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STABILIZATION OF THE VARIABLE-LENGTH PENDULUM

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Abstract

The variable-length pendulum, proved to be non-minimum phase and non static-state-feedback linearizable, is stabilized around an equilibrium point by means of the passivity-based design control technique of *Energy Shaping plus Damping Injection*

1 Introduction

The stabilization and study of systems stability properties by means of their energy characteristics have received a growing attention in systems theory. The concepts of passivity and dissipativity have had an important role in this fact. A *passive system* is such a system which cannot store all the amount of energy that has been given, i.e., it dissipates energy in some way. This definition implies the existence of a *storage function* (representing the energy stored by the system) and a *supply function* (external energy received by the system). The idea of stored energy can be used connected to the system stability, considering the stored energy function as a Lyapunov-like function. Indeed, this fact can be deduced directly from the fact that a passive system has the Kalman-Yakubovich-Popov (KYP) property [1].

Furthermore, if a system is passive, properties referring to its zero dynamics can be followed: passive systems are minimum phase, since if the system output is rendered zero by means of an adequate feedback, the remaining dynamics or zero dynamics is Lyapunov-stable.

The control of a system in terms of the energy it can store or dissipate is known as *Passivity-Based Control* (PBC), and gives us a rather intuitive approach exploiting the system physical properties and the interconnection of the system with its environment. The aim of passivity-based control design methodology is modifying the closed-loop system energy and the addition of the required dissipation, it is the *Energy Shaping plus Damping Injection* (ESDI) control technique.

Passivity concepts will make possible to distinguish different parts or components of the systems dynamics. Depending the stored energy of the system along the trajectories of the system decreases, increases or remains we will speak of the dissipative, non-dissipative or invariant (also called lossless) part of the system, respectively.

We will use passivity concepts in connection to the stabilization problem. We aim to stabilize the *variable-length pendulum* trajectories around a desired equilibria by means of an energy approach. System passivity properties will be used, and the Lyapunov's stability of the system equilibrium point will be guaranteed by means of assuring the existence of a Lyapunov-like energy function.

There are a lot of works relating the control of the variable-length pendulum. A first approach to the problem in the frame of Lagragian dynamics is the one given by Bressand *et al* [2]. In addition to the tracking problem for this particular system recently presented by Sira-Ramírez [14], we could also point out the solution to the regulation problem presented by Fliess *et al* [3].

The application of the passivity approach to systems control is found in a great variety of papers written in the last decades. There are mainly devoted to mechanical (such as robots), electro-mechanical (such as induction motors) and purely electrical systems (such as dc-to-dc power converters), for a survey see Ortega *et al* [9]. Some examples of application to robotics and Lagragian systems are the works of Ortega *et al* [8, 11] and the ones presented by Kelly and coworkers (for example [4]). Electro-mechanical systems were addressed, among others, in Nicklasson *et al* [7]. Passivity-based controllers for power electronics were given, for instance, in Sira-Ramírez *et al* [12]. Passivity-based control design has also been applied to a great variety of nonlinear systems.

The control design methodology to use will be the ESDI one, which consists of modifying the stored energy of the system for the desired equilibrium to be the minimum of the new energy function for the closed-loop system and the addition of the required dissipation. Generalities and details of this technique can be found in Ortega *et al* [10] and the work of Sira-Ramírez *et al* [12]. In the literature, the ESDI idea has been applied in two different ways, the main differences between them are related to the way the *Energy Shaping* is made. On the one hand, we have the approach of Ortega *et al* [11] and Maschke

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et al [6], for which the expression for the closed-loop system Lyapunov-like energy function will not be needed to be known in order to ensure the stability of the system equilibrium point. On the other hand, the work of Sira-Ramírez *et al* [13, 15] is based on the explicit knowledge of the expression for the system energy. Indeed, the energy shaping is made in a different way in the two approaches. In the first one, the new energy function is conceived as a variation of the initial one ($V + \Delta V$, considering V the initial energy function), on the contrary, in the second one, the energy shaping is represented by the energy associated to the error system, which definition is based on the proposal of an auxiliary dynamics. We can also detect a difference between the two mentioned approaches through the purpose of the dissipation injected to the system.

