurement (DDPm).

In section 2 we present the vehicle model which is used for the synthesis of active steering control. In section 3 we consider disturbance decoupling by state-derivative feedback and the conditions under which asymptotic stability of the closed loop dynamics can be guaranteed. In section 4 these results are used to derive a controller for active steering. In section 5 the resulting controller is investigated by simulations with a nonlinear vehicle model. Finally, conclusions are given in section 6.

1 Introduction

Advanced vehicle control covers nearly every degree of freedom in vehicle dynamics, and due to increasing safety requirements in the automobile industry even more progress will be made in this field. The newest technologies for improving driving safety consist of vehicle stability control systems that improve the directional stability and steerability of a vehicle.

In [1] a control law was presented for active car steering which robustly decouples the yaw mode from the lateral dynamics of the car. In driving experiments with side-wind and μ -split braking the safety advantages were demonstrated which result from the disturbance attenuation properties of this control.

Here we present an alternative control design approach for active steering control. The objective is the design of a controller that renders the yaw dynamics of the vehicle independent of disturbance torques. Although the yaw dynamics can not be disturbance decoupled by means of a static or dynamic state feedback, it will be shown that this is possible when using state-derivative information in the feedback loop. By adding state-derivative information

2 Vehicle model

Consider a vehicle with mass m and moment of inertia J about its vertical axis. Its yaw dynamics are influenced by disturbance torques M_{zD} around its vertical axis, resulting from side wind, a flat tire or braking on icy roads. The objective for active steering is the attenuation of this disturbance torque.

A vehicle model in which the wheels of the vehicle are assumed to be lumped into one wheel along the centerline of the vehicle is described in [2]. The longitudinal velocity v_x is available from the rear-wheel ABS, and will be assumed to be constant. Therefore the states of this *nonlinear single-track model* are the yaw rate r and the side-slip angle β_F at the front axle, see Figure 1. The input to the vehicle is the front wheel steering angle $\delta_F = \delta_S + \delta_C$. It is composed of a steering angle δ_S commanded by the driver and a steering angle δ_C from the active steering system.

The lateral force F_{yF} at the front axle is

$$F_{yF}(x, \delta_F) = \cos \delta_F F_{sF}(\delta_F - \beta_F) + \sin \delta_F F_{lF},$$

with state-vector $x = [\beta_F, r]^T$ and longitudinal tire force F_{lF} . This lateral force F_{yF} is linearized for small devia-

This constraint is clearly not feasible since the matrix $M = (I - g(x)\alpha^d(x))$ is singular. The requirement of well-posedness is thus a necessary condition for having a well-defined closed loop system (3, 4).

The problem of disturbance decoupling by state derivative feedback may be formulated as follows

Problem 3.1 (DDPd) Consider the system (3). Under what conditions does there exist a regular and well-posed feedback (4) such that in the closed loop dynamics the disturbance q does not influence the output y , no matter how w is chosen.

As mentioned in the introduction, seeking a disturbance decoupling control law (4) amounts to solving a variant of the disturbance decoupling problem with disturbance measurements (DDPm). In the DDPm, see [6], instead of (4) a feedback is to be found of the form

$$u = \alpha(x) + \beta(x)w + \gamma(x)q, \quad u \in \mathbb{R}. \quad (6)$$

The following result from [5] solves the DDPd as a variant of the DDPm. For the definitions of relative degree and the Lie-derivative $L_f h(x)$ of a function $h : \mathbb{R}^n \rightarrow \mathbb{R}$ along a vector field f on $\mathbb{R}^n$, we refer to [6].

Theorem 3.1 Consider the system (3) with input relative degree ρ and disturbance relative degree σ . The disturbance decoupling problem by state derivative feedback (DDPd) is solvable if and only if either

- (i) $g(x), e(x)$ linearly independent and $\rho \leq \sigma$
- (ii) $g(x), e(x)$ linearly dependent and $\rho = \sigma = \infty$

Corollary 3.1 Consider the system (3) and assume that the input relative degree ρ is finite and (5) holds. Suppose that the feedbacks $\alpha(x), \beta(x)$ and $\alpha^d(x)$ are chosen such that

$$\begin{aligned} \frac{\alpha(x) + \alpha^d(x)f(x)}{1 - \alpha^d(x)g(x)} &= -\frac{L_f^\rho h(x)}{L_g L_f^{\rho-1} h(x)} \\ \frac{\beta(x)}{1 - \alpha^d(x)g(x)} &= \frac{1}{L_g L_f^{\rho-1} h(x)} \\ \frac{\alpha^d(x)e(x)}{1 - \alpha^d(x)g(x)} &= -\frac{L_e L_f^{\rho-1} h(x)}{L_g L_f^{\rho-1} h(x)}. \end{aligned} \quad (7)$$

Then, all feedbacks of the form (cf. (4))

$$u = \alpha(x) + \alpha^d(x)\dot{x} + \beta(x)w + \beta(x)\gamma(h(x), \dots, L_f^{\rho-1} h(x)), \quad (8)$$

solve the disturbance decoupling problem (DDPd). The function $\gamma(h(x), \dots, L_f^{\rho-1} h(x))$ is an arbitrary analytic function and w denotes an arbitrary new input function.

