Joint Subsample Time Delay and Echo Template Estimations for Ultrasound Signals

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***Abstract*—In ultrasound applications, the signal obtained from a real data acquisition system is corrupted by noise and the echoes may have subsample time delays, which in some cases, compromises scatterer localization. Most time delay estimation (TDE) techniques require a precise signal template, otherwise localization deteriorate. In this paper, we propose an alternate scheme that jointly estimates an echo template and time delays for several echoes from noisy measurements. Reinterpreting existing methods from a probabilistic perspective, we extend their functionalities through a joint application of a maximum likelihood estimator (MLE) and a maximum a posteriori (MAP) estimator. Finally, we present simulated results to demonstrate the superiority of the proposed method over traditional ones.**

I. Introduction

Ultrasound is a popular Non-Destructive Testing (NDT) technique. In applications such as defectoscopy, applying the pulse-echo method, the sound applied to a inspected object is reflected by any discontinuity, as it is shown on Figure 1. Since the location of a discontinuity is arbitrary, the comparison of a centered echo used as a template, and the signal obtained from the acquisition system may also have subsample time delays.

The data acquisition on any mode (A-scans, B-scans, C-scans...) can be affected by thermal effects, electrostatic processes and also by noise present on cables and electronic components of the acquisition system [1]. Figure 2 illustrates the effects of noise and subsample time delays on a acquired echo.

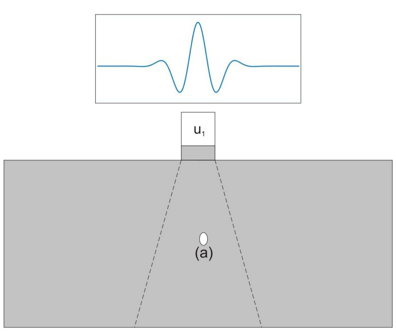

Figure 1. Defect detection with Ultrasound Testing

A Time Delay Estimation (TDE) technique can be utilized to determine the relative time shift between the reference signal and a relatively similar received signal. Aspects like precision, and computational cost of the TDE method are important. Several TDE techniques have been developed, such as the normalized and non-normalized cross correlation [2], sum absolute difference (SAD) [3] and sum squared error (SSE) [4]. A performance comparison of these and other methods can be found in [5]. The method applied in this work was proposed in [4]. It interpolates the reference signal in order to find a continuous time estimate and utilizes it as a pattern-matching function. This function is compared to the discrete sampled signal through SSE. A relative time delay is found when the SSE is minimum. This approach is relatively simple and straightforward and it has a good accuracy at a low computational cost.

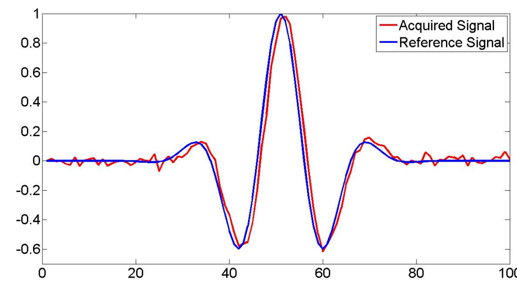

Figure 2. The acquired signal is subject to white gaussian noise and a subsample time delay

The noise present on real acquisition systems can be caused by various reasons. It corrupts the data and lowers the information we get on the original signal. Data reconstruction and restoration techniques are often used to recondition the treated signal in order to extract more information from the acquired data [6]. They can be used in a vast field of applications such as telecom, biomedicine among others, and in all of them, there is a common goal, which is to get more information on the original data. For noiseless sampled data, traditional interpolation methods can be enough to give us the all the information on the original signal. On the other hand, if the data is corrupted by noise these methods may fail, since they consist in trying to fit a determined function through the exact value of the sample. In this case, regularized interpolation methods can give us better results. This paper utilizes cubic B-splines as the interpolation method because of its smoothing properties, and also because of the smooth nature of the considered data. As in [7], we combine the B-spline interpolation with a regularization method, due to the presence of gaussian noise. This procedure is explained in matrix notation in [8] being referred to as *Smoothing Splines*.

