Robust transient stabilization of a synchronous generator in an uncertain power network with transfer conductances

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Robust Transient Stabilization of a Synchronous Generator in an Uncertain Power Network with Transfer Conductances

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Abstract— The robust transient stability problem of a synchronous generator in an uncertain power network with transfer conductances is addressed. A robust adaptive nonlinear feedback control algorithm is designed on the basis of a third order model of the synchronous machine: only two system parameters (synchronous machine damping and inertia constants) along with upper and lower bounds on the remaining uncertain ones are supposed to be known. The conditions to be satisfied by the remote network dynamics for guaranteeing $\mathcal{L}_2$ and $\mathcal{L}_\infty$ robustness and asymptotic relative speed regulation to zero are weaker than those required by the single machine-infinite bus approximation.

I. Introduction

Power networks are among the most complex large-scale, interconnected nonlinear systems ([3], [12]). Mechanical and electrical perturbations such as load shedding, generation tripping or short circuits may force one or more generators to go out of step and to be disconnected from the network. The transient stabilization problem consists in the design of a suitable excitation feedback control keeping each generator close to the synchronous speed when mechanical and electrical perturbations occur (voltage regulation is not considered at this stage). On the basis of linear approximations around operating conditions, decentralized linear controllers were first designed, which however may not be able to guarantee the power grid stability (see [10]) and to handle the severe disturbances and contingencies typically occurring in power networks. In the recent years, several nonlinear algorithms have been proposed for power systems control. For a particular power systems structure $\mathcal{M}$ consisting of a group of generators tied together by a strong network of transmission lines and linked to a single generator by a comparatively weak set of tie lines, the single machine-infinite bus approximation, which models all the remaining network as a fixed voltage source and an impedance, is advantageous in the nonlinear control design: transient stabilization can be achieved even without requiring the knowledge of critical parameters ([5], [6], [7], [10], [14], [15], [20], [22], [26], [27]). In particular, $\mathcal{L}_2$ and $\mathcal{L}_\infty$ robustness and asymptotic relative speed regulation to zero are achieved in [15] despite

uncertainties in all system parameters.

In this paper, we address the transient stabilization problem for the particular power systems structure $\mathcal{M}$ above considered. The single machine-infinite bus model is however not used so that remote network dynamics effects are taken into account. Following the theoretical developments in [16], [17], [21], a robust adaptive nonlinear feedback control is designed which does not assume the knowledge of the system parameters excepting for the machine damping and inertia constants. On the basis of upper and lower bounds on the uncertain model parameters, $\mathcal{L}_2$ and $\mathcal{L}_\infty$ robustness and asymptotic relative speed regulation to zero are guaranteed under a set of assumptions on the network dynamics which are weaker than those required by the single machine-infinite bus approximation, while the controller parameters are adapted using the robust adaptive techniques in [16]. Simulation results with reference to a 3-machine, 9-buses power network illustrate the effectiveness of the adopted approach, showing that the proposed robust nonlinear excitation control prevents each network machine from going out of step in the presence of electrical parameter perturbations and unmodelled dynamics and improves the performance with respect to the controller in [15] based on the single machine-infinite bus approximation. The presented approach in conjuction with the special structure of power systems considered here allows us, without assuming the availability of the machine internal voltage measurement, to solve the robust transient stabilization problem of a synchronous generator in a power network in the presence of transfer conductances and uncertainties in the most of system parameters, which is typically a difficult result to obtain for general structures of power systems (see for instance [9], [11], [13], [18], [20], [23], [24], [25], [28], [29], [30]).

II. System Dynamic Model

A power system consisting of n generators interconnected through a transmission network is described by the $3n$-order nonlinear model in [19] [$1 \leq i \leq n$]

$$\begin{aligned}
\dot{\delta}_i &= \omega_i \\
\dot{\omega}_i &= -\frac{D_i}{2H_i}\omega_i + \frac{\omega_0}{2H_i}P_{mi} - \frac{\omega_0}{2H_i}P_{ei} \\
\dot{E}'_{qi} &= \frac{k_{ci}}{T'_{d0i}}u_{fi} - \frac{E'_{qi}}{T'_{d0i}} - \frac{(x_{di} - x'_{di})}{T'_{d0i}}I_{di}
\end{aligned} \tag{1}$$

in which the first two equations represent the i-th generator mechanical dynamics involving the the power angle δ_i(rad), the relative angular speed ω_i(rad/s), the active electrical power P_{ei}(p.u.), the mechanical input power P_{mi}(p.u.), the

synchronous speed ω_0(rad/s), the damping constant D_i(p.u.) and the inertia constant H_i(s), while the third equation constitutes the i-th generator electrical dynamics involving the transient EMF E'_{qi}(p.u.) in the quadrature axis, the input u_{fi}(p.u.) to the thyristor amplifier, the gain k_{ci} of the excitation amplifier, the direct axis transient open circuit time constant T'_{d0i}(s), the direct axis reactance x_{di}(p.u.) and the direct axis transient reactance x'_{di}(p.u.). The i-th generator electrical equations are