Remark 3.2 Note that the equations (7) are solvable in $\alpha(x), \beta(x)$ and $\alpha^d(x)$.

Besides disturbance decoupling there are additional requirements on the closed loop system, such as asymptotic stability or tracking. From linear control theory it is well-known that the stability respectively instability of the zero dynamics is very important for design purposes, and this may be expected to be true in the nonlinear case as well, see [6].

Consider the SISO nonlinear system (3) with equilibrium $\bar{x}$ and input relative degree ρ at $\bar{x}$. The zero dynamics, see [6], are the dynamics under the restriction $y(t) = 0 \forall t$, provided that $q = 0$. If the DDPm is solvable, then $\rho \leq \sigma$ and we have $y^{(k)} = L_f^k h(x)$ for $k = 0, \dots, \rho-1$. Therefore the zero dynamics must evolve on the subset

$$Z^* = \{x \in \mathbb{R}^n : L_f^k h(x) = 0, k = 0 \dots \rho-1\}.$$

The ρ -th derivative of the output y is

$$y^{(\rho)} = L_f^\rho h(x) + L_g L_f^{\rho-1} h(x)u + L_e L_f^{\rho-1} h(x)q. \quad (9)$$

The additional constraint $y^{(\rho)} = 0$ shows that the input u^* needed for zeroing the output y is given by

$$u^*(x, q) = -\frac{L_f^\rho h(x)}{L_g L_f^{\rho-1} h(x)} - \frac{L_e L_f^{\rho-1} h(x)}{L_g L_f^{\rho-1} h(x)}q. \quad (10)$$

The zero dynamics of (3) are thus given by

$$\dot{x} = f(x) + g(x)u^*(x, 0), \quad x \in Z^*, \quad (11)$$

In [7] a feedback was proposed for the simultaneous achievement of disturbance decoupling and asymptotic stability of the closed loop system for $q = 0$. This feedback is given by

$$\begin{aligned} u &= \frac{1}{L_g L_f^{\rho-1} h(x)} \left(-L_f^\rho h(x) - L_e L_f^{\rho-1} h(x)q \right. \\ &\quad \left. - c_0 h(x) - c_1 L_f h(x) - \dots - c_{\rho-1} L_f^{\rho-1} h(x) \right) \end{aligned} \quad (12)$$

where $c_0, \dots, c_{\rho-1}$ are real numbers. Introduce the polynomial $p(z) = \sum_{k=0}^{\rho-1} c_k z^k$. In [6, 7] the following sufficient condition for solvability of the DDPm with stability is given. It is in fact the nonlinear equivalent of the well-known result in linear systems theory; Ackermann's formula, see [8].

Proposition 3.2 Suppose that the equilibrium $\bar{x} = 0$ of the zero dynamics of the system (3) is locally asymptotically stable and the polynomial $p(z)$ is Hurwitz. Then the feedback law (12) locally asymptotically stabilizes the equilibrium $\bar{x}$ when $q = 0$.

As is done in linear control theory, a nonlinear system with asymptotically stable zero dynamics is said to be minimum phase. For these systems the simultaneous achievement of disturbance decoupling (DDPd) and asymptotic

