Tutoring Reinforcement Learning via Feedback Control

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Abstract—We introduce a control-tutored reinforcement learning (CTRL) algorithm. The idea is to enhance tabular learning algorithms by means of a control strategy with limited knowledge of the system model. By tutoring the learning process, the learning rate can be substantially reduced. We use the classical problem of stabilizing an inverted pendulum as a benchmark to numerically illustrate the advantages and disadvantages of the approach.

I. INTRODUCTION

Reinforcement learning (RL) [1], [2] is increasingly used to learn control policies from data [3]-[5] in a number of different applications. Despite the several advantages of this approach to control, one of its key drawbacks lies is the requirement of performing a typically large number of trials to explore the state-action space and hence learn a suboptimal control policy for the plant of interest. In particular, the control policy is found by exploring the Markov Decision Process encapsulating the control problem, thus accepting possible failures while learning. Unfortunately, in control applications, long training phases are often unacceptable and failures while learning might lead to unsafe situations. Moreover, these applications are often characterized by a continuous state-space, and using RL as is requires a dense discretization of the system state space or a function approximation that is also subject to the learning process.

To overcome these limitations, many different flavours of RL have been developed in the increasingly vast literature on the problem. First and foremost, model-based reinforcement learning where the introduction of some mathematical model of the plant is used to guarantee some degree of stability of the learning process and partially solve some of the issues mentioned above, e.g. [6]–[10]. Other extensions include Deep Learning strategies such as the Deep Q-Network (DQN) approach presented in [11] and the Actor-Critic paradigm [1], [12], [13] among many others.

In this paper, we present an alternative model-based approach, we name Control-Tutored RL (CTRL), where a feedback control strategy designed with only limited or qualitative knowledge of the system dynamics is used to assist the RL algorithm *when needed*. For the sake of clarity, we focus on Q-learning (QL) as a RL algorithm and discuss the resulting *control-tutored Q-learning* (CTQL) algorithm showing that it is well apt to deal with continuous or large state spaces while retaining many of the features of a tabular method. We wish to emphasize that our algorithm is complementary to other existing model-based approaches such as [14], [15]. Indeed, in our setting, the control-tutor supports the process of exploring the optimization landscape by suggesting possible actions based on its partial knowledge of the system dynamics. The learning agent can then deploy the policy suggested by the control-tutor whenever it is unable to find a better action by querying the Q-table. With this respect, the control-tutor supports the process of filling in the elements of the Q-table when it is needed, speeding up as a result the convergence of the learning process when compared to that of the Q-learning when used without the tutor. A related but different idea was independently presented in [10] where RL is mirrored with a Model Predictive Controller (MPC) and a different strategy is used to orchestrate transitions between RL and MPC. To validate our approach, we apply CTQL to solve the classical benchmark problem of stabilizing an inverted pendulum comparing its performance with Q-learning and the feedback strategy used as a tutor. We find that CTQL obtains better performance and convergence than Q-learning or feedback control on their own, solving the stabilization problem even when they are unable to do so by themselves.

II. PRELIMINARIES

Reinforcement learning is an area of machine learning aimed at studying how sub-optimal policies can be computed from data [2], [16] in order to solve dynamic programming problems involving uncertain dynamics [1], [2]. By closely following [17], we formulate the RL control problem as a constrained optimal control problem of the form:

$$\max_{n=1} \mathbb{E}[J_N^{\pi}], \tag{1}$$

s.t.
$$x_{k+1} = f_k(x_k, u_k, w_k),$$
 (2)

$$u_k = \pi_k(x_{0:k}, u_{0:k-1}), \tag{3}$$

$$x_k \in \mathcal{X}, u_k \in \mathcal{U},\tag{4}$$

where $x_k \in \mathcal{X}$ is the state of the system, $u_k \in \mathcal{U}$ is the control input, $w_k \in \mathcal{W}$ is some process noise, $f_k : \mathcal{X} \times \mathcal{U} \times \mathcal{W} \mapsto \mathcal{X}$ is the system vector field, $x_{0:k} = \{x_0, x_1, ..., x_k\}$ is the system state trajectory from step 0 to step k, $u_{0:k} = \{u_0, u_1, ..., u_k\}$ is the sequence of control inputs fed to the system from step 0 to step k. The objective function J_N^{π} in (1) is defined as follows:

$$J_N^{\pi} = r_N(x_N) + \sum_{k=1}^{N-1} r_k(x_{k-1}, u_{k-1}, x_k), \qquad (5)$$

where N is the time horizon, $r_k : \mathcal{X}^2 \times \mathcal{U} \mapsto \mathbb{R}$ is some reward function, $\hat{r}_N : \mathcal{X} \mapsto \mathbb{R}$ is the reward associated to

