A New Leader-follower Model for Bayesian Tracking

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***Abstract*—This paper introduces a novel leader-follower model for tracking a group of manoeuvring objects under a probabilistic framework. The proposed model develops on the conventional leader-follower model in which the followers are driven stochastically towards the velocity and position of the leader. Here we consider the dynamic of followers as a mean-reverting process and express it in a continuous-time stochastic differential equation. Instead of using a standard global Cartesian or polar system, an intrinsic coordinate model is utilised for the leader where piecewise constant forces are applied relative to the heading of the leader. Followers then mean revert towards the heading angle and speed of the leader, leading to a more realistic behavioural modelling than the more conventional global coordinate systems. Such a dynamical model is readily incorporated into tracking algorithms using for example the variable rate particle filtering framework which can accurately capture and estimate the manoeuvres of the leader and followers. The simulation results verify its efficacy under challenging group tracking scenarios and future work will explore automatic identification from groups of moving objects.**

***Index Terms*—leader-follower model, intrinsic coordinates, variable rate particle filter, Bayesian inference**

I. Introduction

Multiple Object Tracking (MOT) aims to estimate the kinematic state (e.g., position, velocity, heading) of objects based on the noisy measurements from sensors. In standard MOT algorithms, objects are assumed to be scattered and tracked independently. However, in real scenarios the interaction among a number of moving objects can often be witnessed, e.g., in bird flocks, fish schools, crowds. Such closed-spaced objects that present certain interaction pattern are defined as a group. Methods for group tracking depart from MOT as the interaction information is taken into account in the group dynamics so as to enhance the tracking performance.

Interaction models have been extensively studied and a thorough overview of these models in the tracking domain is given in [1]. In this paper, we focus only on one particular interaction model, dubbed the leader-follower model, in which the followers steer to the kinematic state of the defined leader. The bulk velocity model [2] and virtual leader model [3] are two classical models which utilise an additional group state, normally the averaged position and velocity of all objects, as the virtual leader of the group. The leader-follower model has been further developed in a continuous-time setting via a multivariate Stochastic Differential Equation (SDE) and shown to be effective in enhancing group tracking accuracy [4]. To identify a leader in groups, a causality reasoning framework has been proposed to rank objects with respect to their dominance effect [5]. An alternative method models the leader-follower relationship using a sparse structure, and interaction strength is deduced by inferring the interaction term using Gibbs sampling [6]. Both these two methods utilise batch inference methods (i.e., assume all related data, sensory measurements, are available) and/or assume that the interaction pattern within groups is fixed over time. However, practically the interactions can vary over time. For example, studies have shown that during migration, geese flying in a V-formation take turns to lead [7], [8]. Therefore a rotated leadership model has been proposed for inferring a dynamically changing group structure, and an online Gibbs sampler and deterministic particle filter (PF) have then been designed to infer sequentially the leadership over time [9]. Nevertheless, these models assume a constant velocity dynamic and do not address the manoeuvrability of the group objects.

Here we specifically address the planar manoeuvring group tracking where the manoeuvre inherently features a curvilinear motion with sudden changes in heading and/or speed. Compared to Cartesian coordinates, such curvilinear motion is more convenient to be expressed in an intrinsic coordinate system where an object is assumed to move along a curved path with nonzero accelerations tangential and normal to the object's trajectory (see fig. 1). This intrinsic coordinate system has been utilised in [10] and proven to be an efficient alternative to the classic coordinated turn [11] and rectilinear models [12]. Specifically in [10], nested multiple model trackers are adopted to estimate the tangential and normal accelerations, which is a variant of the standard interacting multiple model [13]. However, the curvilinear motion model in [10] was approximated on condition that the acceleration is relatively small, consequently limiting its application in practice. A more flexible variable rate tracking model was developed in [14] in which the state sampling rate is not determined by the measurement data. In addition, by assuming a constant force between the manoeuvre time intervals, the movement of the object can be viewed as a piecewise deterministic path, and the sequence of state points can be estimated by a variable rate particle filter (VRPF) [15]–[17].

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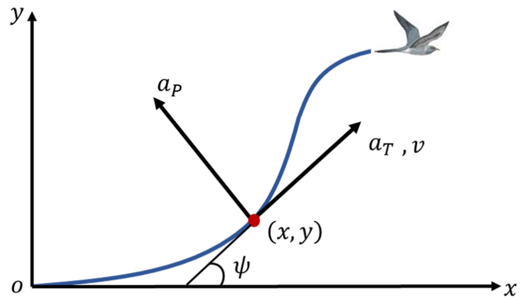

Fig. 1. Illustration of intrinsic coordinates.

In this paper, we take a step further by developing a new leader-follower model for tracking the planar motion of actively manoeuvring group objects; in particular, we adopt an intrinsic dynamic model to describe the manoeuvres of the leader, with the followers reverting towards the leader. Specifically, the followers' motion can be mathematically expressed by a continuous-time stochastic differential equation (SDE). The posterior of the joint states for both the leader and followers can be sequentially estimated by Bayesian inference, and a VRPF is utilised due to the nonlinear property of the model. Future work will explore the identification of the leader in group tracking.