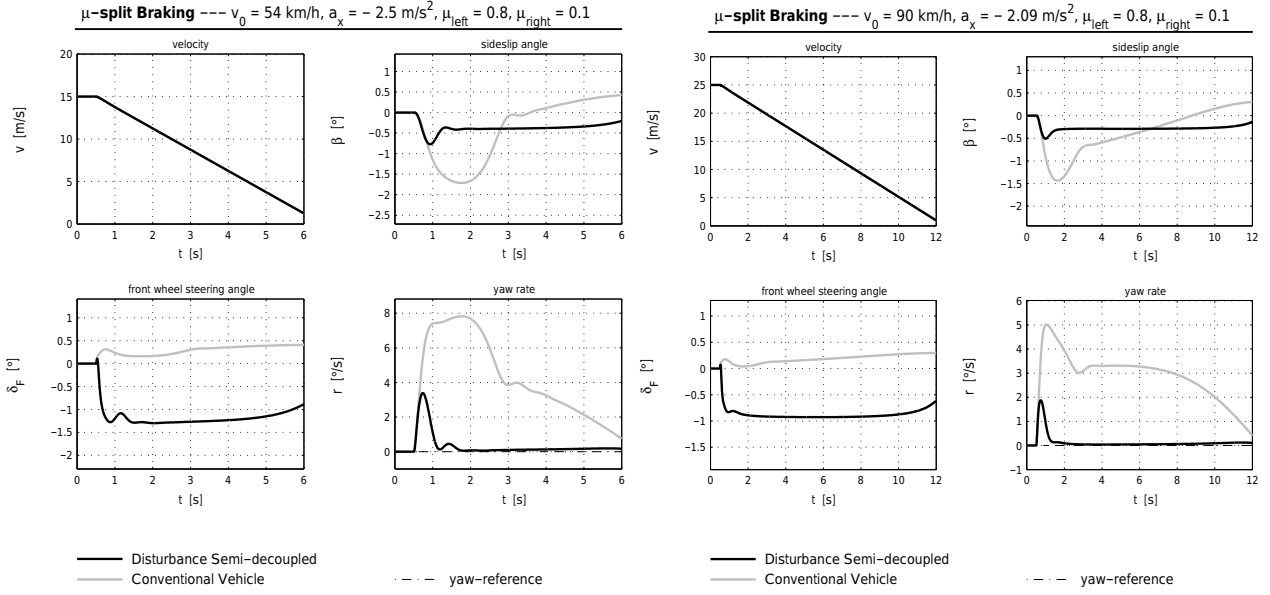


Figure 2: μ -split braking with initial speeds of 54 km/h and 90 km/h respectively

stability of the equilibrium $\bar{x}$ is achieved by simply choosing the function γ in (8) as

$$\begin{aligned} \gamma(y, \dots, y^{(\rho-1)}) &= -c_0 h(x) - c_1 L_f h(x) - \dots \\ &\quad - c_{\rho-1} L_f^{(\rho-1)} h(x) \end{aligned} \quad (13)$$

However the hypothesis that the zero dynamics are asymptotically stable may not be necessary in order to obtain disturbance decoupling and asymptotic stability. In fact, there are systems having unstable zero dynamics in which the simultaneous achievement of these two goals is still possible [6, 7].

4 Application to active steering

As mentioned in the introduction, the objective of the active steering system is to render the yaw rate r of the vehicle independent of the disturbance torque M_{zD} . The vehicle system (2) has input and disturbance relative degree $\rho = \sigma = 1$ and by Theorem 3.1 the DDpd is solvable. The zero-dynamics, thus with $M_{zD} = 0$, are

$$\dot{\beta}_F = -\frac{lc_r}{ml_F} \frac{\cos^2 \beta_F}{v_x} \beta_F - \frac{a_x}{2v_x} \sin(2\beta_F). \quad (14)$$

Consider the positive definite function $V(\beta_F) = \beta_F^2$. For small β_F the time-derivative of $V(\beta_F)$ becomes approximately

$$\dot{V}(\beta_F) = -2 \left(\frac{lc_r}{ml_F} + a_x \right) \frac{\beta_F^2}{v_x}.$$

Given the magnitude of c_r/m , even on an icy road with c_r small, it holds that $a_x > -lc_r/(ml_F)$. When $v_x > 0$ we

have $\dot{V}(\beta_F) < 0, \forall \beta_F \neq 0$, and by the second method of Lyapunov $\beta_F = 0$ is a locally asymptotically stable equilibrium of the zero dynamics (14).

In [5] it is shown that the equations (8) and (13) result in the following controller.

$$\begin{cases} \delta_C = \frac{1}{\mathcal{F}_2(\delta_S, F_{IF})} \left[\beta_F^* + \frac{ml_R}{lc_{F0} \cos \delta_S} \cdot \frac{a_{yP}}{\mu_c} \right] \\ \quad + k_d(x)(\dot{r} - k_p(r - r_{ref})) - H(s)\delta_S + w_d \\ \dot{w}_d = -(r - r_{ref}). \end{cases} \quad (15)$$

The feedback gains k_d and k_p satisfy $k_d \neq 0$ respectively $k_p < 0$. A reference value r_{ref} for the yaw rate is computed by a prefilter $F(s, v_x)$ that gives the controlled vehicle a similar steady-state response to the conventional vehicle [9]. The feedback gain k_p locally asymptotically stabilizes the steady state with yaw rate $r = r_{ref}$. The variable β_F^* denotes the estimated side-slip angle at the front axle and the function $\mathcal{F}_2$, see (1), compensates for the effect of *steering by longitudinal tire-forces*. A filter $H(s)$ has been added to the feedback of δ_S , and makes an improvement of the handling performance of the controlled vehicle possible.

