A novel distributed secondary frequency control scheme for power networks with high order turbine governor dynamics

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Abstract— We consider the problem of distributed secondary frequency regulation such that stability and an economically optimal allocation are attained. We study an arbitrarily connected power network with general linear generation dynamics and use a swing equation to describe frequency dynamics. We propose a distributed averaging dynamic output control (DADOC) scheme that makes use of local frequency and generation measurements and provides stability and optimality guarantees when a broad class of high order turbine governor dynamical systems with quadratic cost functions are considered. The proposed controller includes a number of design parameters that affect the stability and optimality properties of the system. We choose these parameters in a computationally efficient way by solving appropriate linear matrix inequalities (LMIs). Furthermore, we demonstrate how the proposed analysis applies to several examples of turbine governor models. Moreover, to highlight the contribution of our work, we compare and explain the main advantages of DADOC schemes over existing schemes in the literature, in terms of required measurements for their implementation and allowable generation dynamics and cost functions such that stability and optimality are guaranteed. The practicality of our analysis is demonstrated with simulations on the Northeast Power Coordinating Council (NPCC) 140-bus system that verify that our proposed controller achieves convergence to nominal frequency and an economically optimal power allocation.

I. Introduction

Motivation: There is currently a growing attention on renewable sources of generation as a result of environmental concerns, with their penetration in power networks expected to grow over the next years [1], [2]. This will greatly increase the amount of active elements in the power network making its electromechanical behaviour less predictable and centralized control approaches increasingly difficult to implement. This highlights the importance of investigating distributed schemes that will guarantee power network stability when such devices are included. Over the past few years, these concerns have motivated research on distributed schemes with applications on both primary [3], [4] and secondary frequency control [5], [6], [7].

The introduction of highly distributed schemes for frequency control raises an issue of economic optimality in the power allocation. Attempts to resolve this issue in the literature resulted in devising appropriately constructed optimization problems that ensure economic optimality and asking for the system equilibria to be solutions of these problems. It is evident in the literature that a synchronising variable is useful for optimality to be achieved. While in primary control studies frequency is used as the synchronizing signal [3], [8], [9], [10], in secondary control studies some other signal, resulting from a suitably designed controller, has to be employed (e.g. [5], [11], [12]). Therefore, the study of how distributed controllers that achieve optimality for secondary frequency regulation should be designed is an interesting and applicable problem.

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Literature survey: There are many recent studies associated with distributed stability and optimality in secondary frequency control. Many of those, involve control schemes with dynamics that follow from a primal/dual algorithm associated with some optimal power allocation optimisation problem [6], [11], [13], [14], [15]. This approach allows to take into account economic considerations along with the objectives of secondary frequency control. Furthermore, it allows for stability guarantees when high order and non linear generation dynamics are considered. However, such schemes require knowledge of demand, which can in some cases limit their practicality.

A different approach for optimal distributed secondary frequency regulation involves the use of distributed averaging proportional integral (DAPI) controllers [5], [12], [16], [17]. DAPI controllers are simpler than primal/dual inspired ones, requiring only knowledge of local frequency and exchange a synchronization signal without requiring any generation or load measurements. On the other hand, existing results in literature do not accommodate first or higher-order generation dynamics and are limited to the case of proportional active power sharing.

Main contributions: In this paper, we propose a control scheme, that will be referred to as distributed averaging dynamic output control (DADOC), for optimal distributed secondary frequency regulation. The proposed scheme dynamics achieve the synchronization of an exchange variable, which is necessary for optimality, and contain frequency dependent terms that ensure that the frequency attains its nominal value at equilibrium. Furthermore, DADOC schemes include feedback from generation output, which is a key part in their design, enhancing their stability properties, and allowing the inclusion of a broad range of linear generation dynamics.