II. Group Model

In this section, we develop a 2-dimensional mathematical model of group dynamics for depicting the manoeuvre behaviour and leader-follower interaction among group members. Particularly, we introduce a variable rate framework, in which the state arrival rate is not aligned with the observation process. Given the arrival time sequence and motion parameters, the dynamic of the leader between arrival times is a deterministic path under the intrinsic coordinate system; the motion of the followers can be considered as an Ornstein-Uhlenbeck random process which drifts to leader's heading and speed over time.

A. Intrinsic Leader-follower Model

Consider the measurements $\{z_n\}_{n=1}^N$ are observed with a fixed sampling rate at time $\{t_1, ..., t_N\}$. Here we define a variable state as $s_k = [\tau_k, \theta_k]$, which consists of the arrival time τ_k and state vector θ_k at τ_k, where $k = 1...K$ is the index of arrival time and $\tau_K < t_N$. Different from the standard state space models, we assume the states arrive at random and the state sequence follows a Markov process, i.e.,

$$p(s_{k+1}|s_{1:k}) = p(s_{k+1}|s_k). \tag{1}$$

Consequently, the states can be independently generated as follows:

$$s_{k+1} \sim p(s_{k+1}|s_k) = p(\theta_{k+1}|\theta_k, \tau_{k+1}, \tau_k)p(\tau_{k+1}|\tau_k). \tag{2}$$

The distribution of time points is set as

$$\tau_{k+1} - \tau_k \sim \exp(\lambda), \tag{3}$$

where λ is the rate parameter of this exponential distribution.

Conditional on a sequence of arrival time, the entire trajectories of group members are continuously linked by piecewise trajectories between every interval (τ_k, τ_{k+1}).

Assume we have M objects in the group, and the Lth object is the leader of the group with the others being followers, $L \in \{1...M\}$. Define the variable state vector at arrival time τ_k as $s_k = \{s_{k,i}\}_{i=1}^M$, and $s_{k,i} = [\tau_{k,i}, \theta_{k,i}]$. Then the transition density $p(\theta_{k+1}|\theta_k, \tau_{k+1}, \tau_k)$ conditional on the arrival time can be rewritten as

$$\begin{aligned} p(\theta_{k+1}|\theta_k, \tau_{k:k+1}) &= p(\theta_{k+1,L}|\theta_{k,L}, \tau_{k:k+1}) \\ &\times p(\theta_{k+1,f}|\theta_{k,f}, \theta_{k:k+1,L}, \tau_{k:k+1}), \end{aligned} \tag{4}$$

where $f = \{i\}_{i=1}^M, \forall i \neq L$.

Since different dynamical models are applied to the leader and the followers, the continuous-time motion model of them between state arrival time (τ_k, τ_{k+1}) and their transition functions will be discussed separately as follows.

1) The Leader: A 2D intrinsic coordinate system is utilised for modelling the motion of the leader. Specifically, here we define the state for the Lth leader at arrival time τ_k is

$$\theta_{k,L} = [a_{T,k}, a_{P,k}, v_L(\tau_k), \psi_L(\tau_k), x_L(\tau_k), y_L(\tau_k)],$$

where $v_L(\tau_k)$ is the speed tangential to the path; $\psi_L(\tau_k)$ is the heading that is anticlockwise relative to the x- axis, and $x_L(\tau_k)$, $y_L(\tau_k)$ are the positions in Cartesian coordinates. Manoeuvre parameters (i.e. the tangent and normal acceleration, $a_{T,k}$ and $a_{P,k}$) acting on the leader are also included in the state vector.

In particular, we assume the force applied relative to the heading of the leader to be constant between time interval $[\tau_k, \tau_{k+1}]$. Correspondingly, $a_{T,k}$ and $a_{P,k}$ are constants sampled according to $a_{T,k} \sim \mathcal{N}(\mu_T, \sigma_T^2)$, $a_{P,k} \sim \mathcal{N}(0, \sigma_P^2)$. Thus the movement of the leader is a deterministic curvilinear motion between $[\tau_k, \tau_{k+1}]$ and the standard equations of which can be expressed in a continuous-time model as follows:

$$\dot{v_L}(t) = a_{T,k}, \tag{5}$$

$$\dot{\psi}_L(t) = \frac{a_{P,k}}{v_L(t)}, \tag{6}$$

$$\dot{x_L}(t) = v_L(t)cos(\psi_L(t)), \tag{7}$$

$$\dot{y_L}(t) = v_L(t)sin(\psi_L(t)), \tag{8}$$

where $\tau_k < t < \tau_{k+1}$.