Unlike in the linear case, see [5], we have not been able to completely decouple the disturbance M_{zD} . We have therefore added an integrating action w_d , and the controller (15) should be referred to as the nonlinear disturbance *semi-decoupling* controller. More importantly, the controller (15) is not robust with respect to the uncertain road adhesion factor μ . Therefore μ has been replaced by a control parameter μ_c that may be scheduled against the velocity v_x .

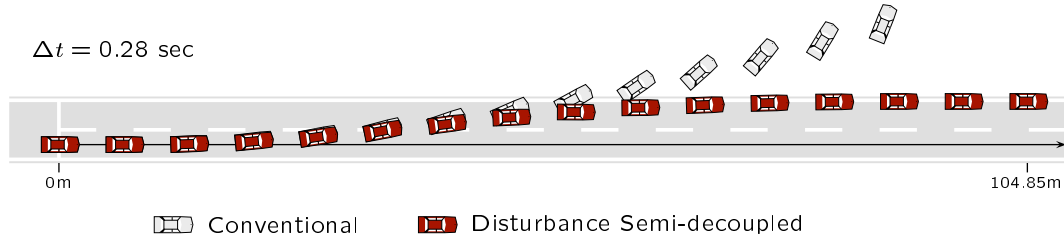


Figure 3: Lane-change maneuver on an icy road: single sinusoidal steering wheel input $\hat{\delta}_L = 100^\circ$ at $v = 90$ km/h with road adhesion coefficient $\mu = 0.3$

Remark 4.3 We have been content with asymptotic stabilization of the steady-state with $r = r_{ref}$. One might want to achieve asymptotic output tracking, see ([6, 7]). In that case, asymptotic stability of the tracking dynamics for $y \equiv r_{ref}$ is needed. However, tracking control requires feed-forward of $\dot{r}_{ref}$ with unity gain, resulting in rather large controller steering angles δ_C . In general this will result in poor performance of the tracking controller due to saturation of the tire side-forces, since no saturation is incorporated in the linearization (1) of the lateral force F_{yF} at the front axle.

5 Simulation results

The simulations are performed using a 27-th order nonlinear double-track model that has been developed in [10]. The nonlinear tire characteristics are described by the HSRI tire model [11]. This nonlinear double-track model has been extended with a PID cruise speed controller that distributes the drive/brake torques scheduled by the wheel slips. This makes it possible to incorporate the effect of the anti-lock braking system (ABS).

m	1916 kg	l_F	1.514 m
J	3654 kg m ²	l_R	1.323 m
c_{F0}	101600 N rad ⁻¹	l_P	1.441 m
c_{R0}	213800 N rad ⁻¹	l_L	16.2

Table 1: Data for the single-track model

The front steering angle of the conventional car $\delta_S = \delta_L/l_L$ is completely determined by the steering wheel angle δ_L . In modelling the actuator dynamics, we assume a third order actuator model

$$G_a(s) = \frac{\omega_a}{2s + \omega_a} \cdot \frac{\omega_a^2}{s^2 + \sqrt{2}\omega_a s + \omega_a^2}, \quad w_a = 2\pi f,$$

where $f = 10$ Hz. The single-track model data are taken from Table 1 and correspond to a BMW 735i. The controller parameters are empirically set to

$$k_d = -1/20, \quad k_p = -7, \quad \mu_c = 0.95.$$

The first maneuver to be investigated is μ -split braking. When a braking maneuver is performed on a road surface with asymmetric road adhesion coefficients, then this results in so-called μ -split braking. This means that the difference between the braking forces at the left and right wheels of the vehicle induce a disturbance torque around the vertical axis which needs to be compensated. Figure 2 shows μ -split braking maneuvers of the conventional and controlled vehicle at two different initial velocities. At $t = 0.5$ s braking is started and the induced disturbance torque results in an undesired yaw motion of the vehicle. From the course of the yaw rate r it can be clearly seen how the disturbance semi-decoupling controller assists the driver in the attenuation of the disturbance impact. Since the disturbance semi-decoupling controller is provided with disturbance information, it is able to react fast to the disturbance torque and it preserves the directional stability of the vehicle.