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the terminal state $x_N \in \mathcal{X}$ and $\pi = \{\pi_0, \pi_1, ..., \pi_{N-1}\}$ is the control policy with $\pi_k : \mathcal{X}^{k+1} \times \mathcal{U}^k \mapsto \mathcal{U}$ being a randomized mapping. The expected value $\mathbb{E}[J_N^{\pi}]$ in (1) is taken with respect to the control policy π and the random variables $(x_0, w_{0:N})$ assumed to be defined over a common probability space with known and independent distributions. In many applications of RL to control problems, the system dynamics encoded by f_k is often unknown so the problem (1)-(4) cannot be directly solved. In this situation, strategies can be used such as the Q-learning algorithm [18]. This algorithm seeks to learn an approximation of the rewardto-go function. Such approximation is often carried by a tabular representation, known as the Q-table, which is filledin online by successive trial and error experiments. In order to succeed, the method requires enough trials so that the learning agent can explore a large enough region of the optimization landscape and hence identify an approximation of the optimal policy.

In what follows, we discuss how a feedback control law with limited knowledge of the plant model can be embedded in the learning process assisting the learning agent in identifying a sub-optimal solution to the control problem in a lesser number of trials.

III. THE CONTROL-TUTORED REINFORCEMENT LEARNING

Our approach starts from the observation that in many control problems it is reasonable to assume the knowledge of a mathematical model that partially describes the dynamics of the system. That is, we make the following:

Assumption 1. Only an estimate, say $\hat{f}_k(x_k, u_k)$, of the system vector field $f_k(x_k, u_k, w_k)$ is available and

$$f_k(x_k, u_k, w_k) = f_k(x_k, u_k) + \delta_k(x_k, u_k, w_k)$$

where $\delta_k(x_k, u_k, w_k)$ is a vector field encompassing all the unknown terms in the dynamics (e.g. higher order terms, unmodelled dynamics etc.)

Using \tilde{f} we construct a feedback control strategy (or control-tutor policy) that the learning agent can use to decide what the next action to take should be. Namely, as shown in Figure 1, the algorithm will pick either the action suggested by the control-tutor policy, π^C or that proposed by the RL algorithm, π^R , according to which yields the lower expected value of the objective function, J_N^{π} .

Remark 1. A key difference between the approach presented here and the one of e.g. [19], [20] is that we do not seek to learn an approximation for $\delta_k(x_k, u_k, w_k)$. Interestingly, as we shall see, the presence of the tutor makes it possible to learn an optimal policy without learning the uncertainty.

Given the set-up described above, the problem of designing a model-based control-tutor for the RL algorithm (simply termed as the *CTRL problem* in what follows) can be stated

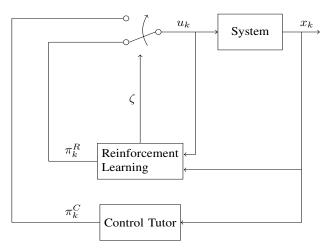


Fig. 1: Schematic of the Control-Tutored Reinforcement Learning (CTRL) algorithm. At each k, the agent selects its next control action u_k from a given system state x_k . This is done by choosing either the control action suggested by the control-tutor policy π_k^C or the one suggested by the RL policy π_k^R . The choice is made in accordance to a boolean variable, ζ , that will be formally defined in Section IV.

by adapting the formulation (1)-(4) as follows:

$$\max_{-} \mathbb{E}[J_N^{\pi}], \tag{6}$$

s.t.
$$x_{k+1} = f_k(x_k, u_k, w_k),$$
 (7)

$$u_k = \pi_k(x_{0:k}, u_{0:k-1}, \zeta) \tag{8}$$

$$= \begin{cases} \pi_k^R(x_{0:k}, u_{0:k-1}) & \text{if } \zeta \text{ is true,} \\ \pi_k^C(x_{0:k}, u_{0:k-1}) & \text{otherwise,} \end{cases}$$
(9)

$$x_k \in \mathcal{X}, u_k \in \mathcal{U},\tag{10}$$

where the control policy π_k is now selected between π_k^R : $\mathcal{X}^{k+1} \times \mathcal{U}^k \mapsto \mathcal{U}$, i.e. the randomized mapping defined according to the RL algorithm, and $\pi_k^C : \mathcal{X}^{k+1} \times \mathcal{U}^k \mapsto \mathcal{U}$ which is a randomized mapping defined by the feedback control law, and ζ is a Boolean condition based on the expected value of the reward-to-go.