By integration, the model for tangential speed $v_L(t)$ is written as:

$$v_L(t) = v_L(\tau_k) + a_{T,k}(t - \tau_k), \tag{9}$$

and the heading $\psi_L(t)$ is expressed as

$$\psi_L(t) = \psi_L(\tau_k) + \frac{a_{P,k}}{a_{T,k}} \ln \left| \frac{v_L(t)}{v_L(\tau_k)} \right|. \tag{10}$$

Subsequently, the location of the leader in a Cartesian coordinates can be obtained by integrating (7)-(8) from time

τ_k to t, and hence the position on the x-axis and y-axis at time t can be computed as:

$$\begin{aligned} x_L(t) &= x_L(\tau_k) \\ &+ \frac{v_L(t)^2}{4a_{T,k}^2 + a_{P,k}^2}[a_{P,k}sin(\psi_L(t)) + 2a_{T,k}cos(\psi_L(t))] \\ &- \frac{v_L(\tau_k)^2}{4a_{T,k}^2 + a_{P,k}^2}[a_{P,k}sin(\psi_L(\tau_k)) + 2a_{T,k}cos(\psi_L(\tau_k))], \end{aligned} \tag{11}$$

$$\begin{aligned} y_L(t) &= y_L(\tau_k) \\ &+ \frac{v_L(t)^2}{4a_{T,k}^2 + a_{P,k}^2}[-a_{P,k}cos(\psi_L(t)) + 2a_{T,k}sin(\psi_L(t))] \\ &- \frac{v_L(\tau_k)^2}{4a_{T,k}^2 + a_{P,k}^2}[-a_{P,k}cos(\psi_L(\tau_k)) + 2a_{T,k}sin(\psi_L(\tau_k))]. \end{aligned} \tag{12}$$

Therefore, the transition function $p(\theta_L(t)|\theta_{k,L}, \tau_{k:k+1})$ for the leader can be described explicitly by (9)-(12).

2) The follower: The motion of the followers can be considered as a mean-reverting process which steers towards leader's heading and speed. The state for the ith follower at arrival time τ_k is

$$\theta_{k,i} = [v_i(\tau_k), \psi_i(\tau_k), x_i(\tau_k), y_i(\tau_k)],$$

where i is the index of the followers, $\forall i \neq L$. This dynamic can be easily expressed in a continuous-time SDE:

$$dv_i(t) = \beta\left(v_L(t) - v_i(t)\right)dt + dB_{i,v}, \tag{13}$$

$$d\psi_i(t) = \gamma\left(\psi_L(t) - \psi_i(t)\right)dt + dB_{i,\psi}, \tag{14}$$

where $B_{i,v}$ and $B_{i,\psi}$ are independent Brownian motions with variance $\sigma_{i,v}^2$ and $\sigma_{i,\psi}^2$, and β, γ are parameters that control the strength of the force the leader has on the followers. All these parameters are assumed to be known scalars in this paper.

The speed at time t can be calculated as

$$\begin{aligned} v_i(t) &= e^{-\beta h}v_i(\tau_k) + \int_{\tau_k}^{t} e^{\beta(u-\tau_k-h)}dB_{i,v} \\ &+ \int_{\tau_k}^{t} \beta e^{\beta(u-\tau_k-h)}\left(v_L(\tau_k) + a_{T,k}(u-\tau_k)\right)du, \end{aligned} \tag{15}$$

where $h = t - \tau_k$ and $\tau_k < t < \tau_{k+1}$.

The transition density $p(v_i(t)|v_i(\tau_k))$ is a Gaussian with mean $m_{i,v}(t)$ and covariance $Q_{i,v}(t)$ being

$$\begin{aligned} m_{i,v}(t) &= e^{-\beta h}v_i(\tau_k) + \left(v_L(\tau_k) - a_{T,k}\tau_k\right)\left(1 - e^{-\beta h}\right) \\ &+ \frac{a_{T,k}}{\beta}\left(\beta\tau_k + \beta h - 1 - \mathrm{e}^{-\beta h}\left(\beta\tau_k - 1\right)\right), \end{aligned} \tag{16}$$

$$\begin{aligned} Q_{i,v}(t) &= \int_0^h e^{-\beta u}\sigma_{i,v}\sigma_{i,v}e^{-\beta u}du \\ &= -\frac{\sigma_{i,v}^2}{2\beta}\left(e^{-2\beta h} - 1\right). \end{aligned} \tag{17}$$

Similarly, the heading $\psi_i(t)$ at jump time t can be calculated as

$$\begin{aligned} \psi_i(t) &= e^{-\gamma h}\psi_i(\tau_k) + \int_{\tau_k}^{t} e^{\beta(u-\tau_k-h)}dB_{i,\psi} \\ &+ \int_{\tau_k}^{t} \gamma e^{\gamma(u-\tau_k-h)}\left(\psi_L(\tau_k) + \frac{a_{P,k}}{a_{T,k}}\ln\left|\frac{v_L(u)}{v_L(\tau_k)}\right|\right)du. \end{aligned} \tag{18}$$

Hence, the transition density $p(\psi_i(t)|\psi_i(\tau_k))$ is a Gaussian with mean $m_{i,\psi}(t)$ and covariance $Q_{i,\psi}(t)$.