This work follows the ESDI approach which does not require the knowledge of the new Lyapunov-like energy function. Section 2 presents a model for the variable-length pendulum. In section 3, we will apply the ESDI methodology for the studied system and derive a controller. The study of the properties of the resulting closed-loop system is dealt with in section 4. Some simulations results, the conclusions and suggestions for further research are presented in the last two sections.

2 The variable-length pendulum

Consider the normalized model of the variable-length pendulum studied by Bressand and Rampazzo [2], given by

$$
\begin{array}{rcl}
\dot{q} & = & p \\
\dot{p} & = & -cosu + qv^2 \\
\dot{u} & = & v
\end{array}
\tag{1}
$$

where q denotes the distance from the center of the sliding ball to the origin of coordinates around which the rod rotates. The variable p is the velocity of the ball, as measured along the rod, and u is the angle formed by the rod and the vertical line passing through the origin (see figure 1). The angular velocity v will be considered as the control.
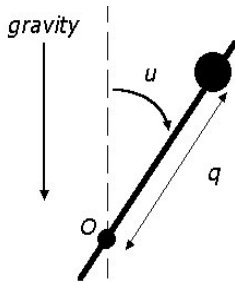

Figure 1: The inverted variable-length pendulum.

This system has interesting properties which make its stabilization a very attractive problem. We can check this system to be non-feedback linearizable by static state feedback and, hence, it is not differentially flat.

As it is proposed in Fliess [3], we will use an averaged model

for the variable-length pendulum, the relation between the new model and the one presented in (1) will be

$$
v = v_1 + v_2 cos(t/\varepsilon)
\tag{2}
$$

where v_1 and v_2 are auxiliary controls and ε is a constant $0 < \varepsilon \ll 1$. Then, substituting (2) in (1) and considering the averaged control systems, it is obtained

$$
\begin{array}{rcl}
\dot{\overline{q}} & = & \overline{p} \\
\dot{\overline{p}} & = & -cos\overline{u} + \overline{q}v_1^2 + \overline{q}\dfrac{v_2^2}{2} \\
\dot{\overline{u}} & = & v_1
\end{array}
\tag{3}
$$

Controls v_1 and v_2 are supposed to depend on the averaged states variables in a continuous and differentiable way.

The above input redefinition and the use of the averaged model overcome the drawback for the original system (1) to be non-flat.

3 ESDI-based stabilization

3.1 Stabilization of the variable-length pendulum

The ESDI approach is proved to stabilize the variable-length pendulum around a desired equilibrium point. Consider the dynamics (3) and the energy function for the system as V, we have

$$
V = \overline{q}cos\overline{u} + \frac{1}{2}\overline{p}^2,
\tag{4}
$$

which gradient takes the following expression

$$
\frac{\partial V}{\partial \overline{x}} = (cos\overline{u}, \overline{p}, -\overline{q}sin\overline{u})
$$

where $\overline{x}$ is the averaged state vector ($\overline{x} = (\overline{q}, \overline{p}, \overline{u})^T$). Thus, system (3) can be rewritten as

$$
\begin{pmatrix} \dot{\overline{q}} \\ \dot{\overline{p}} \\ \dot{\overline{u}} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \left(\frac{\partial V}{\partial \overline{x}} \right)^T +
$$

$$
+ \begin{pmatrix} 0 \\ \overline{q}\left(v_1^2 + \frac{1}{2}v_2^2\right) \\ v_1 \end{pmatrix} =
$$

$$
= J(\overline{x})\left(\frac{\partial V}{\partial \overline{x}} \right)^T + D(\overline{x}, v_1, v_2),
\tag{5}
$$