$$
\begin{aligned}
P_{ei} &= E'^2_{qi}G_{ii} + E'_{qi}\sum_{j=1,j\neq i}^{n}\Big[E'_{qj}G_{ij}\cos(\delta_{ij}) \\
&\quad + E'_{qj}B_{ij}\sin(\delta_{ij})\Big] \\
Q_{ei} &= -E'^2_{qi}B_{ii} + E'_{qi}\sum_{j=1,j\neq i}^{n}\Big[E'_{qj}G_{ij}\sin(\delta_{ij}) \\
&\quad - E'_{qj}B_{ij}\cos(\delta_{ij})\Big] - x'_{di}\Big(I^2_{di} + I^2_{qi}\Big) \\
I_{di} &= -E'_{qi}B_{ii} + \sum_{j=1,j\neq i}^{n}\Big[E'_{qj}G_{ij}\sin(\delta_{ij}) \\
&\quad - E'_{qj}B_{ij}\cos(\delta_{ij})\Big] \\
I_{qi} &= E'_{qi}G_{ii} + \sum_{j=1,j\neq i}^{n}\Big[E'_{qj}G_{ij}\cos(\delta_{ij}) \\
&\quad + E'_{qj}B_{ij}\sin(\delta_{ij})\Big] \\
\delta_{ij} &= \delta_i - \delta_j \\
V_{ti} &= \sqrt{\big(x'_{di}I_{qi}\big)^2 + \big(E'_{qi} - x'_{di}I_{di}\big)^2}
\end{aligned}
$$

in which (for the i-th generator): Q_{ei}(p.u.) is the reactive electrical power, I_{di}(p.u.) is the direct axis current, I_{qi}(p.u.) is the quadrature axis current, V_{ti}(p.u.) is the terminal voltage, G_{ij}(p.u.) and B_{ij}(p.u.) are the i-th row and the j-th column element of nodal conductance and susceptance matrices, respectively, at the internal nodes after eliminating all physical buses, which depend on x'_{di}(p.u.), on the transformer reactance x_{Ti}(p.u.), on the loads and on the transmission line reactance x_{ij}(p.u.) between the i-th generator and the j-th generator. The nodal conductance and susceptance matrices represent the full power network of transmission lines and loads connecting the power generators.

In this paper, we study a particular structure of power systems, i.e., a group G_M of $n-1$ generators, tied together by a strong network of transmission lines, which is linked to a generator g_m (in the following referred to as r-th generator) by a comparatively weak set of tie lines. Accordingly, we explicitly compute $\dot{P}_{er}$ [G_{rj} and B_{rj}, $1 \leq j \leq n$, are assumed to be constant], so that the r-th generator third order dynamic model can be written as [$I_{qr} > c_{Ir} > 0$ (see [20])[1]]

$$
\dot{\delta}_r = \omega_r
$$

[1]We assume that there exist bounded connected open sets $\mathcal{D}_\delta$ and $\mathcal{D}_p$ such that $E'_{qr}(0) > 0$, $\delta_r(t) \in \mathcal{D}_\delta$ and $P_{er}(t) \in \mathcal{D}_p$ imply $I_{qr}(t) > c_{Ir} > 0$, which describes the whole practical operating region of the generator.

$$
\begin{aligned}
\dot{\omega}_r &= -\frac{D_r}{2H_r}\omega_r + \frac{\omega_0}{2H_r}P_{mr} - \frac{\omega_0}{2H_r}P_{er} \quad (2) \\
\dot{P}_{er} &= -\theta_{1r}P_{er} - \theta_{2r}I_{dr}I_{qr} - \theta_{3r}\frac{I_{dr}P_{er}}{I_{qr}} - \theta_{4r}\frac{P^2_{er}}{I^2_{qr}} \\
&\quad + \Big[\frac{\theta_{5r}I^2_{qr} + \theta_{6r}P_{er}}{I_{qr}}\Big]u_{fr} - \Big(Q_{er} + B_{rr}\frac{P^2_{er}}{I^2_{qr}} \\
&\quad + x'_{dr}(I^2_{dr} + I^2_{qr})\Big)\omega_r + \frac{P_{er}}{I_{qr}}R_r
\end{aligned}
$$

in which δ_r, ω_r, P_{er} are the state variables, u_{fr} is the control input, $\theta_{1r} = \frac{1}{T'_{d0r}}$, $\theta_{2r} = \frac{(x_{dr}-x'_{dr})}{T'_{d0r}}$, $\theta_{3r} = \frac{G_{rr}(x_{dr}-x'_{dr})}{T'_{d0r}}$, $\theta_{4r} = \frac{G_{rr}}{T'_{d0r}}$, $\theta_{5r} = \frac{k_{cr}}{T'_{d0r}}$, $\theta_{6r} = \frac{G_{rr}k_{cr}}{T'_{d0r}}$ and the term

$$
\begin{aligned}
R_r &= \sum_{j=1,j\neq r}^{n}\dot{\tilde{E}}'_{qj}\Big[G_{rj}\cos(\delta_{rj}) + B_{rj}\sin(\delta_{rj})\Big] \\
&\quad + \sum_{j=1,j\neq r}^{n}\omega_j\Big[E'_{qj}G_{rj}\sin(\delta_{rj}) - E'_{qj}B_{rj}\cos(\delta_{rj})\Big]
\end{aligned}
$$