$$\begin{aligned} m_{i,\psi}(t) =& e^{-\gamma h}\psi_i(\tau_k) + (1 - e^{-\gamma h})\psi_L(\tau_k) \\ &- \frac{a_{P,k}}{a_{T,k}}e^{-\gamma(h+\frac{1}{m})}\left(\Gamma(0, -\frac{\gamma}{m}) - \Gamma(0, -\gamma(h + \frac{1}{m}))\right) \\ &+ \frac{a_{P,k}}{a_{T,k}}\ln(\frac{v_L(t)}{v_L(\tau_k)}), \end{aligned} \tag{19}$$

$$Q_{i,\psi}(t) = \int_0^h e^{-\gamma u}\sigma_{i,\psi}\sigma_{i,\psi}e^{-\gamma u}du \tag{20}$$

$$= -\frac{\sigma_{i,\psi}^2}{2\gamma}\left(e^{-2\gamma h} - 1\right). \tag{21}$$

where $m = \frac{a_T}{v_L(\tau_k)}$, and $\Gamma(0, \cdot)$ is the incomplete Gamma function.

The position for the ith follower in Cartesian coordinates can be calculated by using the first-order Euler approximation, which assumes that the speed and heading are constants over a small enough time interval dt. Subsequently, the position in Cartesian coordinates can be computed by

$$x_i(\tau_k + dt) = x_i(\tau_k) + v_i(\tau_k)cos(\psi_i(\tau_k))dt, \tag{22}$$

$$y_i(\tau_k + dt) = x_i(\tau_k) + v_i(\tau_k)sin(\psi_i(\tau_k))dt. \tag{23}$$

By substituting dt with h, we can obtain the approximated position of the follower at time t in Cartesian coordinates, nevertheless dt can be set to other value as required for precision. Therefore, the transition function $p(\theta_f(t)|\theta_{k,f}, \theta_{k:k+1,L}, \tau_{k:k+1})$ continuously on interval (τ_k, τ_{k+1}) can be deduced by (15)-(23).

B. Measurement Model

In this paper, the measurement equation takes the following linear Gaussian form in the 2-d Cartesian coordinates:

$$z_n = H\theta(t_n) + w_n, \tag{24}$$

where $z_n \in \Re_{2M\times 1}$ denotes the measurement at observation time step n, $n = 1, ..., N$. $\theta(t_n)$ at time t_n represents the joint states of leader and followers, $\theta(t_n) = [\theta_1(t_n), ..., \theta_M(t_n)]'$, and the observation noise $w_n \sim \mathcal{N}(0, Q_w\boldsymbol{I}_{2M\times 2M})$. Q_w is the noise coefficient. When only position measurements are available, the observation matrix is:

$$H = \begin{bmatrix} H_1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & H_2 & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & H_M \end{bmatrix}, \tag{25}$$

where H_i, $\forall i \neq L$ for the followers is

$$H_i = \begin{bmatrix} \mathbf{0}_{2\times 2} & \boldsymbol{I}_{2\times 2} \end{bmatrix}, \tag{26}$$

and H_L for the leader is

$$H_L = \begin{bmatrix} \mathbf{0}_{2\times 4} & \boldsymbol{I}_{2\times 2} \end{bmatrix}. \tag{27}$$

Therefore, the likelihood can be expressed as $p(z_n|\theta(t_n)) = \mathcal{N}(z_n|H\theta(t_n), Q_w \boldsymbol{I}_{2M\times 2M})$.

III. Variable Rate State Estimation

The aim of group tracking in a variable rate setting is to compute the posterior probability distribution $p(s_{1:k_n}|z_{1:n})$, given all of the observations up to time t_n. The k_n indicates the maximum index of state which satisfies $k_n \in \{1, ..., K\}$ and $0 < \tau_{k_n} < t_n$. This problem can be effectively solved by Bayesian filtering under the Markovian assumption as stated in (2).

The predict step is formulated as:

$$p(s_{1:k_n}|z_{1:n-1}) = p(s_{k_n}|s_{k_{n-1}}) \times p(s_{1:k_{n-1}}|z_{1:n-1}), \tag{28}$$

in which $p(s_{k_n}|s_{k_{n-1}})$ is the state transition density expressed as

$$p(s_{k_n}|s_{k_{n-1}}) = \prod_{j=1}^{J} p(s_{k_{n-1}+j}|s_{k_{n-1}+j-1}). \tag{29}$$

due to the fact that state may arrive one or more times between observation time step $n-1$ and n. Let $J = k_n - k_{n-1}$ denotes the number of manoeuvre; thus $J = 0$ indicates there is no manoeuvre between time t_{n-1} and t_n.

