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***Abstract*—In a context of autonomous driving, perception of the surrounding is a crucial task. It characterizes the vehicle's ability to simultaneously model its surroundings accurately and maintain its position in the environment. In this article, a new framework of mobile robot perception and risk assessment is proposed. Our approach aims to leverage the simultaneous combination of the standard occupancy grid map with a new map that we have called "knowledge map". This proposal was motivated by the fact that risk arises not only from obstacles but also from the lack of knowledge. Using this framework, we are able to assess the risk, mainly of collision, over a given path $\mathscr{P}$ and therefore compute an optimal navigation control of the robot. Thanks to the proposed Bayesian framework the paper also shows how we can combine both local measurements and existing map (eg. OpenStreetMap) and also take account of the robot's localization errors.**

***Index Terms*—Knowledge map, Occupancy grid, Bayesian inference, Risk assessment**

I. Introduction

Before vehicles can navigate safely and efficiently in a given environment to reach expected positions without collisions for executing various tasks, e.g., transportation, surveillance, or exploration, many challenges must be overcome. A crucial part is finding the drivable area to plan a movement over and around the obstacles. The most effective robot navigation algorithms for occupancy grid mapping are based on geometrical principles, such as the recognition of occupied zones or the identification of boundaries between unknown and unoccupied space. However, a map is fundamentally most of the time a field of binary random variables and a sensor is a probabilistic channel that links robot motion in the physical world to information gain for the Bayesian inference. Regarding robot navigation, it is expected that geometric-based intuition and information-based reasoning will agree; nevertheless, up until now, this expectation has only been a conjecture.

If we review the state of the art of navigation and perception algorithms in the context of self-driving cars, we find that the informational aspect is barely addressed. Therefore, there are a few works that use fundamental information metrics (entropy, mutual information, etc.) as a reward for their information-based exploration algorithms [1] [2]. The use of knowledge or informational aspect is often limited to exploration applications only.

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In this article, we consider the fact that risk stems not only from the incoming obstacles but also from the lack of knowledge in the robot navigation area. Thus, our goal is to rigorously include the notion of lack of knowledge in the risk assessment equation. Our approach simultaneously exploits two complementary robot-centered maps: the occupancy grid map (O map) and the "knowledge map" (K map). These two maps combine not only the local robot information coming mainly from the onboard sensors (Lidar, camera, etc.) with their uncertainties but also the absolute ones that can be extracted, for instance, from a prior high-level semantic map with a GNSS (Global Navigation Satellite Systems) or from the infrastructure, etc. The proposed framework thus allows Bayesian evolution and updating processes, handles multi-sensor measurements considering robot evolution but also prior data such as an absolute semantic map that can be projected in the locals maps having a rough robot localization estimation, and this in the same way than for local sensors measurements.

To illustrate the influence of knowledge on risk analysis, we simultaneously build the knowledge and the occupancy maps and update them in an iterative manner using the Bayesian inference. Then we use them to evaluate the risk along a trajectory.

To summarize, our research contributions are as follows:

- We propose a new grid mapping technique that stores for each cell a probability of being known.
- We develop an analytical approach that allows to evolve and update the maps whatever the measurements (local sensors or prior maps for instance) while respecting the real evolution of the information.
- We provide a method for assessing the risk of collision along a path using both maps.
- We present results from simulation experiments of a vehicle crossing an intersection.

The paper is organized as follows: section II describes the related works, section III brings details about the standard Bayesian occupancy grid map and the used Bayesian filter. Section IV gives the main principles of our approach explaining how the maps evolve and how they are updated. The risk assessment is also described in this section. Section V shows experiments on an intersection case where both Lidar data and prior map are used to build the maps.

II. Related works

The current navigation algorithms can be divided in two types that are used in contexts with perfect or limited knowledge, respectively. The first type includes the majority of most early works [3]. In these works, a collision free path for achieving the objective is previously planned in a map constructed by the knowledge of the workspace. However, complete knowledge is unrealistic and hence relatively recent works mainly focus on the second scenario, i.e., how to navigate the robot in environments that aren't totally known. One of the proposed solutions is reactive navigation [4] whose concept is directly mapping the sensory data for on-line obstacles avoidance. Such algorithms don't require any prior information, but they also can't account for any potential prior knowledge of the workspace. Having partial knowledge beforehand is actually the situation that is most common in real-world situations, as opposed to either having complete knowledge or none at all. Another approach called hybrid navigation [5] [6] looks to design systems with both the deliberative and reactive layers in order to get around these constraints. This is because, intuitively, the deliberative layer can use the available knowledge to generate long-term plans for avoiding trap situations, and the reactive layer can offer good reactivity for carrying out the plan.