$J(\overline{x})$ is the $n \times n$-skew-symmetric matrix and $J(\overline{x})\left(\frac{\partial V}{\partial \overline{x}}\right)^T$ represents the *invariant component* of the system, D is a vector field grouping the dissipative and non-dissipative terms for the system dynamics. For stability purposes, we would like to convert the terms appearing in D into passive ones for some energy function, which may be different from the initial one V. Hence, the stabilization of the variable-length pendulum for an equilibrium point $\overline{x}^*$ will consist of finding controls $v_1 = \beta_1(\overline{x})$

and $v_2 = \beta_2(\overline{x})$ such that the system dynamics can be expressed as

$$\begin{pmatrix} \dot{\overline{q}} \\ \dot{\overline{p}} \\ \dot{\overline{u}} \end{pmatrix} = [J(\overline{x}) - R(\overline{x})] \left(\frac{\partial V^T}{\partial \overline{x}} + \frac{\partial \Delta V^T}{\partial \overline{x}} \right), \qquad (6)$$

where $R(\overline{x})$ an $n \times n$-positive-semi-definite symmetric matrix.

Now, $J(\overline{x}) \left[\frac{\partial (V + \Delta V)}{\partial \overline{x}} \right]^T$ represents the *invariant component* of the closed-loop system and, the dissipative one, that is, the one which implies a loss of energy as time goes forward, is given by $R(\overline{x}) \left[\frac{\partial (V + \Delta V)}{\partial \overline{x}} \right]^T$.

$V_d = V + \Delta V$ is the new Lyapunov-like function to be used in order to ensure the stability of the system around the equilibrium point $\overline{x}^* = (\overline{q}^*, 0, \overline{u}^*)^T$.

Remark 3.1 V_d *is certainly a Lyapunov-like function. The function ΔV will be chosen in such a way for $\overline{x}^*$ to be a strict local minimum of V_d, i.e., there exists an open neighbourhood $\overline{\mathscr{X}}$ of $\overline{x}^*$ such that $V_d(\overline{x}^*) < V_d(\overline{x}), \forall \overline{x} \in \overline{\mathscr{X}}$. This condition is met whenever the derivative of V_d with respect to $\overline{x}$ has an isolated zero at $\overline{x}^*$ and in addition to, the hessian of V_d is a positive-definite quadratic form, namely*

$$\left(\frac{\partial V_d}{\partial \overline{x}} \right)_{\overline{x} = \overline{x}^*} = 0 \qquad (7)$$

$$\left(\frac{\partial V_d}{\partial \overline{x}} \right)_{\overline{x} \neq \overline{x}^*} \neq 0 \qquad (8)$$

$$\left(\frac{\partial^2 V_d}{\partial \overline{x}^2} \right)_{\overline{x} = \overline{x}^*} \quad be \quad positive\text{-}definite \qquad (9)$$

Moreover, the derivative of V_d along the trajectories of the desired dynamics (6) takes the form,

$$\dot{V}_d = \frac{\partial V_d}{\partial \overline{x}} [J(\overline{x}) - R(\overline{x})] \left(\frac{\partial V_d}{\partial \overline{x}} \right)^T$$

Since $\frac{\partial V_d}{\partial \overline{x}} J(\overline{x}) \left(\frac{\partial V_d}{\partial \overline{x}} \right)^T = 0$, then

$$\dot{V}_d = -\frac{\partial V_d}{\partial \overline{x}} R(\overline{x}) \left(\frac{\partial V_d}{\partial \overline{x}} \right)^T \leq 0$$

Thus, we can assure that $\overline{x}^*$ is a locally stable equilibrium point for (6).

Expression (5) together with (6) leads us to

$$\left(\frac{\partial \Delta V}{\partial \overline{x}} \right)^T = [J(\overline{x}) - R(\overline{x})]^{-1} \left[R(\overline{x}) \left(\frac{\partial V}{\partial \overline{x}} \right)^T + \right.$$

$$\left. + D(\overline{x}, v_1, v_2) \right] \qquad (10)$$

We will design R for expression (10) to be met, and for this purpose we need not know the expression for ΔV, it is enough to know it exists.