The update step is expressed as

$$\begin{aligned} p(s_{1:k_n}|z_{1:n}) =& \int p(s_{1:k_n}, \theta(t_n)|z_{1:n}) d\theta(t_n) \\ \propto& p(s_{1:k_n}|z_{1:n-1}) \int p(z_n|s_{1:k_n}, \theta(t_n), z_{1:n-1}) \\ &\times p(\theta(t_n)|s_{1:k_n}, z_{1:n-1}) d\theta(t_n). \end{aligned} \tag{30}$$

Note that the density $p(\theta(t_n)|s_{1:k_n}, z_{1:n-1})$ is a delta function, i.e.

$$p(\theta(t_n)|s_{1:k_n}, z_{1:n-1}) = \delta(\theta(t_n) - \hat{\theta}(t_n)). \tag{31}$$

where $\hat{\theta}(t_n)$ is the predicted state vector at exactly time t_n specified in II-A. The likelihood at t_n only depends on its corresponding state vector $\theta(t_n)$:

$$p(z_n|s_{1:k_n}, \theta(t_n), z_{0:n-1}) = p(z_n|\theta(t_n)). \tag{32}$$

Therefore, (30) is further deduced as

$$\begin{aligned} p(s_{1:k_n}|z_{1:n}) \propto& p(s_{1:k_n}|z_{1:n-1}) \int p(z_n|\theta(t_n)) \\ &\times \delta(\theta(t_n) - \hat{\theta}(t_n)) d\theta(t_n) \\ =& p(s_{1:k_n}|z_{1:n-1}) p(z_n|\hat{\theta}(t_n)). \end{aligned} \tag{33}$$

In this paper, we adopt a variable rate particle filter to sequentially estimate the arrival time and states of the group members. The posterior $p(s_{1:k_n}|z_{1:n})$ is utilised as the target distribution, approximated by a set of weighted particles. Here, a bootstrap particle filter is employed where the transition density $p(s_{k_n}|s_{k_{n-1}})$ in (29) is used as the proposal $q(s_{k_n}|s_{k_{n-1}}, z_{1:n})$. The steps of VRPF are as follows:

1) Suppose at observation time step $n-1$, we have N_p particles $\{s^{(p)}_{1:k_{n-1}}, \omega^{(p)}_{n-1}\}_{p=1}^{N_p}$, then the posterior distribution $p(s_{1:k_{n-1}}|z_{1:n-1})$ can be approximated as

$$p(s_{1:k_{n-1}}|z_{1:n-1}) \approx \sum_{p=1}^{N_p} \omega^{(p)}_{n-1} \delta(s_{1:k_{n-1}} - s^{(p)}_{1:k_{n-1}}). \tag{34}$$

where $\sum_{p=1}^{N_p} \omega^{(p)}_{n-1} = 1$.

2) at time n, for $p = 1$ to N_p, $s^{(p)}_{k_n}$ is sampled from $p(s_{k_n}|s^{(p)}_{k_{n-1}})$ which can be expressed as follows by considering (2) and (29):

$$\begin{aligned} p(s_{k_n}|s^{(p)}_{k_{n-1}}) =& \prod_{j=1}^{J} p(\tau_{k_{n-1}+j}|\tau^{(p)}_{k_{n-1}+j-1}) \\ &\times p(\theta_{k_{n-1}+j}|\theta^{(p)}_{k_{n-1}+j-1}, \tau_{k_{n-1}+j}, \tau^{(p)}_{k_{n-1}+j-1}). \end{aligned} \tag{35}$$

Hence, we first sample a sequence of arrival times $\tau^{(p)}_{k_n}$ according to distribution in (3), and then the state vector $\theta^{(p)}_{k_n}$ can be obtained by the transition function deduced in (II-A).

The corresponding importance weight is updated as

$$\omega^{(p)}_n \propto \omega^{(p)}_{n-1} p(z_n|\hat{\theta}^{(p)}(t_n)), \tag{36}$$

where $p(z_n|\hat{\theta}^{(p)}(t_n))$ is the likelihood function.

3) Resample step is taken if effective sample size $\hat{N}_{eff}$ is smaller than the chosen threshold N_{thres}; particles will be resampled according to their weight $\omega^{(p)}_n$ under the multinomial resampling scheme [18], and $\omega^{(p)}_n$ will be set to $\frac{1}{N_p}$.

4) Subsequently, the posterior distribution $p(s_{1:k_n}|z_{1:n})$ at observation time step n can be approximated by particles $\{s^{(p)}_{1:k_n}, \omega^{(p)}_n\}_{p=1}^{N_p}$, and $\sum_{p=1}^{N_p} \omega^{(p)}_n = 1$.

IV. Results

Synthetic and real scenarios are presented to verify the performance of the proposed intrinsic leader-follower model in group tracking.

A. Synthetic Data

In this part, we consider a synthetic scenario with manoeuvring objects moving in a group. Specifically, we simulate 100 independent trajectories of 3 objects over 200 time steps by using the proposed group model, where the time interval between observations is $1s$. The object 1 is set as the leader of the group for the whole process, and the motion is initialised with a random velocity and position. The parameters used are $\mu_T = 0.01$, $\sigma^2_T = 0.02$, $\sigma^2_P = 5$, $\sigma^2_{f,v} = 0.01$, $\sigma^2_{f,\psi} = 0.01$, $\beta = 0.9$, $\gamma = 0.9$. The interarrival time is assumed to follow an exponential distribution with rate parameter $\lambda = 10$.