A more generic solution is the occupancy grid mapping [7] that uses a probabilistic tessellated representation of multi-source spatial information and allows to construct safe and collision free paths from a point of interest to another through path planning algorithms [8]. Then, several works implemented a notion of risk in occupancy grids such as [9]. However, Laconte *et al.* showed that the discrete nature of the Bayesian occupancy grid is far from well suited to compute risk along a path [10]. As a result, they proposed a novel framework called Lambda-field, which is a new approach to store the occupancy of the environment. It permits the computation of risk along a given path by keeping its physical sense and not in the form of probabilities, i.e., the risk according to robot's nature is not the same according to the obstacle's nature. In this paper we keep the same risk approach, but we use a Bayesian framework that allows to take into account both exteroceptive sensors errors (lidar, camera), proprioceptive sensors errors (for time evolution: odometers, etc) and also localization error since we eventually want to use a prior map (e.g., OpenStreetMap [11]) and the projection of this map needs will be blurred by the localization errors.

III. PRELIMINARIES

This section is devoted to the introduction of preliminaries and our notations used throughout the article. We briefly review the occupancy grid in §III-A and the used Bayesian filter in §III-B.

A. Occupancy and Knowledge maps

The occupancy map is used to model the environment, where the map is stored as a grid of cells, each of them represents a small fraction of the environment. Additionally, each cell holds a value, corresponding to the occupancy probability of the specific element. We assume that all the occupancy cells are independent and that a Bayesian filter is employed to update the occupancy probabilities.

The occupancy map is denoted by the random variables $O = \{O_1, ..., O_N\}$, where N is the number of cells and $O_i \in [0, 1]$ is the binary random variable that indicates the occupancy of the i^{th} cell. In this case, $O_i = 0$ indicates an empty cell with a total confidence and $O_i = 1$ indicates an occupied cell with a total confidence. The realization of the random variable O_i is denoted by o_i.

The knowledge map is designated by $K = \{K_1, ..., K_N\}$ where N is the number of cells and $K_i \in [0, 1]$ is the random variable indicating the probability that the state of i^{th} cell being known. Thus, $K_i = 0$ indicates an unknown cell with absolute certainty, $K_i = 1$ indicates a well-known cell. The realization of the random variable K_i is denoted by k_i.

Notice O and K maps have same shape and size and are processed in the same way. In the rest of the paper we'll therefore note M these maps ($M \in \{O, K\}$) and $M = \{M_1, ..., M_N\}$. Both maps are robot centered.

B. Binary Bayes filter

The robot is equipped with a set of sensors. Let the observations from both onboard sensors or infrastructure ones be denoted by $Z = Z_1, ..., Z_L$ where L is the number of information sources. Thus, the problem of occupancy grid mapping can be formulated as computing the posterior distribution over the map based on the observations $z_{1:t}$ and given the vehicle's trajectory $x_{1:t}$. Here, we denote by $z_{1:t}$ the measurements obtained up to time t and by $x_{1:t}$ the sequence of states from the initial time through time t.

As we are interested in estimating only the current state of the i^{th} cell o_i independently and if we assume that the robot's pose is known, one can use Bayes rule:

$$p(o_i|z_{1:t}, x_{1:t}) = \frac{p(z_t, x_t|o_i, z_{1:t-1,x_{1:t-1}})p(o_i|z_{1:t-1}, x_{1:t-1})}{p(z_t, x_t|z_{1:t-1}, x_{1:t-1})} \tag{1}$$

using the first Markov assumption that the state o_i is enough to estimate the robot's pose and measurement x_t, z_t. Applying a second time the Bayes rule, we obtain:

$$p(o_i|z_t, x_t) = \frac{p(o_i|z_t, x_t)p(z_t, x_t)p(o_i|z_{1:t-1}, x_{1:t-1})}{p(o_i)p(z_t, x_t|z_{1:t-1}, x_{1:t-1})} \tag{2}$$