IV. CONTROL-TUTORED Q-LEARNING (CTQL) IMPLEMENTATION

As a representative implementation of CTRL, we develop a control-tutored Q-learning (CTQL) approach by extending the Q-learning approach, briefly described in Sec. II, to solve a stabilization problem. We make the standard assumption that the policy is time-invariant and that the Markov property holds.

We select the boolean switching criterion in (9) as

$$\zeta = \begin{cases} 1 & \text{if } \max_{u \in \mathcal{U}} \{Q(x_k, u)\} > 0, \\ 0 & \text{otherwise,} \end{cases}$$
(11)

where $Q(x_k, u)$ is the value stored in the Q-table for the state-action pair (x_k, u) that approximates the reward-to-go from that state. In this way at step k, given the state x_k ,

the learning agent checks the sign of the entries of the Qtable for all actions $u \in \mathcal{U}$. If at least one of these entries is positive, then the control action u_k is selected according to the classic ε -greedy Q-learning algorithm [1]:

$$\pi^{R}(x_{k}) = \begin{cases} \arg\max_{u \in \mathcal{U}} Q(x_{k}, u) & \text{with probability } (1 - \varepsilon), \\ \operatorname{rand}(u) & \text{with probability } \varepsilon. \end{cases}$$
(12)

Intuitively, this means that the approximate reward-to-go from the current state stored in the Q-table is positive and hence a reward increase is possible by taking this choice.

Otherwise, if the current approximation contained in the Q-table has no positive values, the action is chosen according to the one suggested by the control-tutor via the policy $\pi^{C}(x_{k})$ which we also choose as a ε -greedy policy of the form:

$$\pi^{C}(x_{k}) = \begin{cases} \arg\min_{u \in \mathcal{U}} \|v(x_{k}) - u\| & \text{with probability } (1 - \varepsilon) \\ \operatorname{rand}(u) & \text{with probability } \varepsilon, \end{cases}$$
(13)

where v is the control input generated by a feedback controller designed using the model estimate \hat{f} available to the control-tutor. As such input does not necessarily belong to \mathcal{U} , the policy function $\pi^{C}(x_{k})$ selects the action $u \in \mathcal{U}$ which is closest to $v(x_{k})$.

Once the action is selected from either $\pi^R(x_k)$ or $\pi^C(x_k)$, the corresponding expected reward is then computed and used to update the Q-table. The pseudo-code of the CTQL algorithm is given in Algorithm 1.

Note that to preserve the spirit of the Q-learning algorithm both policies $\pi^C(x_k)$ and $\pi^R(x_k)$ contain some degree of randomness to favour exploration. We are currently studying whether this guarantees that, when implemented, the policy selection function of the CTQL is still within the scope of the probabilistic proof of convergence available for the Qlearning algorithm and described in [2], [18].

Algorithm 1 control-tutored Q-learn	iing
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Initialize $Q(x, u) = 0, \forall x \in \mathcal{X}, u \in \mathcal{U}$
Detect initial state x_0
for $k = 0$ to N do
Compute ζ via (11)
if ζ then
$u_k \leftarrow \pi^R(x_k)$
else
$u_k \leftarrow \pi^C(x_k)$
end if
Observe and store x_{k+1} and r_k
$Q(x_k, u_k) \leftarrow (1 - \alpha)Q(x_k, u_k) + \alpha[r_k +$
$+\gamma \max_{u \in U} Q(x_{k+1}, u)]$
end for $u \in U$

To illustrate the viability and effectiveness of CTQL, we apply it to solve the problem of stabilizing an inverted pendulum and discuss its performance by comparing it to a traditional (untutored) Q-learning approach.

V. APPLICATION TO THE INVERTED PENDULUM

We consider the problem of stabilizing the physical pendulum provided by the OpenAI Gym framework [21], [22] in its inverted position.