The measurements and the true trajectories for an example simulation in 2-D Cartesian coordinates are shown in Fig. 2.

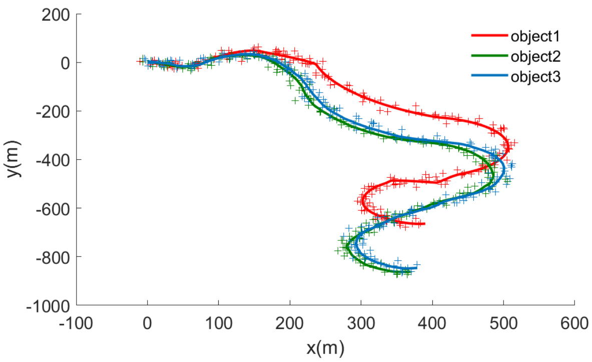

Fig. 2. Simulated tracks of 3 objects; pluses denote measurements, and solid lines are the true tracks

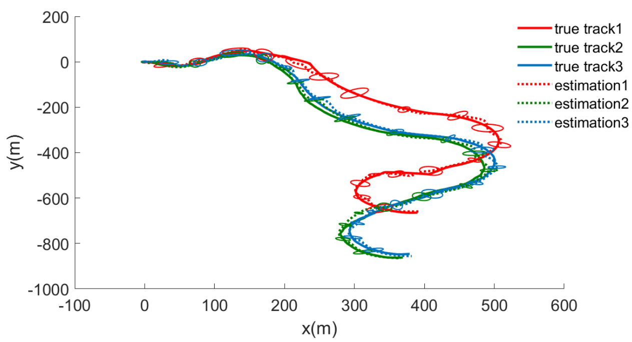

Fig. 3. Estimated tracks and the corresponding 99% confidence ellipses; solid lines are the true tracks, and dotted lines are estimations

It is clear that the motion of the group is highly manoeuvring and presents a leader-follower group behaviour. Meanwhile, we can observe that the followers do not strictly revert to the position of the leader as in the standard leader-follower model; instead, the followers keep in alignment with the heading and the speed of the leader. This setting might be practical in cases with known heading information.

TABLE I
PERFORMANCE AVERAGED ACROSS 100 RUNS.

Methods	RMSE
Intrinsic Leader-follower Model	2.8633
Independent Intrinsic Model	3.8735
Constant Velocity Leader-follower Model	9.9504

A bootstrap VRPF algorithm is utilised with 2000 particles, and is initialised according to a Gaussian distribution around the true initial states. Fig. 3 shows the estimated tracks and their corresponding 99.7% confidence ellipses of a group of 3 objects. It can be seen that the group state are well-estimated over time with high accuracy. Comparisons with other methods are also given in Table. I by using the metric of the Root Mean Square Error (RMSE). Especially, we first consider a non-group based independent intrinsic model which does not consider the leadership information, i.e., objects in groups are moving independently and hence are tracked individually. Particularly, we assume each object follows an intrinsic tracking model as in [15]. We also compare our method with the constant velocity leader-follower model in [9] which assumes the leader follows a constant velocity model with the followers reverting to leader's kinematic position states. The results show that all these methods can track group objects properly, while two models based on the intrinsic coordinate system show better tracking performance in such a curvilinear motion case. Moreover, we can see that our model has comparatively the lowest RMSE, thereby demonstrating its superiority in the challenging manoeuvring group tracking scenarios.

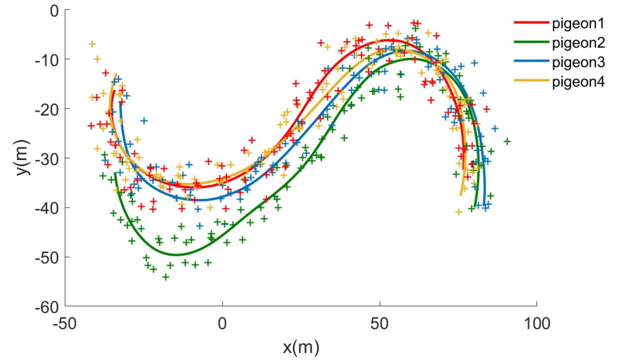

Fig. 4. Pigeons trajectories; solid lines are the true tracks, and pluses denote measurements.