Similarly, we have for the opposite event $\neg o_i$:

$$p(\neg o_i|z_t, x_t) = \frac{p(\neg o_i|z_t, x_t)p(z_t, x_t)p(\neg o_i|z_{1:t-1}, x_{1:t-1})}{p(\neg o_i)p(z_t, x_t|z_{1:t-1}, x_{1:t-1})} \tag{3}$$

To estimate $p(o_i|z_t, x_t)$ and its complement given by eq. (2) and eq. (3), we use its log odds defined by :

$$l_t(o_i) = \ln \frac{p(o_i|z_t, x_t)}{p(\neg o_i|z_t, x_t)} \tag{4}$$

After substitutions and simplifications, we finally obtain:

$$\begin{aligned} l_t(o_i) &= \ln \frac{p(o_i|z_{t-1}, x_{t-1})}{1 - p(o_i|z_{t-1}, x_{t-1})} + \ln \frac{p(o_i|z_t)}{1 - p(o_{it})} - \ln \frac{p(o_i)}{1 - p(o_i)} \\ &= l_{t-1}(o_i) + \ln \frac{p(o_i|z_t)}{1 - p(o_i|z_t)} - l_0(o_i) \end{aligned} \tag{5}$$

Where l_{t-1} is the previous log-odds ratio, the second term is called inverse sensor model since it represent the sensor information and l_0 is the prior of occupancy represented as a log-odds ratio.

IV. Proposed approach

The key step in our framework is the continuous evolution and updating of both maps O and K.

Our approach is a special implementation of the Bayesian filter approach [12]. This approach addresses the general problem of recursively estimating the probability distribution, $P(M|Z)$ conditioned on the observation Z. This posterior distribution is obtained in two stages: evolving the map M (remember M represents both O and K) and estimating the new posterior.

A. Evolving the maps

Since the map M is robot-centered, each movement of the robot is accompanied by the movement of the map but mutual positional relationships between the retained data never change. For this purpose, we update the position shift values x_{shift} and y_{shift} in accordance with the robot's movement after each movement instead of updating the map's current data. This process causes the data to be shifted and blurred due to imperfect odometry or localization.

In the same coordinate system, if we consider that those errors have Gaussian distributions centered on the displacements $\bar{X}_{shift} = (\bar{x}_{shift}, \bar{y}_{shift}, \bar{\theta}_{shift})^\top$ with covariance matrix Σ_e which can be updated from the robot evolution model (geometric model), the resulting map is none other than the previous map convoluted with the 3D Gaussian evolution function $G_e(X_i) = \mathcal{N}(\bar{X}_{shift}, \Sigma_e(t))$. Where X_i is the cell's position in the map frame.

It should be noted that in our case, we have opted to rotate the robot within the map instead of rotating the map to nullify the errors coming from the rotation, i.e. $\bar{\theta}_{shift} = 0$.

For dealing with odometry errors, suppose that at stage t, the state of the robot is $X_t = [x_t \; y_t \; \theta_t]^\top$, which comprises Cartesian coordinates x_t, y_t and orientation θ_t with respect to a global reference frame. A rotation α followed by a translation d moves the robot to a new state X_{t+1}.

$$X_{t+1} = f(X_t) = X_t + \begin{bmatrix} d\,\cos(\theta_t + \alpha) \\ d\,\sin(\theta_t + \alpha) \\ \alpha \end{bmatrix} + w_{t+1} \qquad (6)$$

Where w designates an external observation noise due to deformation of wheel radius, vibrations and other unknown errors. It is assumed to be Gaussian with zero mean and covariance $Q(t)$.

The noise covariance,$Q(t)$, was modelled on the assumption that there are two independent sources of error, angle and translation. Whence x_{t+1} depends upon x_t, α and d, we must translate the uncertainty in α and d into uncertainty in X_t. This is done by partial differentiation of (6) with respect to α and d, what gives the following Jacobian:

$$J_f(t) = \nabla f(X_t) = \begin{bmatrix} -d\,\sin(\theta_t + \alpha) & \cos(\theta_t + \alpha) \\ d\,\cos(\theta_t + \alpha) & \sin(\theta_t + \alpha) \\ 1 & 0 \end{bmatrix} \qquad (7)$$

The complete expression for $Q(t)$ is then:

$$Q(t) = J_f(t) \begin{bmatrix} \alpha^2\sigma_\alpha^2 & 0 \\ 0 & \sigma_d^2 \end{bmatrix} J_f(t)^\top \qquad (8)$$

Where σ_α^2 and σ_d^2 are variances of α and d.