represents the effect of the network remote dynamics (the group G_M of generators) on the r-th generator, where $\tilde{E}'_{qj} = E'_{qj} - E'_{qj0}$, with E'_{qj0} the pre-fault equilibrium value for E'_{qj}. In the practice, the exact values of the model parameters are hard to obtain, and in particular P_m, G_{rr}, B_{rr} are lumped parameters which account for unmodelled dynamics such as turbine and load dynamics. Those parameters may undergo sudden on line variations due to mechanical and electrical perturbations and faults. In the following, we will suppose the parameters ω_0, D_r, H_r to be known and will assume that: the uncertain piecewise continuous parameters $T'_{d0r}(t)$, $x_{dr}(t)$, $x'_{dr}(t)$, $k_{cr}(t)$ and the uncertain constant parameter G_{rr} are within the corresponding known positive bounds (T'_{d0rm}, T'_{d0rM}), (x_{drm}, x_{drM}), (x'_{drm}, x'_{drM}), (k_{crm}, k_{crM}), (G_{rrm}, G_{rrM}); the uncertain constant parameter B_{rr} is within the corresponding known bounds (B_{rrm}, B_{rrM}); the mechanical input power $P_{mr}(t) \in \mathcal{D}_p$ is a class $\mathcal{C}^1$ function satisfying: $P_{mrM} \geq P_{mr}(t) \geq P_{mrm}$ and $|\dot{P}_{mr}(t)| \leq \dot{P}_{Mr}$, with P_{mrm}, P_{mrM}, $\dot{P}_{Mr}$ known positive reals. Physical considerations concerning transmission lines, loads and mechanical turbines make the above assumptions reasonable in the transient stabilization problem of power networks.

III. Problem statement

A suitable control problem formulation is introduced in this section, which will allow us to solve the transient stabilization problem for the synchronous generator g_m by quantitatively characterizing the robustness with respect to both permanent and vanishing model parameter perturbations. Denote by $\delta_{rs} \in \mathcal{D}_\delta$ the pre-fault constant value for the power angle δ_r and let θ_{irm}, θ_{irM} be the known positive bounds on the uncertain parameters $\theta_{ir}(t)$, $1 \leq i \leq 6$, with $\hat{P}_{mr}(t)$ a suitable estimate of the uncertain mechanical power P_{mr}. The transient stabilization problem addressed in this paper is rigorously formulated as follows.

Definition 1 (Transient Stabilizing Control): *Assume that for any bounded piecewise continuous real-valued function*

$u_{fr}(\cdot)$ defined on $\mathbb{R}_0^+$ and for all $t \geq 0$: i) for each $j \neq r$, $1 \leq j \leq n$, the j-th generator power angle $\delta_j(t)$, the relative angular speed $\omega_j(t)$ and the quadrature axis transient EMF $E'_{qj}(t)$ are piecewise differentiable functions of time t and boundedness of the r-th generator variables $\delta_r(t)$, $\omega_r(t)$, $P_{er}(t)$ implies boundedness of $E'_{qj}(t)$; ii) there exist μ_r, ν_r, ρ_r (unknown) non-negative reals, $\bar{\varphi}_{\mu r}(\cdot)$, $\bar{\varphi}_{\nu r}(\cdot)$, $\bar{\varphi}_{\rho r}(\cdot)$ known $\mathcal{K}_\infty$ functions and $g_r(t)$ (unknown) bounded non-negative real-valued function of time t such that

$$
\begin{aligned}
|R_r(t)| \;\leq\; & \sup_{0 \leq \tau \leq t}\{g_r(\tau)\} + \mu_r\bar{\varphi}_{\mu r}\Big(\max_{0 \leq \tau \leq t}\{|\delta_r(\tau) - \delta_{rs}|\}\Big) \\
& + \nu_r\bar{\varphi}_{\nu r}\Big(\max_{0 \leq \tau \leq t}\{|\omega_r(\tau)|\}\Big) \\
& + \rho_r\bar{\varphi}_{\rho r}\Big(\max_{0 \leq \tau \leq t}\{|P_{er}(\tau) - P_{mr}(\tau)|\}\Big).
\end{aligned}
$$

Define [ε_{jr}, ε_{xr}, ε_{Br} ($1 \leq j \leq 6$) are positive reals]:

$$
\begin{aligned}
y_r(t) \;=\;& \big[\delta_r(t) - \delta_{rs}, \omega_r(t), P_{er}(t) - P_{mr}(t)\big]^T \\
\xi_r(t) \;=\;& \big[y_r(t)^T, P_{mr}(t) - \hat{P}_{mr}(t)\big]^T \\
w_{dr}(t) \;=\;& \big[\dot{P}_{mr}(t), \theta_{1rM} - \theta_{1rm} + \varepsilon_{1r}, \theta_{2rM} - \theta_{2rm} \\
& + \varepsilon_{2r}, \theta_{3rM} - \theta_{3rm} + \varepsilon_{3r}, \theta_{4rM} - \theta_{4rm} + \varepsilon_{4r}, \\
& \max\{\theta_{5rM} - \theta_{5rm} + \varepsilon_{5r}, \theta_{6rM} - \theta_{6rm} + \varepsilon_{6r}\}, \\
& x'_{drM} - x'_{drm} + \varepsilon_{xr}, B_{rrM} - B_{rrm} + \varepsilon_{Br}, \\
& \sup_{0 \leq \tau \leq t}\{g_r(\tau)\}, \mu_r, \nu_r, \rho_r\big]^T.
\end{aligned}
$$

A bounded control law u_{fr} is called a transient stabilizing control for the r-th generator if it guarantees the closed loop system to satisfy the following: (S1) $\mathcal{L}_\infty$ disturbance attenuation property, i.e.

$$
\|y_r(t)\|^2 \;\leq\; h_{1r}(\xi_r(0))e^{-c_r t} + \frac{1}{k_r}\gamma_{1r}(\|w_{dr}(\cdot)\|_\infty)
$$

holds for all $t \geq 0$, where $h_{1r}(\xi_r(0)) \geq 0$, $c_r > 0$ and $\gamma_{1r}(r)$ is a class $\mathcal{K}_\infty$ function; (S2) $\mathcal{L}_2$ disturbance attenuation property, i.e.

$$
\int_0^T \|y_r(\tau)\|^2 d\tau \;\leq\; h_{2r}(\xi_r(0)) + \frac{1}{k_r}\int_0^T \gamma_{2r}(\|w_{dr}(\tau)\|)d\tau
$$

holds for any given $T > 0$, where $h_{2r}(\xi_r(0)) \geq 0$ and $\gamma_{2r}(r)$ is a class $\mathcal{K}_\infty$ function.