Our analysis provides a systematic way to deduce stability and optimality for a broad range of linear systems, including those of higher order, when quadratic cost functions are considered. DADOC schemes have the advantage over DAPI schemes that they allow the inclusion of first and high order generation dynamics, by imposing only an additional condition for knowledge of generation output. Although primal dual inspired schemes are also able to provide stability guarantees when high order generation schemes are considered, those impose a requirement for knowledge of demand, which can in some cases be restrictive. DADOC controllers share benefits from both schemes, with mild measurement requirements (generation and frequency) and allow for stability to be deduced when high order generation dynamics are included. Furthermore, DADOC schemes allow the inclusion of highly relevant generation dynamics that are not included in any of the aforementioned control schemes.

The contribution of this work, is not only limited to the design of the new controller, but extends to providing conditions on the design gains, such that stability and optimality are guaranteed. An important feature on the proposed conditions, is that those can be efficiently verified by means of a linear matrix inequality (LMI). Various examples of

relevant generation dynamics are provided to demonstrate the importance of our analysis.

Paper structure: The remainder of the paper is structured as follows: Section II provides some basic notation and in Section III we present the power network model and the generation dynamics. In Section IV we present our proposed controller and provide conditions such that an optimal power allocation is achieved at steady state. In Section V we present the main stability result of this paper, and the relevant conditions imposed to guarantee it. In Section VI we discuss various applications of the proposed results and clarify the contribution of our analysis by comparing it with existing literature. Our results are numerically validated in Section VII, where convergence to nominal frequency and an economically optimal allocation are demonstrated at the presence of high order turbine governor dynamics. Finally, conclusions are drawn in Section VIII.

Note that the proofs of the main results are omitted due to space constrains and will be provided in an extended version of this paper.

II. NOTATION

Real numbers are denoted by $\mathbb{R}$, and the set of n-dimensional vectors with real entries is denoted by $\mathbb{R}^n$. For a function $f(q)$, $f : \mathbb{R} \rightarrow \mathbb{R}$, we denote its first derivative by $f'(q) = \frac{d}{dq}f(q)$, its inverse by $f^{-1}(.)$. A function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is said to be positive definite if $f(0) = 0$ and $f(x) > 0$ for every $x \neq 0$. We write $\mathbf{0}_n$ and $\mathbf{1}_n$ to denote $n \times 1$ vectors with all elements equal to 0 and 1 respectively. We denote by $\mathcal{I}_k$ for some $k > 0$ the identity matrix of rank k. For a discrete set S, the term $|S|$ denotes its cardinality. A matrix A is said to be Hurwitz if all its eigenvalues lie on the open left half plane [18]. A matrix A is said to be positive definite (semi-definite) when $x^T A x > 0$ for all $x \neq 0$ (when $x^T A x \geq 0$ for all x). Finally, $\text{Im}(A)$ denotes the range of a matrix $A \in \mathbb{R}^{m \times n}$.

III. POWER NETWORK DYNAMICS

In this section, we present a mathematical description of the power network and a general class of linear generation dynamics that is considered within the rest of the manuscript. Furthermore, we discuss how stability considerations within primary frequency control timeframe can affect the choice of generation droop coefficients, following existing results from literature. The latter will be associated with the secondary frequency regulation analysis presented later in this manuscript.

A. Network model

We use a connected graph (N, E) to describe the power network, where $N = \{1, 2, \ldots, |N|\}$ is the set of buses and $E \subseteq N \times N$ the set of transmission lines connecting the buses. There are two types of buses in the network, buses with inertia and buses without inertia. Since generators have inertia, it is reasonable to assume that only buses with inertia have non-trivial generation dynamics. We define $G = \{1, 2, \ldots, |G|\}$ and $L = \{|G| + 1, \ldots, |N|\}$ as the sets of buses with and without inertia respectively such that $|G| + |L| = |N|$. Moreover, the term (i, j) denotes the link connecting buses i and j. The graph (N, E) is assumed to be directed with an arbitrary direction, so that if $(i, j) \in E$ then $(j, i) \notin E$. Additionally, for each $j \in N$, we use $i : i \rightarrow j$ and $k : j \rightarrow k$ to denote the sets of buses that precede and succeed bus j respectively. It should be noted that the form of the dynamics in (1)–(2) below is

not affected by changes in graph ordering, and our results are independent of the choice of direction. We make the following assumptions for the network:
1) Bus voltage magnitudes are $|V_j| = 1$ p.u. for all $j \in N$.
2) Lines $(i, j) \in E$ are lossless and characterized by their susceptances $B_{ij} = B_{ji} > 0$.
3) Reactive power flows do not affect bus voltage phase angles and frequencies.