The controller design procedure will be based on assuring the solvability of (10) and the fact that $\overline{x}^*$ is a strict local minimum of the energy function V_d (7)-(9).

3.1.1 Controller design

For simplicity's sake, dynamics (3) will be rewritten as

$$\begin{aligned} \dot{\overline{q}} &= \overline{p} \\ \dot{\overline{p}} &= -cos\overline{u} + \overline{q}w \\ \dot{\overline{u}} &= v_1 \end{aligned} \qquad (11)$$

where $w = v_1^2 + \frac{1}{2}v_2^2, w > 0$. Thus, the original multi-input averaged system of the general form $\dot{\overline{x}} = f(\overline{x}, u)$ has now an affine-in-the-input structure $\dot{x} = f(\overline{x}) + G(\overline{x})u$

$$\dot{\overline{x}} = \begin{pmatrix} \overline{p} \\ -cos(\overline{u}) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ \overline{q} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} w \\ v_1 \end{pmatrix} \qquad (12)$$

Choosing as R the matrix

$$R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & r_3(\overline{q}, \overline{p}, \overline{u}) \end{pmatrix},$$

and taking into account the relation (10), we have for system dynamics (12)

$$\frac{(\partial \Delta V)}{\partial \overline{x}} = \left(-\overline{q}w, 0, \overline{q}sin\overline{u} - \frac{v_1}{r_3} \right) \qquad (13)$$

The existence of such a local function ΔV, which gradient is the one written in (13), is assured if the rotational of ΔV is zero. This lead us to the following three conditions,

$$\frac{\partial}{\partial \overline{p}} \left(\frac{v_1}{r_3} \right) = 0 \Rightarrow \frac{v_1}{r_3} = \phi(\overline{q}, \overline{u}) \qquad (14)$$

$$sin\overline{u} - \frac{\partial}{\partial \overline{q}} \left(\frac{v_1}{r_3} \right) + \overline{q}\frac{\partial w}{\partial \overline{u}} = 0 \qquad (15)$$

$$\frac{\partial w}{\partial \overline{p}} = 0 \Rightarrow w = w(\overline{q}, \overline{u}) \qquad (16)$$

Putting together (14) and (16) into (15), we have

$$v_1 = r_3 \phi(\overline{q}, \overline{u}), \ \omega = \omega(\overline{q}, \overline{u}) \text{ and } sin\overline{u} - \frac{\partial \phi}{\partial \overline{q}} + \overline{q}\frac{\partial w}{\partial \overline{u}} = 0 \ (17)$$

In order to fulfill (17), let us define

$$\begin{aligned} w &= \frac{cos\overline{u}}{\overline{q}} + w_1(\overline{q}, \overline{u}) \\ w_1 &= \frac{1}{\overline{q}}[a_1(\overline{q} - \overline{q}^*) + a_2(\overline{u} - \overline{u}^*)] \\ v_1 &= a_2 r_3(\overline{q} - \overline{q}^*) + a_3(\overline{u} - \overline{u}^*) \end{aligned} \qquad (18)$$

where a_1, a_2, a_3, r_3 are constants. Then, we have

$$\frac{\partial(\Delta V)}{\partial \overline{x}} = (-\overline{q}w, 0, \overline{q}sin\overline{u} - \frac{v_1}{r_3}) =$$
$$= (-cos\overline{u} - a_1(\overline{q} - \overline{q}^*) - a_2(\overline{u} - \overline{u}^*), 0,$$
$$, \quad \overline{q}sin\overline{u} - a_2(\overline{q} - \overline{q}^*) - \frac{a_3}{r_3}(\overline{u} - \overline{u}^*)) \quad (19)$$