Another source of uncertainty is the uncertainty in the position and orientation at time step t, Σ_e, carried forward to time step $t+1$. So we need an another Jacobian to determine how the uncertainty is transferred between the time steps. To compute it we must make an another partial differentiation of (6) with respect to x_t, y_t and θ_t. The resulting Jacobian, $\nabla f'(X_t)$, is given by :

$$J'_f(t) = \begin{bmatrix} 1 & 0 & -d\,\sin(\theta_t + \alpha) \\ 0 & 1 & d\,\cos(\theta_t + \alpha) \\ 0 & 0 & 1 \end{bmatrix} \qquad (9)$$

By assuming that the error at time t is not correlated with other errors, the complete pre-localization covariance matrix at time $t+1$, $\Sigma_e(t+1)$, can be evaluated as follows :

$$\Sigma_e(t+1) = J'_f(t)\,\Sigma_e(t)\,J'_f(t)^\top + Q(t) \qquad (10)$$

In the continuous space, a map is a sum of 2D Dirac delta functions, so we can write:

$$\begin{cases} M(t) & = & \sum_{i=1}^{N} \delta(X_t - X_i) \\ M(t+1) & = & M(t) \circledast G_e(X_i) \end{cases} \qquad (11)$$

Where $\circledast$ is the convolution operator (see Appendix section VII for details about this convolution implementation).

We also note that the probability distribution of each cell i of the map $M(t+1)$ noted m_i verifies the following equation:

$$\iint m_i(t+1)\; dxdy = \iint G_e(X - X_{shift})\; dxdy = 1 \quad (12)$$

which is consistent with the probability density function property.

B. Estimation (Add new measurements to map)

1) Embedded sensors measurements: Applying the Standard Bayes filter to estimate the knowledge probability of each cell given a set of observations $\{Z_1, .., Z_L\}$, we obtain:

$$P(m_i|Z_1, ..., Z_L) \propto P(m_i) \prod_{j=1}^{L} \frac{P(Z_j|m_i)}{P(Z_j)} \qquad (13)$$

As a result, our map can be updated by multiple information sources. In our case, we will not treat the problem of conflicting information fusion because, quite simply, we consider the assumption that the knowledge is cumulative.

At initialisation, since we usually have no prior knowledge, k_i cells of K map will have zero value. For the O map it is 0.5 by convention.

2) Prior map: Our framework allows to combine a prior map.

Since prior map data provides information about the existence and location of on-road objects, it is already used to extract road obstacles. In [13], the authors have taken advantage of the maps for both localization and perception. In [14], the authors utilize Geographical Information System databases for object detection and geo-spatial localisation. In fact, now we can have access to very detailed and accurate geo-referenced databases which provide rich information for autonomous navigation. Using these maps, lane-level attributes describing the structure of the road are available [15]. In [16], an algorithm for occupancy grid map generation from Open Street Map [11] indoor data is proposed for indoor positioning applications.

In this work, we choose to use the approach proposed by [17] where a prior map is considered to be a virtual sensor, providing information about the space occupied by infrastructures, buildings, etc. With an accurate pose estimation, we can convert the extracted information into a local perception map. This perception map will then be fused with a local occupancy grid generated from the on-board sensor data.

After we have extracted both O and K maps from the prior one, we project them on the robot centered frame. Actually this projection can be not very accurate (usually given by GNSS sensors), that's why the prior maps will be projected in the probable robot pose and will be blurred by a Gaussian distribution $G_p(X_i) = \mathcal{N}(0, \Sigma_p(t))$ expressing the localization uncertainty . This is done in the same way detailed in §IV-A.

It should also be highlighted that, in our perspective, the prior map only provides information about permanent obstacles (buildings, walls, etc.), omitting information about unoccupied areas, which may contain temporary or new obstacles. That means that walls in a prior map generate for instance occupied cells in the occupancy map O with high probability in the K map while free areas generates non occupied cells in O but with low probability of knowledge in K map.

C. Risk assessment along a path

Now, having the knowledge map K simultaneously with the occupancy grid map O, the question that arises is how to exploit them.