A. Problem Formulation

To achieve the control objective we define the reward function in (5) as:

$$r_k(x_k, x_{k-1}) = -\left[V(x_k) - V(x_{k-1})\right] + \rho(x_k), \quad (14)$$

where $x_k = [x_{k,1}, x_{k,2}]$ is the system state with $x_{k,1}$ and $x_{k,2}$ being the angular position and angular velocity of the pendulum respectively, meanwhile $V : \mathcal{X} \mapsto \mathbb{R}$ is a scalar quadratic function defined as:

$$V(x_k) = k_1 x_{1,k}^2 + k_2 x_{2,k}^2, \tag{15}$$

 $\rho(x_k)$ is an additional term of the reward function that accounts for a positive prize p that the agent receives if the pendulum falls in a sufficiently small neighborhood of the upward position. Specifically, such a term is defined as:

$$\rho(x_k) = \begin{cases} p & \text{if } x_{1,k} \in [-\epsilon, \epsilon], \\ 0 & \text{otherwise,} \end{cases}$$
(16)

Also, we let $r_N(x_N) = \rho(x_N)$ in (5).

Remark 2. Substituting (14) in (5) we get, in the expression for J_N^{π} , the sum $\sum_{k=0}^{N-1} V(x_{k+1}) - V(x_k) = -(V(x_N) - V(x_0))$. In the special case where the function $V(\cdot)$ is the Lyapunov function for the system, maximizing this term implies finding a solution to the optimization problem in (1)-(4) that minimizes the N-step derivative of the Lyapunov function. We note how, in the example described in this section, the CTQL is able to stabilize the upward equilibrium of the inverted pendulum even if $V(\cdot)$ is not chosen a Lyapunov function for the system¹.

We start by implementing the classical Q-learning algorithm. The state space is defined as $\mathcal{X} := \mathcal{D} \times \mathcal{G}$, where \mathcal{D} is the set of angular positions and \mathcal{G} is the set of angular speeds. The action space \mathcal{U} is the set of possible values of the control input u_k . In our implementation, the sets \mathcal{D}, \mathcal{G} and \mathcal{U} are obtained by discretizing the continuous state space and control input of the pendulum as described in the Appendix. The results of the Q-learning implementation on its own are shown in Fig. 2 and Fig. 3. The design of the tutor control law requires some model of the expected dynamics. We assume that only the linearized version of the inverted pendulum model around the upward position $x_k = [0,0]^T$ is available to the control-tutor. Specifically we assume the following model of the pendulum dynamics is available to the tutor:

$$x_{k+1} = \hat{f}(x_k, v_k) = Ax_k + Bv_k,$$
(17)

¹We leave for future research the problem of finding analytical conditions on $V(\cdot)$ that make it a viable choice for the CTQL algorithm to solve a given problem.

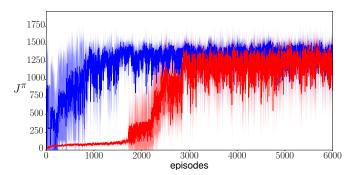


Fig. 2: Value of the objective function obtained by using the CTQL (blue) or the Q-learning (red) algorithms. The figure was obtained by running the algorithms S = 5 times for 6000 episodes. The figure shows the average (dashed) and the standard deviation (shaded areas).

with $B = [0, dt \frac{1}{I}]^T$ and A defined as:

$$A = \begin{bmatrix} 0 & 1+dt\\ g\frac{l}{2I}dt & 1 \end{bmatrix},$$
(18)

where dt is the sampling time, $I = m \frac{l^2}{3}$ is the inertia of the homogeneous rod, g is the gravitational constant and l and m are the length and the mass of the rod respectively. All parameters values chosen for this study can be found in the Appendix.

The control-tutor is then designed as the state-feedback control input:

$$v_k = -Kx_k, \tag{19}$$

with the control gains selected as $K = [5.83, 1.83]^T$ in order to render the origin of the linearized system a stable node with a settling time of 10s (corresponding to about 200 steps in our discretization). The control input v_k defined in (19) is then used in (13) to obtain the control-tutor policy. The results of the CTQL implementation are shown in Fig. 2 and Fig. 4.

B. Comparison between CTQL and Q-learning

The numerical validation is carried out on the Pendulum-V0 environment [22]. We define the training sessions set $S = \{1, ..., S\}$, the episodes set $\mathcal{E} = \{1, ..., E\}$ and the simulation set $\mathcal{N} = \{1, ..., N\}$. Each episode corresponds to a simulation of N steps of the pendulum starting from the initial condition on the stable downward position. More details on the simulation parameters can be found in the Appendix.