It can be checked that conditions (7)- (9) are met, i.e. the equilibrium point $(\overline{q}^*, 0, \overline{u}^*)$ is an isolated strict local minimum for the closed-loop system energy function V_d if and only if,

$$a_1 < 0, a_3 < 0$$
$$a_1 a_3 - r_3 a_2^2 > 0 \quad (20)$$

Note that w is an auxiliary control used for simplifying the averaged model (3). We need the expression of v_1 and v_2 for the computation of the control v, thus, from

$$v_1^2 + \frac{v_2^2}{2} = w(\overline{q}, \overline{u}) = \frac{cos\overline{u}}{\overline{q}} + w_1(\overline{q}, \overline{u})$$

the control v_2 can be obtained,

$$v_2 = \sqrt{2\left(\frac{cos\overline{u}}{\overline{q}} + w_1 - v_1^2\right)}, \quad (21)$$

where

$$v_1 = a_2 r_3(\overline{q} - \overline{q}^*) + a_3(\overline{u} - \overline{u}^*)$$
$$w_1 = \frac{1}{\overline{q}}[a_1(\overline{q} - \overline{q}^*) + a_2(\overline{u} - \overline{u}^*)]$$

The control v will be computed using relation (2).

To conclude with, we have found controls v_1, v_2 such that, the closed-loop averaged system (3) can be written as

$$\dot{\overline{x}} = (J - R)\frac{\partial(V + \Delta V)}{\partial \overline{x}^T}$$

for a new energy function $V_d = V + \Delta V$. Actually, $V_d(\overline{x}) - V_d(\overline{x}^*)$ is a local Lyapunov-like function of this system around the equilibrium point $\overline{x}^*$.

Taking into account the above conditions, we conclude that $\overline{x}^* = (\overline{q}^*, 0, \overline{u}^*)$ is an asymptotically stable equilibrium point for the system dynamics (12), and consequently for the dynamics (1). This fact has been achieved by means of a damping injection through the introduction of the matrix R, and an energy shaping, via the new energy for the closed-loop system $V_d = V + \Delta V$.

Remark 3.2 *The averaging theory (see, for example, Theorem 8.3 from Khalil [5]) assures that the stability properties of the averaged system are the same as the original system ones. Both systems have the same equilibrium point. The error presented in the approximation solution for the system given by the solution of the averaged system depends upon the parameter ε: the smaller ε is, the smaller the steady-state error is.*

4 Study of the system and the stability region around the equilibrium point

The closed-loop system stability can be analized from the conditions for the equilibrium $\overline{x}^*$ to be a strict local minimum of V_d (7)- (9).

From equation (19), the hessian matrix of V_d valued in $\overline{x}^*$ is

$$\left(\frac{\partial^2 V_d}{\partial \overline{x}^2}\right)_{\overline{x}=\overline{x}^*} = \begin{pmatrix} -a_1 & 0 & -a_2 \\ 0 & 1 & 0 \\ -a_2 & 0 & -\frac{a_3}{r_3} \end{pmatrix}, \quad (22)$$

then, inequalities (20) guarantee the Hessian (22) to be positive definite.

In order to design constants a_1, a_2, a_3, equation (21) has to be taken into account, thus the next inequality is also required

$$\frac{cos\overline{u}}{\overline{q}} + \frac{1}{\overline{q}}[a_1(\overline{q} - \overline{q}^*) + a_2(\overline{u} - \overline{u}^*)] >$$
$$> [a_2 r_3(\overline{q} - \overline{q}^*) + a_3(\overline{u} - \overline{u}^*)]^2 \quad (23)$$

4.1 Study of the range of the constants appearing in the controller design

The stability domain, in other words, the range of a_1, a_2, a_3, r_3 which gives the largest stability region around the equilibrium point $\overline{x}^*$ will be established from (20) and (23).