In this paper, both TDE and data reconstruction techniques

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are theoretically reinterpreted as probabilistic tools and then jointly applied in generic noisy sampled data, in order to find a better estimate on the data and on the subsample time delay that affects the echoes. For the simulation, a generic continuous signal is sampled and subject to noise and a subsample time delay. It will be shown that the combined application of both techniques gives us a more accurate TDE then the one presented in [4] specially for low SNR signals, and also gives us a smoother signal estimate.

II. Proposed method

Consider a ultrasound data acquisition system working with a pulse-echo technique. A high resolution signal is generated with K echoes, representing the continuous time original data. The first echo is the one that contains the highest energy, and the following echoes are weaker due to signal attenuation, caused by material granular spreading, absorption, scattering and etc [9]. This signal is sampled, subject to additive white Gaussian noise and also, the echoes have subsample time delays between them. The generic discrete time model for one echo of this signal is given as:

$$f_i[n] = a_i y(nT - \tau_i) \tag{1}$$

where $f_i[n]$ is the acquired signal, a_i represents the amplitude from the echoes, y is the reference, T is the sampling period and τ_i is the subsample time delay

The generic discrete time model for the signal containing numerous echoes is as follows:

$$f[n] = \sum_{k=1}^{K} f_k[n] + w[n] \tag{2}$$

where $f[n]$ is the complete signal containing the K echoes and nK samples and $w[n]$ is the additive white Gaussian noise.

Given that the we have a known time delay, we now need a tool with subsample accuracy in order to get a estimate. The TDE method analyzed and applied in this work was the SSE proposed by [4]. It has subsample precision and compared to the other methods, has the advantage of low computational cost and still, its performance as precise as the cross correlation method. This approach also considers the energy of the two compared signals. The TDE is taken by comparing the highest energy echo to the other ones, giving us a relative time delay between them with a reasonable precision. It can be noted that this accuracy decreases if the data is corrupted by noise. Another interesting remark of this tool, is that it can be interpreted as a Maximum Likelihood Estimator.

Due to the fact that in real acquisition systems the data is corrupted by noise, a regularized data reconstruction method is applied to get a higher SNR signal estimate, lowering the effect of noise on the TDE. The reconstruction technique applied on this work is known as Smoothing Splines with B-spline basis. It consists on a relaxed interpolation method, applying certain smoothing constraints to the data. It improves the estimation of the intensity gradient for noisy signals and as the TDE technique, it can also be interpreted in a probabilistic point of view, giving us a Maximum a Posteriori Estimator. This will be explored further in this section.

This procedure is repeated in a alternate manner. The subsample time delay is used as input on the smoothing splines basis. The signal is reconstructed with lower noise and this enhances the TDE performance as well as the data estimate.

A. Time Delay Estimation

The technique presented in [4] consists in comparing two sampled signals $f[n]$ and $f_k[n]$ with lengths N and M respectively and $N > M$. The first signal $f_1[n]$ is interpolated through cubic splines in order to generate the continuous time estimated signal $\hat{f}[n]$. Then, a pattern-matching function between $\hat{f}(t)$ and $f_k[n]$ is calculated through the SSE between them.

$$e(\tau) = \sum_{n=1}^{M} (\hat{f}(nT + \tau) - f_k[n])^2 \tag{3}$$

Deriving equation (3) with respect to τ, setting the result to zero and solving also for τ, gives us the value of τ that minimizes $e(\tau)$, which is chosen to be the TDE.

$$\frac{de(\tau)}{d\tau} = \frac{d}{d\tau}\left(\sum_{n=1}^{M} (\hat{f}(nT + \tau) - f_k[n])^2\right) = 0 \tag{4}$$

If we take in consideration the probabilistic part of $e(\tau)$ in (4), which is directly related to the noise present on the acquired signals, this method can also be seen as a Maximum Likelihood Estimation (MLE).

