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Three-dimensional Tracking with Angle Measurements without Observer Maneuver

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Abstract—Passive target estimation is a widely investigated problem of practical interest. We are concerned specifically with an autonomous flight system developed onboard the ONERA ReSSAC unmanned helicopter. This helicopter is equipped with a (visible or infrared) camera and so is able to measure azimuths and elevation angles of a target. The latter is supposed to follow a constant velocity motion. It is well known that observer must maneuver in order to insure the observability of the target state. We are interested in tracking partly the target state when both the observer and the target have a constant velocity model in a three-dimensional space. We describe the set of all the trajectories compatible with the angle measurements and we propose a quick method to estimate these trajectories.

I. INTRODUCTION

Passive target estimation is a widely investigated problem of practical interest. We are concerned specifically with an autonomous flight system developed onboard the ONERA ReSSAC unmanned helicopter [1]. This helicopter is equipped with a (visible or infrared) camera and so is able to perform lines of sight of the targets. More generally, we consider a single moving observer which measures the azimuths and the elevation angles of a moving target in a three-dimensional space. If all measurements are derived from a single observer and consist of angles only, it is well known that target state observability is warranted only under specific conditions on the observer trajectory. The observer must maneuver in order to insure the observability of the target state. Optimal maneuvers have been studied for the purpose of improving the accuracy of the target estimation for bearing-only tracking [2], [3], [4]. The aim of the paper is to describe and to estimate the set of all the trajectories (called ambiguous) compatible with the angle measurements in case where both the observer and the target follow a constant velocity motion. This kind of problem has been studied in bearings only tracking in a two-dimensional space like in [5]. In the three-dimensional context, the ambiguous trajectories can be determined by the observer as they depend of three azimuths and two elevation angles. We show how these angles can be quickly estimated. A possible application is to discriminate among the ambiguous trajectories in exploiting an a priori knowledge about the target state.

The paper is organized as follows. The general framework is presented in section II. Section III recall the conditions ensuring the observability of the target. In this section we

calculate the set of all the target trajectories compatible with the angle measurements in the case where both the target and the observer follow a uniform motion. We present in section IV a way to estimate these ambiguous trajectories thanks to a linear estimator. Section V shows some simulation results and finally, some conclusions are drawn in section VI.

II. PROBLEM STATEMENT

It is assumed that the target follows a constant velocity model in a three-dimensional space. In a discrete time model, the target state is defined by

$$X_T(t_k) = [x_T(t_k), \dot{x}_T, y_T(t_k), \dot{y}_T, z_T(t_k), \dot{z}_T]^T \quad (1)$$

The target evolves as follows

$$X_T(t_k) = F X_T(t_{k-1}) + \eta(t_k) \quad (2)$$

where $\eta(t_k)$ is a white Gaussian noise process and where the state transition matrix is defined by

$$F = \begin{bmatrix} 1 & \Delta T & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \Delta T & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \Delta T \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$\Delta T \triangleq t_k - t_{k-1}$ being the sampling period. The noise-free azimuths and elevation angles satisfy the nonlinear equation (Fig. 1)

$$\begin{cases} \theta(t) = \arctan \left[\frac{r_y(t)}{r_x(t)} \right] \\ \phi(t) = \arctan \left[\frac{r_z(t)}{\sqrt{r_x^2(t) + r_y^2(t)}} \right] \end{cases} \quad (3)$$

where

$$r(t) = (r_x(t), r_y(t), r_z(t))^T = (x_T(t) - x_o(t), y_T(t) - y_o(t), z_T(t) - z_o(t))^T \quad (4)$$

is the vector of the relative positions between the target and the observer. First we recall, in a simple way, the necessary conditions under which the target is observable.

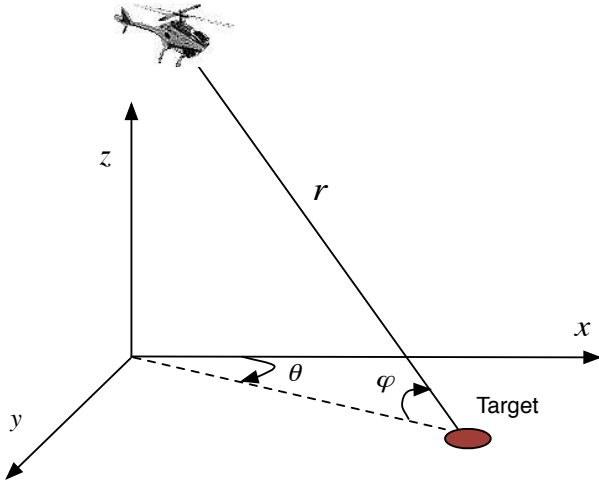


Fig. 1. Observer geometry. For better legibility the target is represented on the ground

III. AMBIGUOUS TRAJECTORIES

A. Observability conditions

The observability conditions of three-dimensional estimations with azimuths and elevation angles have been widely studied [6], [7]. The authors analyze the observability (here in a nonlinear context) in a linear framework, thus allowing direct utilization of a simplified observability criterion. A direct application of the following proposition [7] states the necessary conditions under which the target state is observable.