Following the above, we can make use of swing equations to describe the rate of change of frequency at generation buses. Moreover, power must be conserved at each of the load buses. This motivates the following system dynamics (e.g. [19]),

$$\dot{\eta}_{ij} = \omega_i - \omega_j, \ (i, j) \in E, \tag{1a}$$

$$M_j \dot{\omega}_j = -p_j^L + p_j^M - \Lambda_j \omega_j - \sum_{k:j \rightarrow k} p_{jk} + \sum_{i:i \rightarrow j} p_{ij}, \ j \in G, \tag{1b}$$

$$0 = -p_j^L - \Lambda_j \omega_j - \sum_{k:j \rightarrow k} p_{jk} + \sum_{i:i \rightarrow j} p_{ij}, \ j \in L, \tag{1c}$$

$$p_{ij} = B_{ij} \sin \eta_{ij}, \ (i, j) \in E. \tag{1d}$$

In system (1), the time-dependent variable ω_j represents the deviation of the frequency at bus j from its nominal value, namely 50Hz (or 60Hz). The time dependent variables p_j^M represent the mechanical power injection to the generation bus j. The constant Λ_j represents the frequency damping coefficient at bus j. The time-dependent variables η_{ij} and p_{ij} represent, respectively, the power angle difference and the power transferred from bus i to bus j. The constant $M_j > 0$ denotes the generator inertia. The response of the system (1) will be studied, when a step change p_j^L, $j \in N$ occurs in the uncontrollable demand.

B. Generation Dynamics

To investigate broad classes of dynamics and control policies for generation systems, we consider dynamical systems of the form

$$\begin{aligned} \dot{x}_j^M &= A_j x_j^M + B_j u_j, \\ p_j^M &= C_j x_j^M + D_j u_j, \end{aligned}, j \in G \tag{2}$$

with input $u_j(t) \in \mathbb{R}$, state $x_j^M(t) \in \mathbb{R}^{n_j}$, output $p_j^M(t) \in \mathbb{R}$ and corresponding matrices $A_j \in \mathbb{R}^{n_j \times n_j}, B_j \in \mathbb{R}^{n_j}, C_j \in \mathbb{R}^{1 \times n_j}$ and $D_j \in \mathbb{R}$. We assume in (2) that A_j is Hurwitz which implies that given any constant input $u_j(t) = \bar{u}_j$, there exists an asymptotically stable equilibrium point $\bar{x}_j^M \in \mathbb{R}^{n_j}$, such that $A_j \bar{x}_j^M + B_j \bar{u}_j = 0$. Correspondingly, there exists a constant $K_j \in \mathbb{R}$, satisfying $K_j = -C_j A_j^{-1} B_j + D_j$, such that for any constant input $\bar{u}_j$ and corresponding state $\bar{x}_j$, the output $\bar{p}_j^M$ is given by

$$\bar{p}_j^M = C_j \bar{x}_j^M + D_j \bar{u}_j = K_j \bar{u}_j. \tag{3}$$

Our aim in this paper is to provide conditions on the dynamics described in (2) and the choice of input u_j that allow for stability and optimality guarantees and ensure the satisfaction of secondary frequency control objectives.

C. Stability in primary frequency control

The main objective of primary frequency regulation is to balance generation and demand. Within this subsection, we will consider conditions imposed in literature to provide decentralized stability guarantees for primary frequency regulation. These conditions will later be extended when dynamics associated with secondary frequency control are taken into account.

We let the generation input u_j be described by

$$u_j = -k_{d,j}\omega_j, j \in G \tag{4}$$

where $k_{d,j}$ are positive constants related to the droop gains. To obtain decentralized conditions on the design parameters $k_{d,j}$ and generation dynamics (2) such that stability is guaranteed, we will make use of the results presented in [3]. From the analysis in [3], the following input strict passivity assumption should hold for the system (2),(4) when some frequency damping term $\Lambda_j\omega_j$ is present[1].