Definition 2 (Transient Adaptive Stabilizing Control): *Assume that, in addition to assumptions i)-ii), there exist t_0, M_{Rr}, $\gamma_{\delta r}$, $\gamma_{\omega r}$ and γ_{pr} non-negative reals such that: iii) the uncertain machine parameters $P_{mr}(t)$, $T'_{d0r}(t)$, $x_{dr}(t)$, $x'_{dr}(t)$, $k_{cr}(t)$ are constant for all $t \geq t_0$; iv) the following inequality:*

$$
\begin{aligned}
\lim_{t \to \infty}\int_{t_0}^t R_r^2(\tau)d\tau \;\leq\;& M_{Rr} + \gamma_{\delta r}\lim_{t \to \infty}\int_{t_0}^t [\delta_r(\tau) - \delta_{rs}]^2 d\tau \\
& + \gamma_{\omega r}\lim_{t \to \infty}\int_{t_0}^t [\omega_r(\tau)]^2 d\tau \\
& + \gamma_{pr}\lim_{t \to \infty}\int_{t_0}^t [P_{er}(\tau) - P_{mr}(\tau)]^2 d\tau
\end{aligned}
$$

holds. A transient stabilizing control u_{fr} is called a transient adaptive stabilizing control for the r-th generator if it guarantees, the additional property: (S3) asymptotic regulation, i.e.

$$
\lim_{t \to \infty}\Big\|\big[\delta_r(t) - \delta_{rs}, \omega_r(t), P_{er}(t) - P_{mr}\big]\Big\| = 0.
$$

IV. NONLINEAR DESIGN AND STABILITY ANALYSIS

By virtue of techniques similar to those used in [7], we design, in this section, a transient adaptive stabilizing control for the r-th generator according to Definitions 1 and 2. Define the power angle regulation and relative angular speed tracking errors: $\tilde{\delta}_r = \delta_r - \delta_{rs}$, $\tilde{\omega}_r = \omega_r - \omega_r^*$, with δ_{rs} being the power angle constant reference value and ω_r^* being the relative angular speed time-varying reference signal [$k_{\delta r}$ is a positive control parameter]

$$
\omega_r^* \;=\; -\frac{5}{4}k_{\delta r}\tilde{\delta}_r. \tag{3}
$$

Design the active electrical power time-varying reference signal P_{er}^* as [$k_{\omega r}$, $k_{\omega p r}$ are positive control parameters]

$$
P_{er}^* \;=\; \frac{2H_r}{\omega_0}\Big[\frac{5}{4}k_{\omega r}\tilde{\omega}_r + \frac{5}{4}k_{\delta r}\omega_r - \frac{D_r}{2H_r}\omega_r^* + \tilde{\delta}_r \\
+ \frac{1}{k_{\omega p r}}\tilde{\omega}_r\Big] + \hat{P}_{mr} \tag{4}
$$

which relies on the estimate $\hat{P}_{mr}$ of the uncertain mechanical power P_{mr} satisfying the estimation law [k_{per}, k_r are positive control parameters]

$$
\begin{aligned}
\hat{P}_{mr} \;=\;& \phi_r + \frac{2H_r}{\omega_0}\Big(\frac{5}{4}k_{per} + \frac{k_r}{4} + \frac{1}{k_r} + \frac{\omega_0^2}{16H_r^2}k_{\omega p r}\Big)\omega_r \\
\dot{\phi}_r \;=\;& \Big(\frac{5}{4}k_{per} + \frac{k_r}{4} + \frac{1}{k_r} + \frac{\omega_0^2}{16H_r^2}k_{\omega p r}\Big)\Big[-\phi_r \\
& + \frac{D_r}{\omega_0}\omega_r + P_{er} - \frac{2H_r}{\omega_0}\Big(\frac{5}{4}k_{per} + \frac{k_r}{4} + \frac{1}{k_r} \\
& + \frac{\omega_0^2}{16H_r^2}k_{\omega p r}\Big)\omega_r\Big]
\end{aligned} \tag{5}
$$

$$
P_{mrm} \;\leq\; \hat{P}_{mr}(0) \;\leq\; P_{mrM}
$$

and introduce the active electrical power tracking error: $\tilde{P}_{er} = P_{er} - P_{er}^*$. Design the robust control law u_{fr} with the stabilizing and robustifying terms v_r, ϖ_r as [k_{pr}, k_{Rr} are positive control parameters]

$$
\begin{aligned}
u_{fr} \;=\;& \frac{I_{qr}}{\big(\hat{\theta}_{5r}I_{qr}^2 + \hat{\theta}_{6r}P_{er}\big)}v_r \\
& - \frac{k_r I_{qr}\big(2I_{qr}^4 + 2P_{er}^2\big)v_r^2\tilde{P}_{er}}{4\big(\hat{\theta}_{5r}I_{qr}^2 + \hat{\theta}_{6r}P_{er}\big)^2\big(\theta_{5rm}I_{qr}^2 + \theta_{6rm}P_{er}\big)} \\
v_r \;=\;& -\frac{5}{4}k_{pr}\tilde{P}_{er} + \frac{\omega_0}{2H_r}\tilde{\omega}_r + \frac{5D_r k_{\delta r}}{4\omega_0}\omega_r \\
& + \frac{2H_r}{\omega_0}\omega_r + \hat{\theta}_{1r}P_{er} + \hat{\theta}_{2r}I_{dr}I_{qr} + \hat{\theta}_{3r}\frac{I_{dr}P_{er}}{I_{qr}}
\end{aligned}
$$