As mentioned before, the motivation of our framework is its capability to compute risk along a path taking into consideration both the occupancy information provided by the occupancy grid map and also the information about knowledge provided by the knowledge map. Hence, the risk is defined as the integral over the trajectory of the kinetic energy of contact weighted by the probability of risk R defined for each cell by:

$$R(s) = (1 - K(s)) + (O(s) \times K(s)) \tag{14}$$

Where $K(s)$ and $O(s)$ are respectively the knowledge and occupancy probabilities at abscissa s of a given path $\mathscr{P}$.

We can now express our risk function $\Psi(\mathscr{P})$ along a path $\mathscr{P}$ as follows:

$$\Psi(\mathscr{P}) = \int_{\mathscr{P}} \frac{1}{2} \cdot m \cdot v(s)^2 \cdot R(s) ds \tag{15}$$

Where $v(s)$ and $R(s)$ are respectively the instantaneous speed of the robot and the probability of risk at the abscissa s and m is the mass of the robot.

Furthermore, we seek to minimize the collision risk and the travel time simultaneously; the criterion $J(\mathscr{P})$ to optimize will be:

$$J(\mathscr{P}) = \alpha \Psi(\mathscr{P}) + (1 - \alpha) T(\mathscr{P})$$

Here $T(\mathscr{P})$ is the time to travel the path. Since $v(s) = \frac{ds}{dt}$ we have:

$$T(\mathscr{P}) = \int_{\mathscr{P}} dt = \int_{\mathscr{P}} \frac{ds}{v(s)}$$

And $\alpha \in [0, 1]$ is a weight chosen according to the control objective.

V. Experiments

Now we present a simple use-case of our framework where a vehicle has to drive through a crossroad. The robot is equipped with a lidar sensor and uses a prior map of the environment (OpenStreetMap) with a GPS receiver. The evolution and estimation of both knowledge and occupancy maps are shown in figure 1.

The top figures show the two maps at time $t - 1$ while the middle ones show the evolved maps at time $t - 1$ obtained by equation (11). The lower figures represent the estimated maps at time t from a lidar scan (figure 3) and a prior map of the environment (figure 2).

As we mentioned before, the map is projected with a localization error considered as Gaussian with constant diagonal covariance matrix:

$$\Sigma_p = \begin{bmatrix} \sigma_{px}^2 & 0 \\ 0 & \sigma_{py}^2 \end{bmatrix}$$

such as $\sigma_{px} = \sigma_{py} = 0.35m$.

Both maps are $[299 \times 299]$ in size with $0.2m^2$ area cells, thus the robot is located in $\bar{X} = [150, 150]^\top$. During the iteration the robot has moved 1.55 meters forward following x and -0.13 meters following y with the covariance matrix:

$$\Sigma_e = \begin{bmatrix} \sigma_{ex}^2 & \sigma_{exy} & 0 \\ \sigma_{eyx} & \sigma_{ey}^2 & 0 \\ 0 & 0 & \sigma_{e\theta}^2 \end{bmatrix}$$

with $\sigma_{ex} = 0.70\, m$, $\sigma_{ey} = 0.55\, m$, $\sigma_{ey} = 0.05\, rad$ and $\sigma_{exy} = \sigma_{eyx} = 0.35\, m$

As previously stated, we consider that the prior map informs us about permanent obstacles only (walls, sidewalk, etc.) but since there could be mobile or temporary obstacles in the traversable areas (cars, road temporarily closed, etc.) the road surface is assumed as unknown.This allows us to illustrate the complementary nature of the two maps. For instance, the space behind the obstacles represents a navigable area according to the a priori map we possess, but for the vehicle it is not known until it has been able to perceive it.

We note that the evolving stage describes the real evolution of information over time such that at time t, the information extracted at stage $t - 1$ is not very reliable anymore. Each new measure increases knowledge so the dark color of the perceived area is explained by the fact that each cell is crossed by several beams at each scan (the angle step is $0.05°$). Moreover, the evolving step involves the obstacles blurring because of uncertainties. This is advantageous for the path planning module since it allows to find the farthest trajectory from obstacles thanks to risk assessment equation (15).

The estimation stage shows that our approach is generic and both maps can be updated by multi-sources observations.