The first echo is not subject to the known time delay since it is our intention to use it as a reference, this implies that $\tau_1 = 0$.

$$f_1[n] = a_1 y(nT) \tag{5}$$

The spline based TDE method from [4] is then applied to find the time delay between the first and second echo and then, to find the delay between the first and third echo. In both comparisons, the first echo is divided by the ratio between the maximum value of the first echo and the maximum value of the compared echo, such that its amplitudes are comparable

$$f_k[n] = \frac{f_1[n]}{r_{1k}} \tag{6}$$

where r_{1k} is the ratio between the maximum value of the first echo and the maximum value of the k_{th} echo.

For example, to find the time delay between the template echo and other echo, we compare $f[n]$ with $f_k[n]$, through equation (3). Since in this analysis the only probabilistic variable is the noise that is mixed with the compared signals, and that equation (3) can be directly related to this noise, some conclusions can me made. For instance, if the noise is defined be i.i.d. and to have a gaussian distribution

$$P(e(\tau)) = \prod_{n=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{[e(\tau)]^2}{2\sigma^2}\right\} \tag{7}$$

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And equation (7) can be expressed as follows:

$$P(e(\tau)) = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^N \prod_{n=1}^{N} \exp\left\{-\frac{(\hat{f}(nT+\tau) - f_k[n])^2}{2\sigma^2}\right\} \tag{8}$$

The definition of Maximum likelihood estimation is given by:

$$\hat{\tau}_{\text{ML}} = \arg\max_{\tau} \ \{P(e(\tau))\} \tag{9}$$

Since the Maximum Likelihood method requires the maximization of the likelihood function, and this operation is done through its derivative, for simplicity we utilize the Log Likelihood function.

$$L(e(\tau)) = N\left[\frac{1}{\sqrt{2\pi\sigma^2}}\right] - \sum_{n=1}^{N}\left\{\frac{(\hat{f}(nT+\tau) - f_k[n])^2}{2\sigma^2}\right\} \tag{10}$$

For the minimization, we derive (10) with respect to τ and set it to zero.

$$\frac{dL(\tau)}{d\tau} = \frac{d}{d\tau}\left\{\sum_{n=1}^{N}(\hat{f}(nT+\tau) - f_k[n])^2\right\} = 0 \tag{11}$$

It can be equivalently stated that the τ that maximizes (9), minimizes the term inside the exponential.

$$\hat{\tau}_{\text{ML}} = \arg\min_{\tau} \ \{\hat{f}(nT+\tau) - f_k[n]\} \tag{12}$$

Comparing equations (4) with (3) we can see that, in the case where the random variable is Gaussian i.i.d., the SSE process gives us the same estimation for the τ parameter as the MLE process. In [8], other aspects of the MLE are explored.

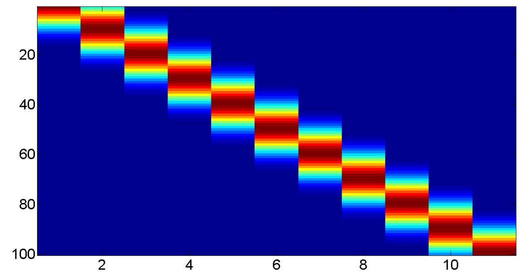
B. Regularized Reconstruction and Smoothing Splines

The technique utilized to perform the regularized reconstruction is an application of the Smoothing Splines interpolation method with an adaptation presented by [8]. This technique consists in creating a dictionary with B-spline basis atoms and finding a set of coefficients that will linearly combine these atoms to find a smoother data estimate.

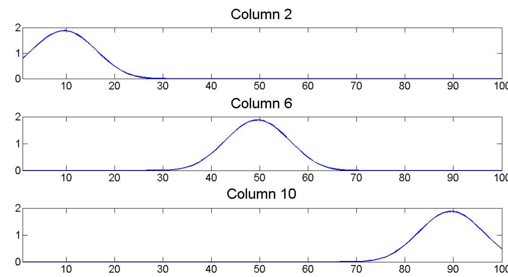
In [10], [11] and [12] the application smoothing splines is analyzed. It can be noted that this technique improves the estimation of the intensity gradient when the signal is noisy. A way to approach the problem of constrained curve fitting is to find the $f(x)$ that minimizes the following cost function:

$$\sum_{i \in Z}\{f_i - \hat{f}(x_i)\}^2 + \lambda \int_{-\infty}^{\infty}\{f^{(d)}(x)\} \tag{13}$$

where $d \leq k-1$ and λ is the smoothing parameter. The first one quantifies the square error between the $\hat{f}(x_i)$ function and the data vector f_i. The second adds the smoothing constraints to the solution.