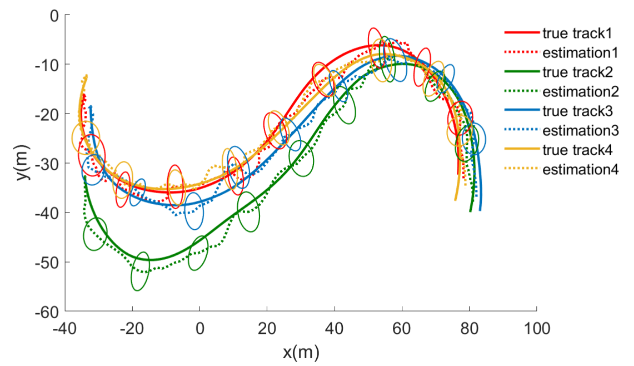

Fig. 5. Estimated tracks and the corresponding 95% confidence ellipses; solid lines are the true tracks, and dotted lines are estimations

B. Pigeon Flock Data

To further prove the efficacy of this algorithm, it is evaluated on the real pigeon flock data presented in [19]. The data we utilise are high-resolution trajectories of four homing pigeons flying in a flock, collected by miniature GPS devices at a rate of 10 Hz. We use a 150 time step sub-segment and the true trajectories and observations of the four pigeons are given in Fig. 4. It can be seen that the flock trajectory features a curvilinear motion, thus being well-suited to be analysed under an intrinsic coordinate system.

Besides manoeuvres of the group, this scenario is especially challenging due to the varying leadership in pigeon flock

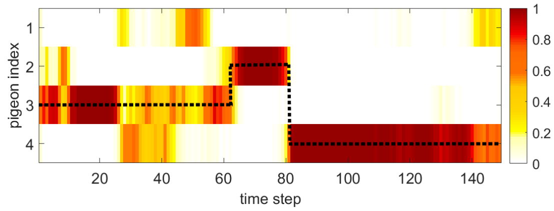

Fig. 6. Estimated leader at each time step and its probability; black dotted line indicates the output, i.e. the most probable leader; color bar denotes the probability of the correctness

TABLE II
PERFORMANCE AVERAGED ACROSS THE 50 RUNS.

Methods	RMSE
Intrinsic Leader-follower Model	1.1434
Independent Intrinsic Model	1.3328
Constant Velocity Leader-follower Model	2.0786

over time. Here we assume the leadership pattern is prior information (e.g., estimated by the leadership identification method in [9]), and the estimated leadership of the flock data over 150 time steps is shown in Fig. 6. The color bar ranging from white to red indicates the probability of this pigeon being the leader, and the dotted line is the most probable leader deduced at each time step. For more details see [9]. To track these highly manoeuvring pigeons with varying dominance hierarchy amongst the flock, the proposed intrinsic leader-follower model is applied to model the dynamics of the group members. The parameters are set as $\mu_T = 0.1$, $\sigma_T^2 = 50$, $\sigma_P^2 = 1000$, $\beta = 10$, $\gamma = 10$. $\sigma_{f,v}^2 = 10$, $\sigma_{f,\psi}^2 = 10$. The interarrival time is assumed to follow an exponential distribution with rate parameter $\lambda = 10$. A VRPF algorithm with 1000 particles is utilised to estimate the group states. From Fig. 5 we can see that our algorithm can accurately capture the manoeuvres of the pigeons. Similarly, the proposed method is compared with the independent intrinsic model and constant velocity leader-follower model. The RMSEs of these methods are listed in Table. II. Likewise, we can see that our method has the smallest RMSE. It is noticeable that the independent intrinsic model is a counterpart in tracking performance to our algorithm with a slightly higher RMSE; however, its computational burden should be considered when the group size grows since this algorithm runs multiple particle filters for objects in a group. Therefore, we can conclude that our algorithm is effective in handling manoeuvring interacting group tracking.

V. CONCLUSION

In this paper, an intrinsic leader-follower model is presented for tracking manoeuvring group objects in a leader-follower formation. Especially, we adopt a variable rate scheme which proves to be more flexible and efficient in modelling manoeuvres of the objects. Correspondingly, the VRPF algorithm is applied to estimate the states of objects in a group. Results show that our proposed model outperforms both the normal leader-follower model and the independent intrinsic tracking model. Future work will refine our model corresponding to realistic tracking settings, e.g., to include frictional resistance force. Moreover, the identification of the group leader in real time will also be studied.

APPENDIX

Here we present the detailed steps to calculate the speed and heading for each follower, respectively.

A. Derivation of the Speed

To solve the SDE in (13), we multiply both sides by $e^{\beta t}$:

$$e^{\beta t} dv_i(t) = -\beta e^{\beta t} v_i(t)dt + \beta e^{\beta t} v_L(t)dt + e^{\beta t} dB_{i,v}. \quad (37)$$

Hence,

$$d\left(e^{\beta t} v_i(t)\right) = \beta e^{\beta t} v_L(t)dt + e^{\beta t} dB_{i,v}. \quad (38)$$

Substitute (9) into (38). The speed at jump time t can be expressed as in (15). Specifically, the steps of calculation the mean $m_{i,v}(t)$ of the transition density $p(v_i(t)|v_i(\tau_k))$ is given here:

$$\begin{aligned} m_{i,v}(t) &= e^{-\beta h} v_i(\tau_k) + \int_{\tau_k}^{t} \beta e^{\beta(u-\tau_k-h)} v_L(u)du \\ &= e^{-\beta h} v_i(\tau_k) + \beta\left(v_L(\tau_k) - a_T\tau_k)\right) \int_{\tau_k}^{t} e^{\beta(u-\tau_k-h)} du \\ &\quad + \beta a_T \int_{\tau_k}^{t} t e^{\beta(u-\tau_k-h)} du \\ &= e^{-\beta h} v_i(\tau_k) + \left(v_L(\tau_k) - a_T\tau_k)\right)\left(1 - e^{-\beta h}\right) \\ &\quad + \frac{a_T}{\beta}\left(\beta\tau_k + \beta h - 1 - \mathrm{e}^{-\beta h}\left(\beta\tau_k - 1\right)\right). \end{aligned} \quad (39)$$

where $h = t - \tau_k$ and $\tau_k < t < \tau_{k+1}$.