The numerical results are used to evaluate both the data efficiency of the learning process and the control performance. In what follows we use the superscript e to denote the episode at which the variables are evaluated. We denote as \mathcal{M} the set of consecutive episodes where the learning agent is able to maintain the pendulum position $x_{1,k}$, and velocity, $x_{2,k}$ in a ball of radius ϵ in the time window $k \in [N/2, N]$.

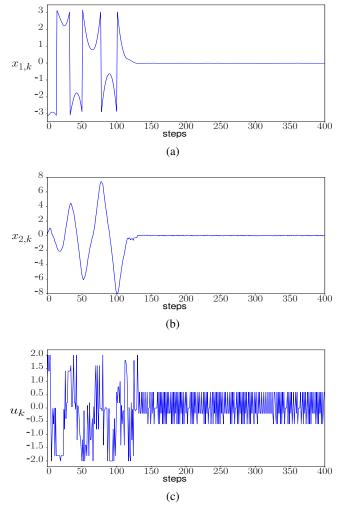


Fig. 3: Time evolution within one episode (consisting of 400 steps) of (a) the angular pendulum position, (b) the angular velocity and (c) the control input when the policy from Q-learning is used. The figure was obtained by controlling the pendulum via Q-learning, with the Q-table obtained after the training of Fig. 2. In the simulation dt = 0.05s.

We then assume the learning phase ends if the following condition is satisfied:

$$\sum_{e \in \mathcal{M}} \left(r_N^e(x_N) + \sum_{k=N/2}^{N-1} r_k^e(x_k, x_{k-1}) \right) \ge M \frac{N}{2} p, \quad (20)$$

where M is the minimum number of successful consecutive episodes we set as a satisfactory threshold (see the Appendix for the value we chose in our implementation).

1) Learning performance: to evaluate the sample efficiency and to make a proper comparison between the strategies proposed, we define the following metrics:

$$\lambda_1 = \max_{e \in \mathcal{E}} J_N^{\pi, e},\tag{21}$$

$$\lambda_2 = \frac{1}{E} \sum_{e \in \mathcal{E}} J_N^{\pi, e}, \tag{22}$$

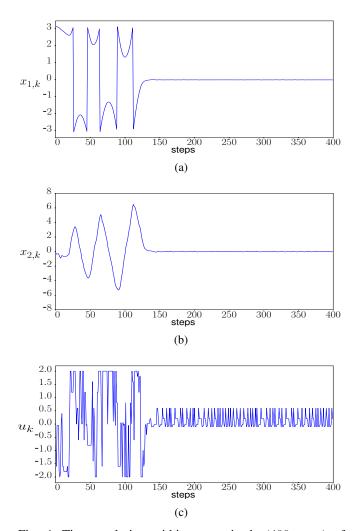


Fig. 4: Time evolution within one episode (400 steps) of (a) the angular pendulum position, (b) the angular velocity and (c) the control input when the controller is trained using a CTQL approach. The control performance is comparable to that shown in Fig. 3 although the CTQL converges to a viable control solution in a much lesser number of episodes (see Table I). In the simulation dt = 0.05s.

where λ_1 is the maximum value of the objective function J_N^{π} , λ_2 is the average value of the objective J_N^{π} with respect to the total number of episodes.

We also define λ_3 as the number of episodes after which the terminal condition (20) is satisfied.

The metrics defined above are averaged over S = 5 runs of the algorithms with E = 6000 episodes each for both Qlearning on its own and CTQL. The results are summarized in Table I showing that both achieve a comparable value of λ_1 but CTQL guarantees a better average value of the objective function across all episodes (metric λ_2), fulfilling the terminal condition (20) after a notably smaller number of episodes (metric λ_3). To further compare the two strategies we show in Fig. 5 the number of times per episode in which

 TABLE I: Data efficiency and learning performance comparison between Q-learning and CTQL

	λ_1	λ_2	λ_3
QL	1552	763	2730
CTQL	1521	1240	417

the action suggested by the control-tutor is taken by the learning agent. We observe that the control-tutor policy is most deployed by the agent during the initial episodes with the number of implemented actions coming from the tutor decreasing as the agent converges towards a suitable control strategy fulfilling the control goal.

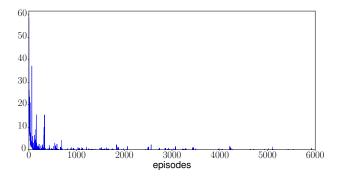


Fig. 5: Number of times, for each episode, where the action suggested by the control-tutor policy is adopted when CTQL is used. Each episode contains a total number of N = 400 steps.