$$+\hat{\theta}_{4r}\frac{P_{er}^2}{I_{qr}^2} + \hat{B}_{rr}\frac{P_{er}^2\omega_r}{I_{qr}^2} + \hat{x}'_{dr}\left(I_{dr}^2 + I_{qr}^2\right)\omega_r$$

$$+Q_{er}\omega_r - \frac{5k_{\delta r}}{4\omega_0}\left[D_r\omega_r - \omega_0(\hat{P}_{mr} - P_{er})\right]$$

$$-\frac{2H_r}{\omega_0}\left(\frac{5}{4}k_{\omega r} + \frac{1}{k_{\omega pr}}\right)\left[\left(\frac{5}{4}k_{\omega r} + \frac{D_r}{2H_r}\right)\tilde{\omega}_r + \tilde{\delta}_r\right.$$

$$\left.+\frac{\omega_0}{2H_r}\tilde{P}_{er} + \frac{1}{k_{\omega pr}}\tilde{\omega}_r\right] + \varpi_r \qquad (6)$$

$$\varpi_r = -\frac{k_{Rr}}{4}\frac{P_{er}^2}{I_{qr}^2}\tilde{P}_{er} - \frac{k_r}{4}\tilde{P}_{er}\left[P_{er}^2 + I_{dr}^2 I_{qr}^2 + \frac{I_{dr}^2 P_{er}^2}{I_{qr}^2}\right.$$

$$+\frac{P_{er}^4}{I_{qr}^4}\left(1 + \omega_r^2\right) + \omega_r^2\left(I_{dr}^2 + I_{qr}^2\right)^2\Big] - \frac{k_r}{4}\left(\frac{5}{4}k_{\omega r}\right.$$

$$+\frac{5}{4}k_{\delta r} + \frac{5}{4}k_{per} + \frac{k_r}{4} + \frac{1}{k_r} + \frac{\omega_0^2}{16H_r^2}k_{\omega pr}$$

$$+\frac{1}{k_{\omega pr}}\Big)^2\tilde{P}_{er} - \frac{k_r}{4}\frac{P_{er}^2}{I_{qr}^2}\tilde{P}_{er}\left[\bar{\varphi}_{\mu r}^2\left(\max_{0\le\tau\le t}\{\tilde{\delta}_r(\tau)\}\right)\right.$$

$$+\bar{\varphi}_{\nu r}^2\left(\max_{0\le\tau\le t}\{\tilde{\omega}_r(\tau)\} + \frac{5}{4}k_{\delta r}\max_{0\le\tau\le t}\{\tilde{\delta}_r(\tau)\}\right)$$

$$+\bar{\varphi}_{\rho r}^2\left(\max_{0\le\tau\le t}\{\tilde{P}_{er}(\tau)\} + P_{mrM} - P_{mrm} + \frac{\dot{P}_{Mr}}{k_{per}}\right)$$

$$+\frac{2H_r}{\omega_0}\left(\frac{5}{4}k_{\omega r} + \frac{1}{k_{\omega pr}} + \frac{5}{4}k_{\delta r}\right)\max_{0\le\tau\le t}\{\tilde{\omega}_r(\tau)\}$$

$$+\left[\frac{2H_r}{\omega_0}\left(1 + \frac{25}{16}k_{\delta r}^2\right) + \frac{5D_r k_{\delta r}}{4\omega_0}\right]\max_{0\le\tau\le t}\{\tilde{\delta}_r(\tau)\}\right)$$

$$+1\Big]$$

which rely on the estimates $\hat{\theta}_{jr}$, $\hat{x}'_{dr}$, $\hat{B}_{rr}$ of the uncertain parameters θ_{jr}, x'_{dr}, B_{rr} whose estimation laws ($1 \le j \le 6$) [functions $\xi_{\theta_j}(t)$, $\xi_x(t)$, $\xi_b(t)$ are yet to be chosen]

$$\dot{\hat{\theta}}_{jr} = \mathrm{Proj}\Big[\xi_{\theta_j}(t), \hat{\theta}_{jr}, \theta_{jrm}, \theta_{jrM}, \varepsilon_{jr}\Big]$$

$$\theta_{jrm} \le \hat{\theta}_{jr}(0) \le \theta_{jrM}$$

$$\dot{\hat{x}}'_{dr} = \mathrm{Proj}\Big[\xi_x(t), \hat{x}'_{dr}, x'_{drm}, x'_{drM}, \varepsilon_{xr}\Big]$$

$$x'_{drm} \le \hat{x}'_{dr}(0) \le x'_{drM} \qquad (7)$$

$$\dot{\hat{B}}_{rr} = \mathrm{Proj}\Big[\xi_b(t), \hat{B}_{rr}, B_{rrm}, B_{rrM}, \varepsilon_{Br}\Big]$$