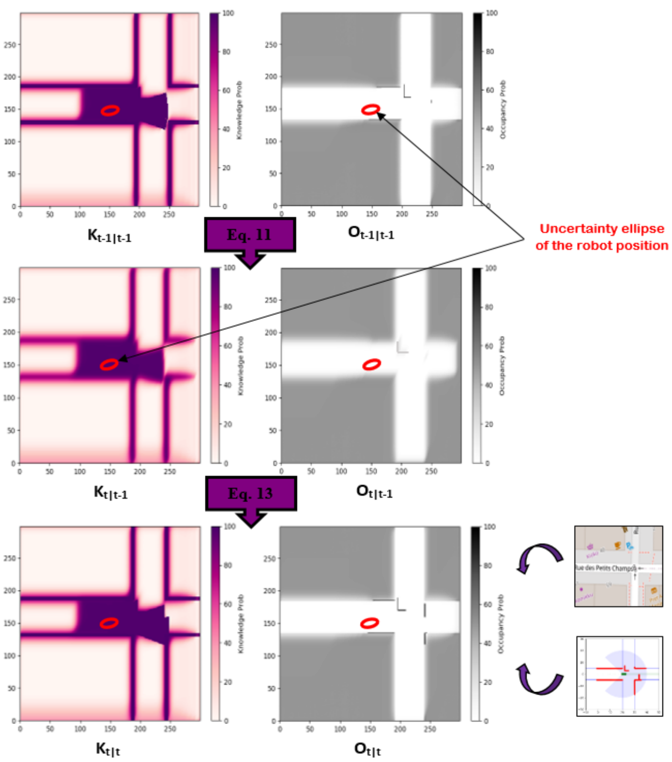

Fig. 1. Simulation results for one iteration of the process. Left: occupancy map O evolution and right: Knowledge map K evolution

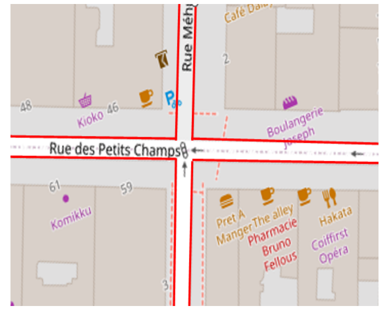

Fig. 2. the portion of OpenStreetMap representing the crossroad and the extracted information in continuous red (measurement)

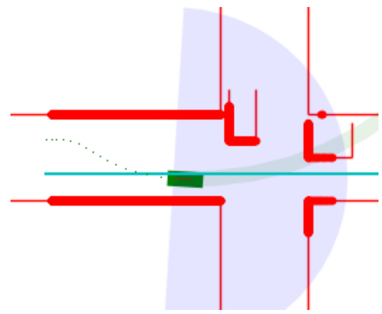

Fig. 3. Lidar measurement used to update the maps, the difference is that for the occupancy grid we distinguish occupied and unoccupied cells whereas for the knowledge map we update all the cells located in the perceptual field in the same way

We also note that the non-updated areas degrade in terms of knowledge and this is an interesting result especially for the next works which aims to address the case of dynamic environments.

VI. Conclusion

We have presented a framework allowing to compute an optimal path minimizing both risk and travel time. This framework uses two dynamic maps: a traditional occupancy map O and a knowledge map K since the risk stems from obstacles (occupancy map) and from a lack of information (knowledge map). We have shown how we can combine not only embedded sensors data but also information coming from an existing map of the environment. Knowing an estimation of the robot pose will simply move and blur this map that will be combined with the two dynamic ones O and K. Future works will be dedicated to demonstrate the effectiveness of our approach on real data with our real experimental vehicles (Renault Zoé).

VII. Appendix: maps convolution

We saw in section IV-A that we need to convolve the maps (O and K) by Gaussian function G_e. This is done here using Fourier-Mellin Transform (see [18] for details). Since both x, y and θ are subject to errors we need to take all these components into account. This can be done by a numerical traditional time-consuming convolution. It is therefore better to compute the FFT (Fast Fourier Transform) because convolution becomes multiplication in frequency domain. In our case, $F[G_e]$ remains a Gaussian function, so it can be easily literally computed. However, we need also to convolve along θ axis. We assume as null the correlations between θ and both x and y, so a polar conversion is achieved on the map and we convolve this polar map with the 1D Gaussian representing θ. All the process takes approximately $50ms$ for 512×512 cells maps (C language and Python implementations on a standard laptop).

VIII. Acknowledgements

This work was supported by the International Research Center "Innovation Transportation and Production Systems" of the I-SITE CAP 20-25.

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