Assumption 1: Consider the systems with input $-\omega_j$ and output $p_j^M - \Lambda_j\omega_j$ described by (1),(2),(4). Then, for each of these systems, there exists a symmetric positive definite matrix $P_j = P_j^T \in \mathbb{R}^{n_j \times n_j}$ such that the functions $V_j(x_j^M) = \frac{1}{2}(x_j^M)^T P_j x_j^M$ satisfy

$$\dot{V}_j \leq -\omega_j p_j^M - \phi_{u,j}(-\omega_j)^2, \tag{5}$$

for some $\phi_{u,j} > 0$, for all $j \in G$.

Assumption 1 follows directly from the conditions imposed in [3]. It has the important property that it can be efficiently verified with appropriate LMI conditions that follow from the KYP lemma [20]. In particular, it can be verified that Assumption 1 holds for given gain $k_{d,j}$ when there exists a positive definite symmetric matrix P_j, such that

$$\hat{Q}_j = \begin{bmatrix} \frac{P_j A_j + A_j^T P_j}{2} & \frac{k_{d,j} P_j B_j - C_j^T}{2} \\ \frac{k_{d,j} B_j^T P_j - C_j}{2} & -\hat{\Lambda}_j - D_j k_{d,j} \end{bmatrix} \leq 0 \tag{6}$$

holds for some $\hat{\Lambda}_j < \Lambda_j$.

We will see in the following sections how the condition in (6) extends when dynamics related with secondary frequency regulation are also included. This will demonstrate that some of the imposed decentralized conditions on stability for secondary frequency regulation can be seen as existing stability requirements for primary frequency control.

IV. DISTRIBUTED AVERAGING DYNAMIC OUTPUT CONTROLLER

In this section we propose a novel secondary control scheme, called distributed averaging dynamic output controller (DADOC) which, as discussed in Section VI-B, offers advantages over existing distributed schemes for secondary frequency regulation.

We consider a communication network described by a connected graph $(G, \tilde{E})$. We propose the following DADOC scheme

$$\gamma_j \dot{p}_j^c = p_j^M - K_j u_j - k_{f,j}\omega_j + \sum_{i:(i,j)\in\tilde{E}} \alpha_{ij}(p_i^c - p_j^c), j \in G \tag{7}$$

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[1] Note that the condition in [3] also applies to non linear systems and is defined locally. However, we simplify it here to fit the purpose of this study.

where $\alpha_{ij} = \alpha_{ji}, \gamma_j$ and $k_{f,j}$ are positive constant gains of the controller, and p_j^c is a power command signal. Furthermore, the generation input in (4) is extended to

$$u_j = k_{c,j}p_j^c - k_{d,j}\omega_j, j \in G \tag{8}$$

where $k_{c,j}, k_{d,j}$ are positive constants. Note that $k_{d,j}$ follows from primary frequency control design, while $k_{c,j}$ is a design parameter for secondary frequency control.

The controller dynamics in (7) ensure that power command variables synchronize at steady state, a necessary feature for optimality interpretation. Furthermore, the frequency dependent term in (7) ensures that when generation and power command variables reach steady state and power command variables synchronize, then steady state frequency must attain its nominal value. The term $p_j^M - K_j u_j$ in (7) does not affect the steady state of the power command variables. However, as shall be seen in the subsequent analysis, it has a pivotal role in providing stability guarantees when high order generation dynamics of the form (2) are considered.

We choose the generation input u_j to be a weighted sum of frequency and power command, allowing the weight coefficients $k_{c,j}$ to be design parameters. In what follows, we provide conditions on how these parameters should be chosen such that convergence to nominal frequency is achieved while also taking economic considerations into account.

A. Equilibrium analysis

We now describe what is meant by an equilibrium of the interconnected system (1), (2), (7), (8).

Definition 1: The point $\beta^* = (\eta^*, \omega^*, x^{M,*}, p^{c,*})$ defines an equilibrium of the system (1), (2), (7), (8) if all time derivatives of (1), (2), (7), (8) are equal to zero at this point.