Let $\rho_{\overline{q}}, \rho_{\overline{u}}$ be real positive numbers and $\overline{q}, \overline{u}$ such that

$$|\overline{q} - \overline{q}^*| < \rho_{\overline{q}}$$
$$|\overline{u} - \overline{u}^*| < \rho_{\overline{u}}$$

Then,

$$\frac{cos\overline{u}}{\overline{q}} + w_1 - v_1^2 >$$
$$> \frac{cos\overline{u}}{\overline{q}} - \frac{1}{\overline{q}}[|a_1|\rho_{\overline{q}} + |a_2|\rho_{\overline{u}}] - (|a_2|r_3\rho_{\overline{q}} + |a_3|\rho_{\overline{u}})^2$$
$$> \frac{cos\overline{u}}{\overline{q}^* + \rho_{\overline{q}}} - \frac{1}{\overline{q}^* - \rho_{\overline{q}}}[|a_1|\rho_{\overline{q}} + |a_2|\rho_{\overline{u}}] -$$
$$- (|a_2|r_3\rho_{\overline{q}} + |a_3|\rho_{\overline{u}})^2$$
$$> \frac{cos(\overline{u}^* - \rho_{\overline{q}})}{\overline{q}^* + \rho_{\overline{q}}} - \frac{1}{\overline{q}^* - \rho_{\overline{q}}}[|a_1|\rho_{\overline{q}} + |a_2|\rho_{\overline{u}}] -$$
$$- (|a_2|r_3\rho_{\overline{q}} + |a_3|\rho_{\overline{u}})^2$$

and (23) holds if

$$\frac{cos(\overline{u}^* - \rho_{\overline{q}})}{\overline{q}^* + \rho_{\overline{q}}} > \left[|a_1|\frac{\rho_{\overline{q}}}{\overline{q}^* - \rho_{\overline{q}}} + |a_2|\frac{\rho_{\overline{u}}}{\overline{q}^* - \rho_{\overline{q}}}\right] +$$
$$+ (|a_2|r_3\rho_{\overline{q}} + |a_3|\rho_{\overline{u}})^2 \quad (24)$$

Finaly, let us define

$$\alpha = \frac{cos(\overline{u}^* - \rho_{\overline{q}})}{\overline{q}^* + \rho_{\overline{q}}}$$

and choose a_1, a_2, a_3 and r_3 in such a way that

$$|a_1|\frac{\rho_{\overline{q}}}{\overline{q}^* - \rho_{\overline{q}}} + |a_2|\frac{\rho_{\overline{u}}}{\overline{q}^* - \rho_{\overline{q}}} \quad < \quad \frac{\alpha}{2}$$

$$(|a_2|r_3\rho_{\overline{q}} + |a_3|\rho_{\overline{u}})^2 \quad < \quad \frac{\alpha}{2}$$

then (24) holds.

A possible solution for a_1, a_2, a_3 and r_3 is

$$
\begin{aligned}
|a_1| &< \left| \frac{\alpha(\overline{q}^* - \rho_{\overline{q}})}{4\rho_{\overline{q}}} \right| \\
|a_2| &< \left| \frac{\alpha(\overline{q}^* - \rho_{\overline{q}})}{4\rho_{\overline{u}}} \right| \\
|a_3| &< \left| \frac{\sqrt{\alpha}}{2\rho_{\overline{u}}\sqrt{2}} \right| \\
|r_3| &< \left| \frac{\sqrt{\alpha}}{2a_2\rho_{\overline{q}}\sqrt{2}} \right|
\end{aligned}
\tag{25}
$$

>From (20) we should consider the constants a_1, a_3 as negative values, the sign of a_2 can be either positive or negative, and the constant r_3 is a positive one, since the matrix R is needed to be positive semi-definite. Then, in addition to conditions (25), we will also take into account $a_1a_3 - r_3a_2^2 > 0$.

Note that from our particular solution of (13) the averaged dynamics becomes linear. One can take benefit from this closing the design process as a pole placement problem.