(a) Smoothing Matrix.



(b) Kernels.

Figure 3. Construction of the Smoothing Matrix. In (a) we can see that the Smoothing Matrix is almost banded, and this indicates that the kernels have local support, as expected since it is a B-spline basis.

The interpolation function can be bumpy and unstable for a small λ or overly smooth for a big λ value. A $\lambda \in (0, \infty)$ is to be chosen for a reasonably good data reconstruction. This approach is analog to the reconstruction problem [13].

The reconstruction problem is considered to be equivalent to solving a linear equation system for a unknown parameter vector $\mathbf{x}$ for a given data vector $\mathbf{f}$ and a operation matrix $\mathbf{B}$, thus

$$\mathbf{f} = \mathbf{B}\mathbf{x} \tag{14}$$

in our case, $\mathbf{B}$ is a B-spline basis dictionary and $\mathbf{x}$ is the B-spline coefficient vector. Figure 3 illustrates the structure of the dictionary. The colors indicate the amplitude of the basis. It can be seen that the Smoothing Matrix has a banded nature. This happens because smoothing splines interpolation is a local fitting method [8].

Since equation (14) does not consider any prior knowledge about a possible solution, a reasonable way to solve (14), would be to find a *least-squares fit* set of solutions, relative to the data vector [14].

$$\hat{\mathbf{f}}_{\mathbf{LSF}} = \arg\min_{x} \ ||\mathbf{f} - \mathbf{B}\mathbf{x}||_2^2 \tag{15}$$

where $||z||_2^2$ is the ℓ_2 norm, and the arg refers to the argument which produces the minimum norm solution. The $\mathbf{x}$ vector which solves (15) can be found through the following equation

$$(\mathbf{B^T B})\hat{\mathbf{x}}_{\mathbf{LSF}} = \mathbf{B^T f} \tag{16}$$

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The solution of (15) through (16) gives us the least square fit with the smallest l_2 norm, and this is the generalized solution. In this type of reconstruction, non observable objects in the data are neglected. This solution is also sensitive to small data variations, and thus, responsive to noise. These problems can be solved through regularization concepts proposed by [15].

Basically, constraints are applied to the solution in order to mitigate the oscillatory nature of the noise present in the acquired data. This constraints can be controlled through the regularization parameter λ, and as previouly mentioned this parameter controls the balance between a stable solution and the generalized solution.

Regularization is vastly researched and it has plenty of literature on this subject such as [12], [14], [6] and [8]. In this work, the Tikhonov regularization was utilized. This method is based on the problem's prior information incorporation into the solution. This can be performed through the inclusion of a new term on equation (15). This gives us

$$\hat{\mathbf{f}}_{\mathbf{Tik}}(\mathbf{x}) = \arg\min_{x} ||\mathbf{f} - \mathbf{B}\mathbf{x}||_2^2 + \lambda||\mathbf{L}\mathbf{x}||_2^2 \tag{17}$$

where the first term is the general least square solution, which guarantees the reconstruction fidelity to the data; The second term refers to the prior knowledge that we have on the real signal. $\mathbf{L}$ is the parameter which captures the prior information on the data behavior and incorporates it onto the solution through an additional l_2 penalty term, controled by the regularization parameter λ. In our case, $\mathbf{L}$ is the second derivative of the cubic B-spline basis set. The solution that minimizes (17) is also the solution of the following equation for $\hat{\mathbf{f}}_{\mathbf{Tik}}$:

$$(\mathbf{B}^{\mathbf{T}}\mathbf{B} + \lambda\mathbf{L}^{\mathbf{T}}\mathbf{L})\hat{\mathbf{f}}_{\mathbf{Tik}} = \mathbf{L}^{\mathbf{T}}\mathbf{g} \tag{18}$$

When compared, (18) and (16) illustrate the difference between the generalized solution and the regularized one. It is clearly shown that for a bigger λ value, the equation (18) solution deviates from the generalized solution, and for a smaller λ value, they get closer. There are plenty of ways to find a solution to (18), including $(\mathbf{B}^{\mathbf{T}}\mathbf{B} + \lambda\mathbf{L}^{\mathbf{T}}\mathbf{L})$ inversion and iterative methods. In this work the utilized method was the *preconditioned conjugate gradient* [16] due to its good convergence power and precision.