In our analysis, we shall consider conditions on controller design variables such that the generation equilibrium values solve an economic optimisation problem, as will be described in Section IV-B.

Throughout the paper, it is assumed that there exists some equilibrium of (1), (2), (7), (8), denoted by $\beta^* = (\eta^*, \omega^*, x^{M,*}, p^{c,*})$, as defined in Definition 1. Note that the study on the existence of equilibria is beyond the scope of this paper and the interested reader is referred to [21]. Furthermore, we use $(p^*, p^{M,*}, u^*)$ to represent the equilibrium values of respective quantities in (1), (2), (7), (8).

The following lemma characterizes the equilibria of the system (1), (2), (7), (8). It demonstrates that the frequency attains its nominal value at steady state and that the power command variables synchronize, a useful property when an optimality interpretation of the equilibria is desired.

Lemma 1: Any equilibrium point β^* given by Definition 1 satisfies $\omega^* = \mathbf{0}_{|N|}$ and $p^{c,*} \in \text{Im}(1_{|G|})$.

Furthermore, the power angle differences at the considered equilibrium are assumed to satisfy the following security constraint. It will be seen that Assumption 2 is required for the convergence results presented in Section V.

Assumption 2: $|\eta_{ij}^*| < \frac{\pi}{2}$ for all $(i,j) \in E$.

The stability and optimality properties of such equilibria will be studied in the following sections.

B. Optimality analysis

We aim to study how generation should be adjusted in order to meet the step change in frequency independent demand and simultaneously minimize the cost that comes from the deviation in the power generated. We now introduce an optimization problem, which we call the optimal generation

regulation problem (OGR), that can be used to achieve this goal.

A quadratic cost is supposed to be incurred when the generation output at bus j is p_j^M. Note that quadratic cost functions are frequently used in the literature, [22], [23], motivated by the fact that a convex function can be locally approximated by a quadratic one. The problem is to find the vector p^M that minimizes this total cost and simultaneously achieves power balance. More precisely, the following optimization problem is considered

OGR:

$$\min_{p^M} \sum_{j \in G} \frac{1}{2} q_j (p_j^M)^2, \tag{9}$$

$$\text{subject to } \sum_{j \in G} p_j^M = \sum_{j \in N} p_j^L,$$

where q_j are positive cost coefficients associated with the generation cost at bus j. Note that more general quadratic cost functions can be considered, following similar approaches as in relevant literature (e.g. [24]). However, we opt for the cost functions in (9) for simplicity. The equality constraint in (9) requires all the frequency-independent loads to be matched by the total generation. This ensures that when system (1) is at equilibrium, then frequency will be at its nominal value, as follows from summing (1b)–(1c) at equilibrium over all buses.

Within the paper, we aim to specify properties on the control dynamics of p^M, described in (2), that ensure that those quantities converge to values at which optimality can be guaranteed for (9). Below, we demonstrate how the controller gains in (7)–(8) can be chosen to ensure that the equilibrium points of the system are solutions to the OGR problem (9). We will then demonstrate in the subsequent section, how convergence to optimality can be achieved.

Proposition 1: Consider equilibria of (1), (2), (7), (8), characterized by Lemma 1. Then, if the control dynamics in (8) are chosen such that

$$k_{c,j} = \frac{1}{q_j K_j}, j \in G, \tag{10}$$

holds, then the equilibrium values $p^{M,*}$ are optimal for the OGR problem (9).

Remark 1: Proposition 1 provides conditions on the choice of the design variable $k_{c,j}$ such that the equilibria of system (1), (2), (7), (8), described by Lemma 1 are solutions to the OGR problem (9). Note that the design variables $k_{f,j}$ and $k_{d,j}$ do not appear in the optimisation problem, since those are gains on frequency deviation, which becomes zero at equilibrium. However, it will be seen in the following section that their values have a significant impact on the stability properties of the system.

V. STABILITY ANALYSIS

This section contains our main convergence results. We provide appropriate conditions on the choice of gains in (7)–(8), applicable to highly relevant generation schemes, and show that when those are satisfied, then convergence is guaranteed.