$$B_{rrm} \le \hat{B}_{rr}(0) \le B_{rrM}$$

are designed by using the projection algorithm $\mathrm{Proj}[\zeta, \hat{z}_r, z_{rm}, z_{rM}, \varepsilon_{zr}]$ in [15] $[(z_{rm} - \varepsilon_{zr}) > 0]$ whose properties are the following [which hold provided that the uncertain constant z_r and the initial condition $\hat{z}_r(0)$ belong to the compact set $[z_{rm}, z_{rM}]$]: 1. $z_{rm} - \varepsilon_{zr} \le \hat{z}_r(t) \le z_{rM} + \varepsilon_{zr}$, for all $t \ge 0$; 2. $\mathrm{Proj}[\zeta, \hat{z}_r, \cdot, \cdot, \cdot]$ is Lipschitz continuous; 3. $|\mathrm{Proj}[\zeta, \hat{z}_r, z_{rm}, z_{rM}, \varepsilon_{zr}]| \le |\zeta|$; 4. $(z_r - \hat{z}_r)\mathrm{Proj}[\zeta, \hat{z}_r, z_{rm}, z_{rM}, \varepsilon_{zr}] \ge (z_r - \hat{z}_r)\zeta$. Defining the estimation errors ($1 \le j \le 6$): $\tilde{\theta}_{jr} = \theta_{jr} - \hat{\theta}_{jr}$, $\tilde{x}'_{dr} = x'_{dr} - \hat{x}'_{dr}$, $\tilde{B}_{rr} = B_{rr} - \hat{B}_{rr}$ and considering the quadratic function

$$\mathcal{V}_r = \frac{1}{2}\left(\tilde{\delta}_r^2 + \tilde{\omega}_r^2 + \tilde{P}_{er}^2 + \tilde{P}_{mr}^2\right)$$

whose the time derivative of function $\mathcal{V}_r$ along the trajectories of the error system satisfies

$$\dot{\mathcal{V}}_r \le -\frac{5}{4}\left(k_{\delta r}\tilde{\delta}_r^2 + k_{\omega r}\tilde{\omega}_r^2 + k_{pr}\tilde{P}_{er}^2 + k_{per}\tilde{P}_{mr}^2\right)$$

$$+\frac{1}{k_r}\left[\dot{P}_{mr}^2 + \tilde{\theta}_{1r}^2 + \tilde{\theta}_{2r}^2 + \tilde{\theta}_{3r}^2 + \tilde{\theta}_{4r}^2\right.$$

$$+\max\left\{\tilde{\theta}_{5r}^2, \tilde{\theta}_{6r}^2\right\} + \tilde{x}'^2_{dr} + \tilde{B}_{ir}^2$$

$$\left.+\sup_{0\le\tau\le t}\{g_r(\tau)\}^2 + \mu_r^2 + \nu_r^2 + \rho_r^2\right]$$

we can establish [recall property 1. of the projection algorithm] that the robust adaptive nonlinear feedback control algorithm (3)-(7) is a transient stabilizing control for the r-th generator. Suppose that, in addition to assumptions i)-ii), assumptions iii)-iv) hold and consider the quadratic function [β_{jr}, β_{xr}, β_{Br} ($1 \le j \le 6$) are positive reals]

$$\mathcal{W}_r = \mathcal{V}_r + \frac{1}{2}\sum_{j=1}^{6}\beta_{jr}\tilde{\theta}_{jr}^2 + \frac{1}{2}\beta_{xr}\tilde{x}'^2_{dr} + \frac{1}{2}\beta_{Br}\tilde{B}_{ir}^2$$

whose time derivative along the trajectories of the error system, for all $t \ge t_0$, satisfies [recall property 4. of the projection algorithm]

$$\dot{\mathcal{W}}_r \le -\frac{5}{4}k_{\delta r}\tilde{\delta}_r^2 - \frac{5}{4}k_{\omega r}\tilde{\omega}_r^2 - \frac{5}{4}k_{pr}\tilde{P}_{er}^2$$

$$-\frac{5}{4}k_{per}\tilde{P}_{mr}^2 + \frac{R_r(t)^2}{k_{Rr}} \qquad (8)$$

provided that the yet to be defined functions $\xi_{\theta_j}(t)$, $\xi_x(t)$, $\xi_b(t)$ ($1 \le j \le 6$) are chosen as

$$\xi_{\theta_1} = -\frac{P_{er}\tilde{P}_{er}}{\beta_{1r}}, \quad \xi_{\theta_2} = -\frac{I_{dr}I_{qr}\tilde{P}_{er}}{\beta_{2r}},$$

$$\xi_{\theta_3} = -\frac{I_{dr}P_{er}\tilde{P}_{er}}{I_{qr}\beta_{3r}}, \quad \xi_{\theta_4} = -\frac{P_{er}^2\tilde{P}_{er}}{I_{qr}^2\beta_{4r}},$$

$$\xi_{\theta_5} = \frac{I_{qr}^2 v_r\tilde{P}_{er}}{(\hat{\theta}_{5r}I_{qr}^2 + \hat{\theta}_{6r}P_{er})\beta_{5r}}, \qquad (9)$$

$$\xi_{\theta_6} = \frac{P_{er}v_r\tilde{P}_{er}}{(\hat{\theta}_{5r}I_{qr}^2 + \hat{\theta}_{6r}P_{er})\beta_{6r}},$$

$$\xi_x = -\frac{\left(I_{dr}^2 + I_{qr}^2\right)\omega_r\tilde{P}_{er}}{\beta_{xr}}, \quad \xi_b = -\frac{P_{er}^2\omega_r\tilde{P}_{er}}{\beta_{Br}I_{qr}^2}.$$