5 Simulation results

Previous results are particularized to $x^* = (0.5, 0, 1.2)$, $\rho_{\overline{q}} = 1$, $\rho_{\overline{u}} = 0.5$. Then a_1, a_2, a_3 and r_3 has been chosen as $a_1 = -0.05$, $a_2 = -0.11$, $a_3 = -0.48$, $r_3 = 1.8$.

Figures 2, 3, 4 and 5 display the dynamics of system (1), actually, the trajectories of the states, the control v, the auxiliar controls v_1 and v_2 and the new energy, respectively. Initial conditions are $(0, 0, 0)$ and $\varepsilon = 0.02$

6 Conclusions

In this paper, we have locally stabilized the variable-length pendulum around an equilibrium point by means of an energy approach. The passivity-based control design methodology of *Energy Shaping and Damping Injection* is applied, and a study of the coefficients appearing in the obtained controller in connection to the closed-loop system stability is provided.

The closed-loop system obtained has a well-defined energy structure in which it is distinguished a conservative and a dissipative components derived from the new closed-loop system energy. Therefore, the proposed stabilization method is a Lyapunov-based design where the construction of the closed-loop system Lyapunov function which ensures the system stability is based upon the ESDI methodology.

The results here obtained can be completed by analyzing the stability region of the resulting controlled system, which will

lead us to global stability results. A study of stability depending on the values for $\rho_{\overline{q}}$, and $\rho_{\overline{u}}$ is required.

It will also be interesting to extend the stabilization problem to solve the tracking problem, for which we may use time-depending Lyapunov-like functions.
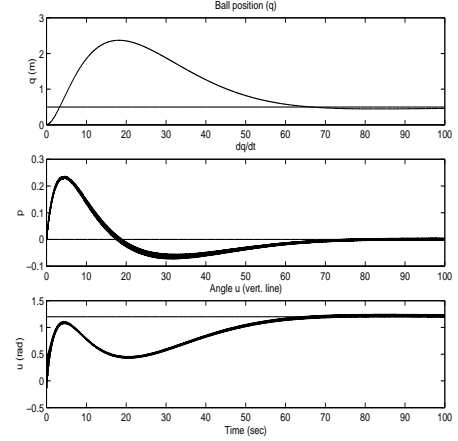
7 Figures
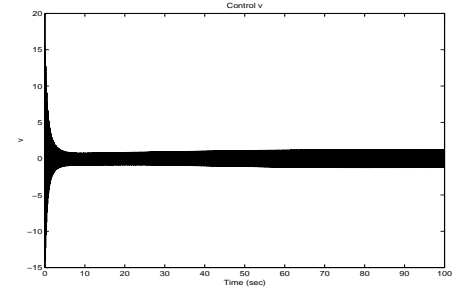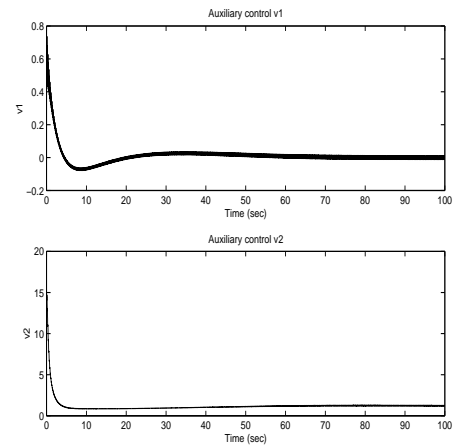

Figure 2: State variables.

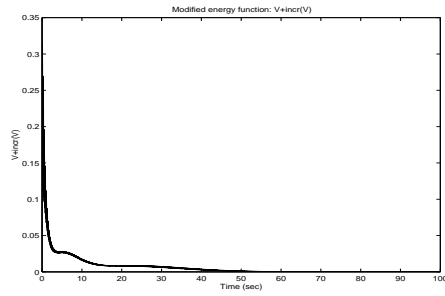Figure 3: Control v.

Figure 4: Auxiliary controls v_1, v_2.

Figure 5: Modified energy $V + \Delta V$.

Acknowledgement

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