A. Controller design conditions

In this section, we impose a condition involving design constants $k_{c,j}, k_{d,j}$ and $k_{f,j}$, which is used in the convergence theorem presented in Section V-B below. We then explain how this condition can be numerically tested in a computationally efficient way. The considered condition is presented below.

Design condition 1: For each generation bus j, with dynamics described by (1), (2), (7), (8), the controller parameters $k_{c,j}, k_{d,j}$ and $k_{f,j}$ are such that

$$\begin{bmatrix} -K_j k_{c,j} + D_j k_{c,j} & r_j^T \\ r_j & \hat{Q}_j \end{bmatrix} \leq 0, \tag{11}$$

where $r_j = \begin{bmatrix} \frac{k_{c,j} B_j^T P_j + C_j}{2} & \frac{k_{f,j} - k_{d,j} K_j + D_j k_{d,j} - D_j k_{c,j}}{2} \end{bmatrix}^T$, holds for some $P_j = P_j^T > 0$ and some $\hat{\Lambda}_j < \Lambda_j$.

Design condition 1 is the main stability condition imposed on this paper, and is feasible for a broad class of linear systems, as discussed in Section VI.

Design condition 1 can be interpreted as an extension of Assumption 1 to secondary frequency control. This is since Assumption 1 is a necessary condition for Design condition 1 to hold, as follows from noting that the matrix $\hat{Q}_j$ in (6) is a principal submatrix of the one considered in (11). Therefore, part of the stability conditions imposed for secondary frequency regulation can be seen as conditions for stability in primary frequency control.

Remark 2: The inequality condition in (11) is an LMI with respect to the matrix P_j and design variables $k_{f,j}$ and can therefore be verified in a computationally efficient way. Furthermore, the condition in (11) can be used to formulate various optimization problems that may make use of the flexibility in choosing the matrix P_j and the design variable $k_{f,j}$. An example of such problem would be to obtain the minimum frequency damping Λ_j that is required for (11) to hold for particular generation dynamics. Hence, (11) can be also useful in system design.

B. Main result

We are now in a position to state our main result, demonstrating convergence to an optimal point of (9) where frequency attains its nominal value.

Theorem 1: Consider an equilibrium of (1), (2), (7), (8) such that Assumption 2 holds and let Design condition 1 and (10) be satisfied. Then, there exists an open neighborhood of initial conditions about this equilibrium such that the solutions of (1), (2), (7), (8) asymptotically converge to a set of equilibria that solve the OGR problem (9) with $\omega^* = \mathbf{0}_{|N|}$.

Theorem 1 demonstrates local convergence to an optimal solution of the OGR problem (9) that, following Lemma 1, satisfies $\omega^* = \mathbf{0}_{|N|}$. The main conditions for stability are Assumption 2, which is abundant in power literature, and Design condition 1. In the following section, we demonstrate the relevance of Design condition 1, explaining how it applies to various generator models.

VI. DISCUSSION

In this section we demonstrate the applicability of our main results with examples of highly relevant generation schemes that fit within the considered analysis. Moreover, we explain the contribution of our results relative to the existing literature.

A. Applications of main results

To demonstrate the relevance of our analysis, we provide an example of second order turbine governor dynamics and explain how our proposed conditions apply to it.

An important aspect of the proposed framework is that it allows the inclusion of high order generation dynamics. A

significant example of such, is the following second order model describing turbine governor dynamics (e.g. [19]),

$$\dot{\alpha}_j = -\frac{1}{\tau_{a,j}}(\alpha_j - K_j(k_{c,j}p_j^c - k_{d,j}\omega_j)), \qquad (12a)$$

$$\dot{p}_j^M = -\frac{1}{\tau_{p,j}}(p_j^M - \alpha_j), \qquad (12b)$$

where α_j is the internal state of the model and $\tau_{a,j}, \tau_{b,j} > 0$ are time constants associated with the generation dynamics. We considered the case where (12) is coupled with the power command dynamics described by (7) and some frequency damping Λ_j. The following lemma provides a sufficient condition for the frequency damping Λ_j such that Design condition 1 holds for the considered system.