Since $\tilde{\delta}_r$, $\tilde{\omega}_r$, $\tilde{P}_{er}$, $\tilde{P}_{mr}$ are bounded, $\dot{\tilde{\delta}}_r$, $\dot{\tilde{\omega}}_r$, $\dot{\tilde{P}}_{er}$, $\dot{\tilde{P}}_{mr}$ are bounded so that $\tilde{\delta}_r(t)$, $\tilde{\omega}_r(t)$, $\tilde{P}_{er}(t)$, $\tilde{P}_{mr}(t)$ are uniformly continuous for all $t \ge t_0$. On the other hand, by virtue of assumptions iii)-iv), if $k_{Rr} > \max\left\{\frac{4\bar{\gamma}_{\delta r}}{k_{\delta r}}, \frac{4\bar{\gamma}_{\omega r}}{k_{\omega r}}, \frac{16\gamma_{pr}}{k_{pr}}, \frac{16\gamma_{pr}}{k_{per}}\right\}$ with

$$\bar{\gamma}_{\delta r} = \gamma_{\delta r} + \frac{25}{8}\gamma_{\omega r}k_{\delta r}^2 + 4\left[\frac{2H_r}{\omega_0}\left(\frac{25}{16}k_{\delta r}^2 + 1\right)\right.$$

$$\left.+\frac{5}{4}\frac{D_r k_{\delta r}}{\omega_0}\right]^2\gamma_{pr}$$

$$\bar{\gamma}_{\omega r} = 2\gamma_{\omega r} + 4\left[\frac{2H_r}{\omega_0}\left(\frac{5}{4}k_{\omega r} + \frac{5}{4}k_{\delta r} + \frac{1}{k_{\omega pr}}\right)\right]^2\gamma_{pr}$$

then, according to (3), (4), (8) and Barbalat's Lemma, we can establish that the robust adaptive nonlinear feedback control algorithm (3)-(7), (9) is a transient adaptive stabilizing control for the r-th generator. The result holds for any initial condition (of the r-th generator) and positive control parameter k_r maintaining, according to (S1), $\delta_r(t) \in \mathcal{D}_\delta$ and $P_{er}(t) \in \mathcal{D}_p$ (guaranteeing $I_{qr}(t) > c_{Ir} > 0$) for all $t \geq 0$. The main result of this paper can be summarized in the following theorem, which somehow[2] extends the recent theoretical contribution in [15].

Theorem: *The robust adaptive nonlinear feedback control algorithm (3)-(7), (9) is:*

- *a transient stabilizing control for the r-th generator;*
- *a transient adaptive stabilizing control for the r-th generator when*

$$k_{Rr} > \max\left\{ \frac{4\bar{\gamma}_{\delta r}}{k_{\delta r}}, \frac{4\bar{\gamma}_{\omega r}}{k_{\omega r}}, \frac{16\gamma_{pr}}{k_{pr}}, \frac{16\gamma_{pr}}{k_{per}} \right\},$$

for any initial condition (of the r-th generator) and positive control parameter k_r maintaining, according to (S1), $\delta_r(t) \in \mathcal{D}_\delta$ and $P_{er}(t) \in \mathcal{D}_p$ (guaranteeing $I_{qr}(t) > c_{Ir} > 0$) for all $t \geq 0$.

V. SIMULATION RESULTS

In this section we illustrate the performance and the robustness of the feedback control algorithm (3)-(7), (9) in the presence of unmodelled dynamics: the proposed control is applied to each generator of the popular Western System Coordinating Council (WSCC) 3-machine, 9-bus system reported in [19] and [1] [$D_i = 0$, $1 \leq i \leq 3$] and described by the two-axis model in [19] from which the model (1) has been derived by neglecting the dynamics of the fast damper-winding E'_{di} and by using the simplification $x_{qi} = x'_{di}$ with x_{qi}(p.u.) being the quadrature axis reactance. The initial conditions for the state variables are computed by systematically solving the load-flow equations of the network and by computing the values of the algebraic variables. For $1 \leq i \leq 3$, the functions $\bar{\varphi}_{\mu i}(\cdot)$, $\bar{\varphi}_{\nu i}(\cdot)$, $\bar{\varphi}_{\rho i}(\cdot)$ are set equal to $\mathbb{I}(\cdot)\big|_{[0,\infty)}$ [$\mathbb{I}(\cdot)$ is the identity function], the control parameters are chosen as $k_{\delta i} = k_{\omega i} = k_{pei} = k_{\omega pi} = 1$, $k_{pi} = 720$, $k_i = 0.001$, $k_{Ri} = 0.1$, $\beta_{1i} = \beta_{2i} = \beta_{3i} = \beta_{4i} = \beta_{5i} = \beta_{6i} = \beta_{xi} = \beta_{Bi} = 348000$ while the initial conditions for the parameter estimates are set equal to the corresponding nominal values. In order to avoid division by zero, for $1 \leq i \leq 3$, $\eta(P_{ei})$ and $\eta(I_{qi})$ replace P_{ei} and I_{qi} in the control algorithm (3)-(7), (9), respectively, where $\eta(\xi) = \begin{cases} \xi, & \text{if } \xi \geq 0.05 \\ 0.05, & \text{otherwise.} \end{cases}$ The goal of the simulation is to verify the effects of a three-phase fault occurring near bus 7 at the end of line 5-7 at $t = 0.001$ s, which is cleared at 0.084 s by opening line 5-7 (circuit breakers reclose at $t = 1$ s). Figure 1-2 show a satisfactory performance of the proposed control even with the physical saturations (which are hit several times according to Fig. 2(b)): despite the considered
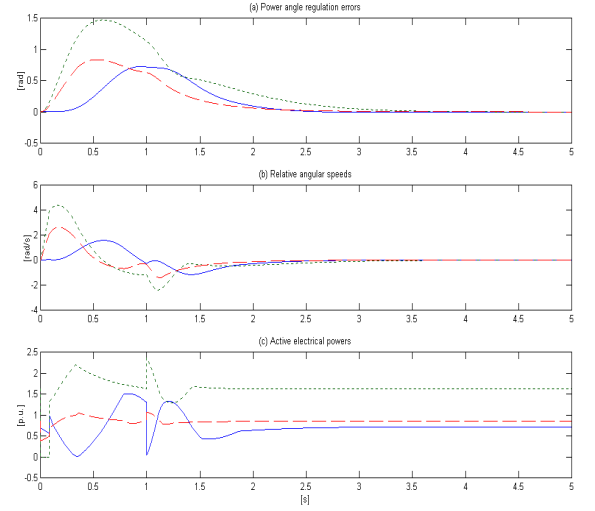