2) Control performance: finally, we define a set of metrics to evaluate and compare the performance of the controller obtained at the end of the learning stage using the CTQL and QL algorithms. Specifically, the control metrics are defined as follows:

• the settling time η_1 defined as the first step such that

$$\|x_k\| \le 0.05 \quad \forall k \ge \eta_1; \tag{23}$$

 the average value η₂ of the mismatch between the state and the target equilibrium (i.e. the origin) over the last Γ = 100 steps:

$$\eta_2 = \frac{1}{\Gamma} \sum_{k=N-\Gamma}^{N} \|x_k\|;$$
 (24)

 the value of the objective computed over the trajectory as defined in (5), i.e. η₃ := J^π_N.

The control performances metrics were evaluated by running the controller after the end of the learning phase to swing up the pendulum from its stable downward position. The results are summarized in Table II where we see that the control performance of controller trained using the CTQL is comparable to that of the controller obtained by running the QL algoritm; the notable difference being therefore the much quicker learning times of CTQL as summarized in Table I.

TABLE II: Control performance comparison between the controller trained using Q-learning and that trained using a Control-Tutored Q-learning approach

	η_1	η_2	η_3
QL	125	0.00968	1384
CTQL	129	0.00901	1359

VI. CONCLUSIONS

We introduced an extension of reinforcement learning where the policy selection function is enhanced by means of a control-tutor that, using a feedback control law with limited knowledge of the system dynamics, is able to support the exploration of the optimization landscape guaranteeing better convergence and shorter learning times. To illustrate the effectiveness of the approach, we tutored the Q-learning algorithm via a state feedback controller to solve the classical benchmark problem of stabilizing an inverted pendulum. In our experiments, the tutor only had access to a linearized model of the pendulum about its inverted position. In this situation, we showed that our CTQL strategy is able to swing up the pendulum and globally stabilize its inverted position with the learning process converging towards a viable control solution after a much shorter number of episodes than the QL when deployed on its own. Ongoing work is focused on refining this approach with the aim of giving theoretical guarantees, obtaining a better understanding of its advantages and limitations for future applications.

ACKNOWLEDGEMENTS

The authors wish to acknowledge the contributions of Prof Pietro De Lellis, University of Naples Federico II, and Ms. Fabrizia Auletta, University of Bristol and Macquarie University, to an earlier version of the CTQL algorithm and its application to the solution of an herding problem that was reported earlier in [23].

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APPENDIX

We give here all the parameters that were used for the numerical simulations reported in the paper. We define the training set composed of S = 5 learning sessions each of them composed of E = 6000 episodes, each corresponding to a simulation of N = 400 steps of the pendulum starting from its stable downward position. The Q-learning update rule is implemented as showed in Algorithm 1 with parameters set to $\alpha = 0.8$ and $\gamma = 0.97$ while the randomness parameter in the policies $\pi^{R}(\cdot)$ and $\pi^{C}(\cdot)$ is set to $\varepsilon = 0.02$. The parameters of the reward function are set to $k_1 = 1, k_2 =$ 0.1 while the price for reaching a state in the neighborhood of the upward position is set as p = 5 and it is given in the region $x_1 \in [-\epsilon, \epsilon]$ with $\epsilon = 0.05$. The terminal condition is met when the agent stabilizes the pendulum for at least half of the N = 400 steps for M = 20 consecutive episodes. The state space X discretization is non-uniform and chosen as follows. The angular position is discretized in 16 equally spaced values when $x_1 \in [-\pi, -\frac{\pi}{9}] \cup (\frac{\pi}{9}, \pi]$ while it is split in 14 discrete values for $x_1 \in (-\frac{\pi}{9}, -\frac{\pi}{36}] \cup [\frac{\pi}{36}, \frac{\pi}{9})$ and 10 equally spaced values for $x_1 \in (-\frac{\pi}{36}, \frac{\pi}{36})$. The angular velocity is discretized in 20 equally spaced values when $x_2 \in [-8, -1) \cup (1, 8]$ and 18 equally spaced values when $x_2 \in [-1, 1]$. The control action space \mathcal{U} discretization is discretized as follows: 18 equally spaced values for $u \in [-2, -0.2] \cup [0.2, 2]$ and 8 equally spaced values for $u \in (-0.2, 0.2)$.