In this work, the reconstruction problem can also be probabilistically interpreted. The first term of Equation (17) can be written as the probability density function $P(\mathbf{f}|\mathbf{x})$.

$$P(\mathbf{f}|\mathbf{x}) \propto \exp\left\{-\frac{||\mathbf{f} - \mathbf{B}\mathbf{x}||_2^2}{2\sigma_\epsilon^2}\right\} \tag{19}$$

The second term of equation (17) can be written as $P(x)$:

$$P(\mathbf{x}) \propto \exp\left\{-\lambda||\mathbf{L}\mathbf{x}||_2^2\right\} \tag{20}$$

this equation brings the prior information that we have on the posterior distribution.

The process of finding a coefficient vector $\hat{\mathbf{x}}$ that maximizes the product of (19) and (20), can also be interpreted as the posterior distribution $P(\mathbf{x}|\mathbf{f})$ maximization, which means we are finding a maximum a posteriori estimate.

$$\hat{\mathbf{x}}_{\mathbf{MAP}} = \arg\max_{\mathbf{x}}\ P(\mathbf{x}|\mathbf{f}) = \arg\max_{\mathbf{x}}\ \big\{P(\mathbf{f}|\mathbf{x})P(\mathbf{x})\big\} \tag{21}$$

Similarly to the MLE process, we can minimize the minus log of (19) and (20) product, taking the derivative of the result with respect to $\mathbf{x}$ and setting to zero.

$$\hat{\mathbf{x}}_{\mathbf{MAP}} = \arg\min_{\mathbf{x}}\ \big\{ - log[P(\mathbf{x}|\mathbf{f})]\big\} \tag{22}$$

$$\frac{d}{d\mathbf{x}}\big\{ - log[P(\mathbf{x}|\mathbf{f})]\big\} = \frac{d}{d\mathbf{x}}\big\{||\mathbf{f} - \mathbf{B}\mathbf{x}||_2^2 + \lambda||\mathbf{L}\mathbf{x}||_2^2\big\} = 0 \tag{23}$$

It is clear that (23) produces the same result as (17). This means that, in our case, since the random variables are i.i.d, with gaussian distribution, the regularized signal reconstruction can be seen as a MAP estimation [17].

After the time delay estimation, the signal is reconstructed through smoothing splines, which in this case, is a B-spline penalized interpolation with a coefficient for every interpolated data point. The TDE is inserted into the B-spline basis set, corresponding to the respective echo, forcing the point spreading to be delayed by this estimate.

III. Simulation Results and Discussion

The simulations made for this paper consists in applying the time delay estimation algorithm on the noisy data. After it we apply a Tikhonov regularized signal reconstruction routine in order to get a smoother data estimate and then the TDE taken again. The reconstruction is now made with a slight subsample shift in the B-spline basis. Note that the waveforms on the simulations have the same shape, with no distortion. This is considered for simplicity, since any discrepancy between the waveforms would introduce more errors to the results.

Gaussian-modulated sinusoidal pulses were generated to simulate the signal echoes. This pulses had the following characteristics: center frequency of $5\ MHz$; Fractional bandwidth of 0.8; period of $1\ \mu s$. The first echo has unity amplitude and the latter ones are multiplied by 0.6 and 0.3 respectively. The noisy sampled signal, or the acquired data had a SNR of 20.