Lemma 2: Consider a bus j with generation dynamics described by (12) coupled with the power command dynamics (7) and some frequency damping Λ_j. Then, Design condition 1 holds for any positive values of $\tau_{a,j}, \tau_{p,j}$ if

$$\Lambda_j > \frac{K_j}{3k_{c,j}}(k_{c,j}^2 - k_{c,j}k_{d,j} + k_{d,j}^2))$$

holds.

Lemma 2 provides a sufficient condition for the value of frequency damping Λ_j such that Design condition 1 holds at a bus with generation and power command dynamics described by (12) and (7) respectively. Note that this condition can be made less conservative when particular values for $\tau_{a,j}, \tau_{p,j}$ are considered.

Note that numerical analysis demonstrates that our proposed results also apply to higher than second order generation schemes. However, these results are omitted due to space constraints.

B. Comparison with existing literature

The problem of addressing issues of stability and optimality for secondary frequency regulation in a distributed way has been widely studied over the last years. Most studies focused on two particular schemes which are briefly discussed below.

A trend in secondary frequency control literature is to consider a distributed averaging proportional integral (DAPI) controller [5], [12], [16], [17]. The benefits of this scheme lie in its simplicity, since further than exchanging a synchronizing variable, it only requires knowledge of the local frequency which is easily obtainable. However, existing stability results along this setting are limited to static generation models and quadratic cost functions.

Another approach in literature is to consider power command dynamics that follow from a primal/dual algorithm associated with some optimization problem [6], [11], [13], [14], [15], which ensures secondary frequency control objectives are met. For convenience, we shall refer to those schemes as 'Primal-Dual'. Primal-Dual schemes have the significant advantage that they allow for stability guarantees when high order and non linear generation dynamics are included. Furthermore, they allow for economic optimality when general convex cost functions are considered. However, these controllers are more complicated than DAPI schemes, requiring knowledge of demand, which can be difficult to obtain in many cases, and generation.

DADOC schemes share benefits of both mentioned control schemes, allowing for stability guarantees when high order generation dynamics are considered and requiring measurements of frequency and generation output, which

	DAPI	DADOC	Primal-Dual
Allowable cost function models	Quadratic	Quadratic	Convex
Allowable generation dynamics	Static	High Order	High Order and Non linear
Required measurements	Frequency	Frequency Generation	Generation Demand

TABLE I: Comparison between our proposed (DADOC) controller and the two dominant schemes in literature on allowable cost function models and generation dynamics and required measurements for their implementation.

are not restrictive to obtain. In particular, DADOC schemes have the advantage over DAPI schemes that they allow the inclusion of high order generation dynamics. This comes in the expense of an additional requirement for generation output measurements. Both Primal-Dual and DADOC schemes allow for stability and optimality guarantees when high order generation dynamics and quadratic cost functions are considered. However, Primal-Dual schemes additionally allow to incorporate non linear generation dynamics and general convex cost functions. The main disadvantage of Primal-Dual schemes is their requirement for measurements of local demand, which can in some cases be difficult to obtain. On the other hand, DADOC schemes allow for stability guarantees for high order generation dynamics without requiring knowledge of demand. It is important to also note that DADOC schemes allow the inclusion of classes of generation dynamics that cannot be incorporated when a Primal-Dual controller is implemented. One such example is described by (12) where, unlike when Primal-Dual schemes are considered, no static dependence on power command is required. Table I summarizes the comparison between DADOC, DAPI and Primal-Dual schemes.

Remark 3: A further attempt to address distributed secondary frequency regulation issues has been made in [24], where the authors propose a controller which allows for stability guarantees when first or second order generation dynamics are considered, by making use of measurements of generators internal states. The main differences of DADOC schemes compared to those in [24], are the different measurement requirements for their implementation (generation output instead of internal states) and that the former provides design conditions for power networks with higher than second order generation dynamics.

VII. SIMULATION ON THE NPCC 140-BUS SYSTEM

In this section we verify our analytic results with numerical simulations on the Northeast Power Coordinating Council (NPCC) 140-bus interconnection system, performed using the Power System Toolbox [25]. This model is more detailed and realistic than our analytical one, including line resistances, a DC12 exciter model, a subtransient reactance generator model, and higher order turbine governor models.