Apart from the directional stability, the handling performance and steerability of the controlled vehicle is very important. In the automotive industry, a standard test for the analysis of the handling performance is an emergency lane-change maneuver. A single lane-change maneuver can be simulated by applying a single sinusoidal wave at the steering wheel. Here, a single sinusoidal wave

$$\delta_L(t) = \begin{cases} \hat{\delta}_L \sin(\pi(t - 0.2)) & \text{if } t \in (0.2 \text{ s}, 2.2 \text{ s}) \\ 0 & \text{else} \end{cases}$$

with amplitude $\hat{\delta}_L$ is applied to the steering wheel.

As mentioned in the previous section, the disturbance decoupling controller is not robust with respect to μ . Therefore an important aspect to be investigated is the sensitivity of the controller (15) to variations of the parameter μ . Therefore we consider a severe lane-change maneuver on an icy road surface, i.e. with $\mu = 0.3$. In Figure 3 the result is presented in a true-scale stroboscopic visualization. The safety advantage of active steering control over conventionally steered vehicles is clearly shown. The tire side forces of the conventional vehicle reach their saturation limit and the vehicle loses its directional stability. The controlled vehicle, however, adjusts the steer-

ing angle δ_F such that the directional stability is preserved and the tire forces do not reach their saturation limit.

6 Conclusions

In this paper a control design approach based on an application of nonlinear disturbance decoupling by state-derivative feedback was presented for active steering control. Unlike in the linear case the resulting controller does not achieve complete disturbance decoupling, and it is not robust with respect to the changing road adhesion coefficient. However, the simulations have shown that the presented controller may considerably enhance the safety of motion, in particular under adverse conditions such as icy roads.

The main advantage of the presented controller is its ability to react fast to disturbances that affect the vehicle dynamics. However, the influence of actuator rate limitations has not been considered, while the use of the yaw acceleration signal in the feedback loop of the controller may put too heavy load on the actuator.

On the other hand, the use of the yaw acceleration $\dot{r}$, which can be computed using noise-sensitive accelerometers, may cause severe noise problems. Therefore advanced accelerometers or sophisticated filtering techniques may be needed to overcome these noise problems.

Acknowledgements

This work was done with financial support from the European Science Foundation (ESF) in the framework of the Scientific Programme on Control of Complex Systems (COSY).

References

- [1] Ackerman J., "Robust control prevents car skidding", *IEEE Control Systems Magazine*, Bode Lecture 1996, **17**, pp. 23-31, (1997).
- [2] Rieker P., Schunck T.E., "Zur Fahrmechanik des gummibereiteten Kraftfahrzeugs", *Ingenieur Archiv*, **11**, pp. 210-224, (1940).
- [3] Mari J., "Rational Modeling of Time Series and Applications of Geometric Control", *Phd. thesis*, Royal Institute of Technology, Stockholm, Sweden, (1998).
- [4] Mari J., "Use of state-derivatives for disturbance rejection and its application to automatic control of vehicles", Tech. report DLR Oberpfaffenhofen, **IB 515-95-10**, (1995).
- [5] Aneke N.P.I., "Application of disturbance decoupling to active car steering", University of Twente, Faculty of Mathematical Sciences, Netherlands, *MSc. thesis, Tech. report DLR Oberpfaffenhofen*, **IB 515-94-2**, (1998).
- [6] Nijmeijer H., van der Schaft A. J., *Nonlinear Dynamical Control Systems*, Heidelberg: Springer-Verlag, (1990).
- [7] Isidori A., *Nonlinear control systems*, London: Springer-Verlag, third. ed., (1995).
- [8] Ackermann J., "Der Entwurf linearer Regelungssysteme im Zustandsraum", *Regelungstechnik*, **20**, pp. 297-300, (1972).
- [9] Ackermann J., Bünte T., "Handling improvement of robust car steering", *Proc. International Conference on Advances in Vehicle Control and Safety*, **July**, Amiens, France, (1998).
- [10] Wohlfarth J., "Aufbau eines Simulationsmodells eines allradgelenkten Fahrzeugs und Einsatz zur Analyse sicherheitskritischer Fahrmanöver", *MSc. thesis, Tech. report DLR Oberpfaffenhofen*, **IB 515-94-2**, (1993).
- [11] Dugoff H., Fancher P.S., Segel L., "Tire performance characteristics affecting vehicle response to steering and braking control inputs", *Final Report National Bureau of Standards Contract*, **CST-460**, Highway Safety Research Institute (HSRI), University of Michigan, Ann Arbor, (1969).