The known time delay applied to the signal so that the analysis could be made, was actually considered as a subsample measure. Since we are trying to simulate a real signal acquisition situation, the acquired data is a sampled signal subject to noise. For simplicity then, the known time delay considered is of 0.5 samples applied in the second and third echoes. This process is illustrated on the following algorithm:

The output of the proposed algorithm can be seen in Table I. It is clear that, combining both TDE and reconstruction methods alternately we get a smoother template estimate as well as a better time delay estimate with lower bias and variance. Note that the first considered TDE is obtained directly from the noisy signal. Note that the first considered TDE is obtained directly from the noisy signal. Since the TDE

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Algorithm 1

Create a generic high resolution signal $y(t)$
Input: $f[n] = \sum_{k=1}^{K} f_k[n] + w[n]$
Find TDE : $\arg\min_{\tau} \left\{ \hat{f}(nT + t) - f_1[n] \right\}$
Find estimate : $\arg\min_{\mathbf{x}} \left\{ ||\mathbf{f} - \mathbf{B}\mathbf{x}||_2^2 + \lambda ||\mathbf{L}\mathbf{x}||_2^2 \right\}$
for $k = 1 : N$ **do**
  Find TDE on the signal estimate
  Find signal estimate with subsample dislocation
end for
Output: TDE, MSE, Smoother signal estimate

technique is noise sensitive, we can get better estimations for smoother signal estimates.

Table I
RESULTS FOR TDE AND MSE BETWEEN ECHOES 1 AND 2 WITH DIFFERENT SNR. THE ACTUAL TIME DELAY IS 99.5

SNR	FirstTDE	FinalTDE	FirstMSE	FinalMSE
20	99.5632	99.5224	0.0007183	0.0004215
25	99.5215	99.5017	0.0004479	0.0003287
30	99.4781	99.4920	0.0004253	0.0003257
35	99.5120	99.5009	0.0002598	0.0002447
40	99.5000	99.5000	0.0002550	0.0002245

Figure 4 illustrates the difference between the acquired data and the treated signal. As expected, the reconstructed signal is smoother due to the regularization. The time delay improvements can hardly be seen here since the plot in *MATLAB* matches the samples positions.

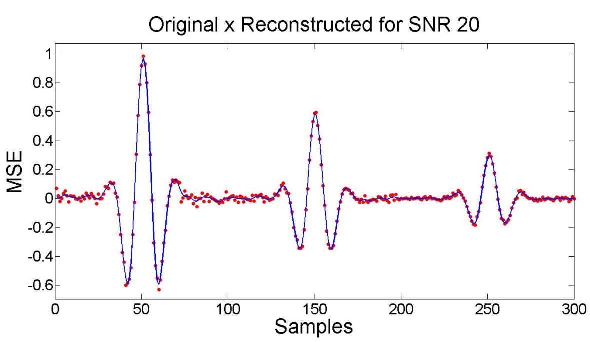

Figure 4. MSE between the generic continuous signal and the reconstructed signal. In red we have the acquired data, and in blue we have the reconstructed signal from the last iteration.

The signal estimate has a high fidelity to the noiseless generic continuous signal, preserving the echoes forms and its amplitudes and also having a very good trade-off between the acquired data points and smoothness.

The effects of regularization are clearly seen in the data smoothing as well as in the MSE comparison. The TDE effect on the reconstruction can also be seen on the MSE improvements, since for each iteration, we have a better time delay estimate for the next delayed reconstruction.

The developed method and results presented in this paper show that the proposed algorithm can lead to slight better results when compared to one of the reference methods all alone. It can be clearly seen that the utilization of both of them in a alternate way can lead to better time delay estimates as well as data estimates.

The results presented here can be reproduced with small changes since we are dealing with white Gaussian noise as a random variable. Note that, even though other techniques are available to estimate time delays and also to reconstruct signals, in this paper we have conveniently chosen methods that could also be analyzed from the probabilistic perspective, which strengthen the theoretical background of the method presented in this work.

A number of issues can be explored in future research. One important aspect that should be addressed is the utilization of the proposed method in real data for different applications. The analysis presented here could also be extended for distorted wavefront cases, since time-domain TDE methods are strongly related to the shape of the signal. For further research, other TDE and reconstruction methods can be tested.

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