The test system consists of 93 load buses serving different types of loads including constant active loads and 47 generation buses. The overall system has a total real power of 28.55GW. For our simulation, we added three loads on units 2, 9, and 17, each having a step increase of magnitude 1 p.u. (base 100MVA) at $t = 1$ second.

To demonstrate the applicability of the proposed controller, the dynamics in (7)–(8) where implemented on 16 generators with third, fourth and fifth order turbine governor dynamics. Furthermore, in order to verify our optimality analysis, we considered quadratic cost functions with cost coefficients
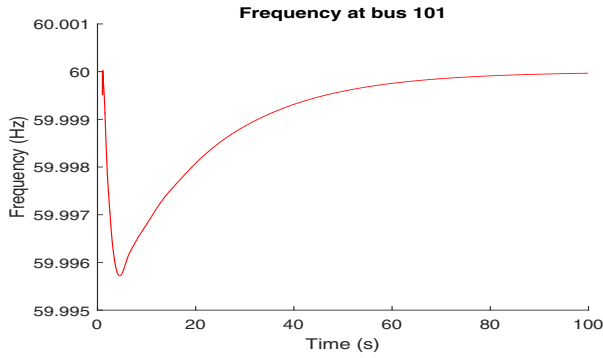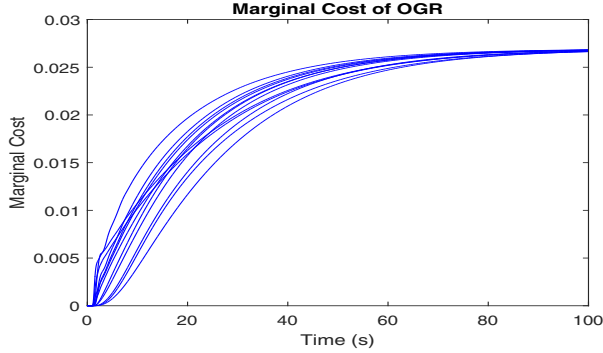
Fig. 1: Frequency at bus 101 of the NPCC network.

Fig. 2: Marginal cost of generation buses contributing to secondary frequency control.

equal to K_j^{-1} that penalised deviations on generation outputs. The choice of cost coefficients relates high cost coefficients with small droop gains, and is consistent with current results in literature on optimal frequency regulation [3]. However, it has been numerically verified that the stability and optimality properties of the system, demonstrated below, are retained for a broad range of cost coefficient values. Controller parameters were selected such that Design condition 1 and optimality codition (10) were satisfied.

The frequency response of a randomly selected bus is depicted on Figure 1. There, it is demonstrated that the frequency returns to its nominal value, hence numerically validating the analytic convergence results of Theorem 1.

Furthermore, from Fig. 2, it is observed that the marginal costs at all 16 generators that contribute to secondary frequency control converge to the same value. This illustrates the optimality in the power allocation among generators, since equality in the marginal cost is necessary to solve (9) when the power generated does not saturate to its maximum/minimum value.

VIII. CONCLUSION

We have considered the problem of designing distributed control schemes such that stability and an optimal power allocation can be guaranteed for secondary frequency control. We proposed a distributed averaging dynamic output controller (DADOC) which ensures stability and optimality when high order linear generation dynamics and quadratic cost functions are considered and also that frequency attains its nominal value at steady state. DADOC controllers are advantageous compared to existing schemes since they allow for stability and optimality guarantees for a broad class of generation dynamics and are also easy to implement, in terms of measurement requirements. Moreover, we demonstrated the applicability of our results with examples of high order generation dynamics and explained how the design parameters can be selected in a computationally efficient

way by appropriate LMI conditions. Our results have been numerically verified with simulations on the NPCC 140-bus system. Interesting potential extensions in the analysis include considering controllable loads in the proposed framework, incorporating non linear generation dynamics and general convex cost functions, and extending the analysis to include voltage dynamics.

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