B. Derivation of the Heading

Similarly, we can deduce (14) by multiplying $e^{\gamma t}$ on both sides and substituting (10) into (14). In particular, here we give the steps of calculating the mean $m_{i,\psi}(t)$ of transition density $p(\psi_i(t)|\psi_i(\tau_k))$. The expression of $m_{i,\psi}(t)$ is

$$\begin{aligned} m_{i,\psi}(t) &= e^{-\gamma h}\psi_i(\tau_k) \\ &\quad + \int_{\tau_k}^{t} \gamma e^{\gamma(v-\tau_k-h)}\left(\psi_L(\tau_k) + \frac{a_P}{a_T}\ln\left|\frac{v_L(v)}{v_L(\tau_k)}\right|\right) dv \\ &= e^{-\gamma h}\psi_i(\tau_k) + \int_{\tau_k}^{t} \gamma e^{\gamma(v-\tau_k-h)}\psi_L(\tau_k)dv + C \end{aligned} \quad (40)$$

where C is further deduced as follows:

$$\begin{aligned} C &= \int_{\tau_k}^{t} \gamma e^{\gamma(v-\tau_k-h)}\left(\frac{a_P}{a_T}\ln\left|\frac{v_L(v)}{v_L(\tau_k)}\right|\right) dv \\ &= \gamma\frac{a_P}{a_T} e^{-\gamma t}\int_{\tau_k}^{t} e^{\gamma v}\ln\left|\frac{v_L(v)}{v_L(\tau_k)}\right| dv \\ &= \gamma\frac{a_P}{a_T} e^{-\gamma t}\int_{\tau_k}^{t} e^{\gamma v}\ln\left|\frac{v_L(\tau_k) + a_T(v-\tau_k)}{v_L(\tau_k)}\right| dv \end{aligned} \quad (41)$$

Define $m = \frac{a_T}{v_L(\tau_k)}$. Then

$$\begin{aligned} C &= \gamma \frac{a_P}{a_T} e^{-\gamma t} \int_{\tau_k}^{t} e^{\gamma v} \ln|1 + m(v - \tau_k)| \, dv \\ &= \frac{a_P}{a_T} e^{-\gamma t} \left(e^{\gamma v} \ln|1 + m(v - \tau_k)| \Bigg|_{\tau_k}^{t} \right) \\ &- \frac{a_P}{a_T} e^{-\gamma t} \int_{\tau_k}^{t} e^{\gamma v} \ln|1 + m(v - \tau_k)|' \, dv \\ &= \frac{a_P}{a_T} \ln|1 + mh| \\ &- \frac{a_P}{a_T} e^{-\gamma t} \int_{\tau_k}^{t} e^{\gamma v} \ln|1 + m(v - \tau_k)|' \, dv \end{aligned} \tag{42}$$

Note that

$$\begin{aligned} [\ln|1 + m(v - \tau_k)|]' &= \frac{1}{|1 + m(v - \tau_k)|} \times [|1 + m(v - \tau_k)|]' \\ &= \frac{\frac{1+m(v-\tau_k)}{|1+m(v-\tau_k)|} \times [1 + m(v - \tau_k)]'}{|1 + m(v - \tau_k)|} \\ &= \frac{m}{1 + m(v - \tau_k)} \end{aligned} \tag{43}$$

Hence,

$$\begin{aligned} &\int_{\tau_k}^{t} e^{\gamma v} \ln|1 + m(v - \tau_k)|' \, dv \\ &= \int_{\tau_k}^{t} \frac{m e^{\gamma v}}{1 + m(v - \tau_k)} dv \\ &= e^{\frac{-\gamma(1-\tau_k m)}{m}} \int_{u_1}^{u_2} \frac{e^u}{u} du \\ &= e^{\frac{-\gamma(1-\tau_k m)}{m}} (\Gamma(0, -u_1) - \Gamma(0, -u_2)) \end{aligned} \tag{44}$$

where $u = \gamma v + \frac{\gamma(1-\tau_k m)}{m}$, and $u_1 = \frac{\gamma}{m}$, $u_2 = \gamma(h + \frac{1}{m})$. $h = t - \tau_k$ and $\tau_k < t < \tau_{k+1}$. In this way, the C can be calculated, and the final expression for the mean of the heading is given in (19).

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