Fig. 1. Proposed control algorithm [Generator 1 (solid), Generator 2 (dot), Generator 3 (dash)]: a) Power angle regulation errors $\delta_i - \delta_{is}$ ($1 \leq i \leq 3$); b) Relative angular speeds ω_i ($1 \leq i \leq 3$); c) Active electrical powers P_{ei} ($1 \leq i \leq 3$).

severe perturbation, synchronous speeds are quickly restored while fast regulation of both power angle and electrical power is guaranteed. For comparison, the same simulation is performed by applying (to each generator of the network) the nonlinear robust adaptive control (with control parameters $k_\delta = 0.1$, $k_\omega = 3$, $k_p = 120$, $k = 0.001$, $\beta_i = 0.0000001$, $1 \leq i \leq 7$ and initial conditions[3] $\hat{\theta}_i(0) = 0$ $i = 1, 2, 4, 5$, $\hat{\theta}_6(0) = \frac{u_{f0} \sin(\delta(0))}{\bar{v}_2(0)}$, $\hat{\theta}_3(0) = P_m$, $\hat{\theta}_7(0) = 1$) designed in [15] on the basis of a (third order) single machine-infinite bus model. As illustrated by the simulation results reported in Fig. 3, neglecting the transient behaviour of the other generators and the interconnections between them may be critical for the control design in [15]: in the presence of the considered perturbation, synchronous speeds can not quickly restored.

CONCLUSIONS

A robust adaptive nonlinear feedback control (3)-(7), (9) has been designed on the basis of a third order model (1) of a synchronous generator in an uncertain power network with transfer conductances. The proposed controller guarantees the $\mathcal{L}_2$ and $\mathcal{L}_\infty$ disturbance attenuation and asymptotic regulation properties (S1)-(S3) under assumptions on the network dynamics generalizing those required by the single machine-infinite bus approximation used in [15].

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[2]Here the machine damping and inertia constants are assumed to be known.
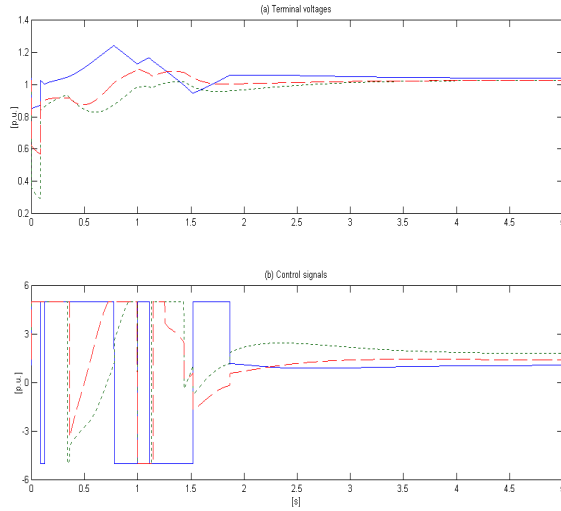
[3]We denote by u_{f0} the nominal input value.

Fig. 2. Proposed control algorithm [Generator 1 (solid), Generator 2 (dot), Generator 3 (dash)]: a) Terminal voltages V_{ti} ($1 \leq i \leq 3$); b) Control signals u_{fi} ($1 \leq i \leq 3$).
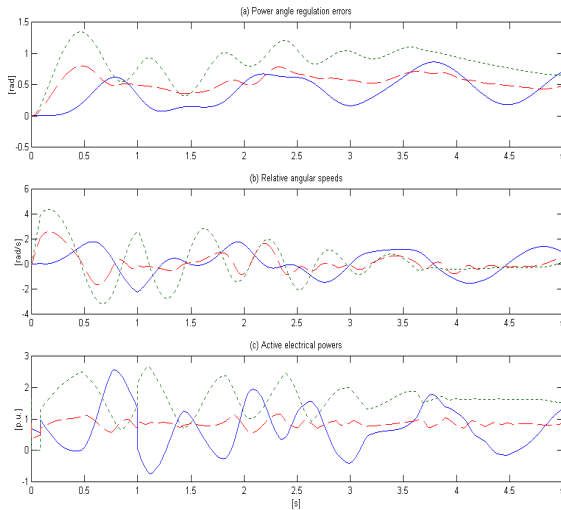

Fig. 3. Robust adaptive control in [15] [Generator 1 (solid), Generator 2 (dot), Generator 3 (dash)]: a) Power angle regulation errors $\delta_i - \delta_{is}$ ($1 \leq i \leq 3$); b) Relative angular speeds ω_i ($1 \leq i \leq 3$); c) Active electrical powers P_{ei} ($1 \leq i \leq 3$).

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