Proposition 1. Let $r(t) = (r_x(t), r_y(t), r_z(t))^T$ be the relative position vector (4). If the target state is observable then

$$\begin{cases} r_x(t) \neq \alpha(t)P_1(t) \\ r_y(t) \neq \alpha(t)P_2(t) \\ r_z(t) \neq \alpha(t)P_3(t) \end{cases} \quad (5)$$

where $P_1(t), P_2(t), P_3(t)$ are any polynomials of degree the order of the target model dynamics and where $\alpha(t)$ is an arbitrary scalar function

The order of the dynamics should be understood as the M th-order of the series expansion in time of the dynamics. For example $M = 0$ corresponds to a stationary target, $M = 1$ corresponds to a trajectory of constant target. In others words, the proposition states that the order of the observer dynamics must be different of the order of the target dynamics. On the contrary, if the target and observer have a constant velocity model,

$$r(t) = r(t_0) + (t - t_0)\dot{r}(t_0) \quad (6)$$

$r_x(t), r_y(t), r_z(t)$ are polynomials of degree one which is also the degree of the target dynamics. So the target state is not observable in this case. Any maneuver of the observer ($M \geq 2$) enables generally observability. However, the conditions of

the Proposition 1 are necessary but not sufficient. Indeed, for example, if the target is stationary and the observer follows a constant velocity model along the line of sight, the target is unobservable while the order of the observer dynamics is greater than the one of the target.

We are now interested in showing all the trajectories compatible with the angle measurements when both the observer and the target have a constant velocity model.

B. Ambiguous trajectories

If both the observer and the target have a constant velocity model (2), there exists different constant velocity trajectories of the target which match with the angle measurements. The next proposition establishes the set of all of such trajectories called ambiguous trajectories.

Let $X_o(t_0)$ be the initial 6-dimensional state vector of the observer. Let t_1, t_2, t_3 be three distinct times and t'_1, t'_2 be two distinct times. Let us introduce some notations

$$\begin{cases} \mathbf{t} = [t_1 - t_0, t_2 - t_0, t_3 - t_0]^T \\ B = \begin{bmatrix} 1 & t'_1 - t_0 \\ 1 & t'_2 - t_0 \end{bmatrix} \\ B_1 = \begin{bmatrix} 1 & t'_1 - t_0 & 0 & 0 \\ 0 & 0 & 1 & t'_1 - t_0 \end{bmatrix} \\ B_2 = \begin{bmatrix} 1 & t'_2 - t_0 & 0 & 0 \\ 0 & 0 & 1 & t'_2 - t_0 \end{bmatrix} \\ \Theta = [\theta(t_1), \theta(t_2), \theta(t_3)]^T ; \Phi = [\phi(t'_1), \phi(t'_2)]^T \end{cases} \quad (7)$$

The vector (Θ, Φ) consists in 3 azimuths and 2 elevation angles evaluated exactly at the times (t_1, t_2, t_3) and (t'_1, t'_2) respectively.

Proposition 2. Assume that both the observer and the target follow a constant velocity motion. The ambiguous target trajectories which are compatible with the azimuths and elevation angles $(\theta(t), \phi(t))$ (3) are determined by the following initial target state $X_T(t_0, \mu)$ (1)

$$X_T(t_0, \mu) = X_o(t_0) + \mu \begin{bmatrix} A_{\Theta}^{-1} \mathbf{t} \\ 1 \\ B^{-1} s_{\Theta, \Phi} \end{bmatrix} \quad (8)$$

where μ is an arbitrary non-zero scalar and where

$$A_{\Theta} = \begin{bmatrix} \tan \theta(t_1) & (t_1 - t_0) \tan \theta(t_1) & -1 \\ \tan \theta(t_2) & (t_2 - t_0) \tan \theta(t_2) & -1 \\ \tan \theta(t_3) & (t_3 - t_0) \tan \theta(t_3) & -1 \end{bmatrix} \quad (9)$$

$$s_{\Theta, \Phi} = \begin{pmatrix} \|B_1 u_{\Theta}\| \tan \phi(t'_1) \\ \|B_2 u_{\Theta}\| \tan \phi(t'_2) \end{pmatrix} \quad (10)$$

with $u_\Theta = \begin{pmatrix} A_\Theta^{-1} \mathbf{t} \\ 1 \end{pmatrix}$. The ambiguous trajectories defined by (8) are determined by the 5 angles (Θ, Φ) (7) and are set by a non-zero scalar μ .

PROOF OF PROPOSITION 2.

An ambiguous trajectory in the three-dimensional space remains ambiguous in the xy plane. Therefore we analyze first the xy ambiguous trajectories only taking into account the azimuths. For $i = 1, 2, 3$ we have

$$\tan \theta(t_i) = \frac{r_y(t_0) + (t_i - t_0)r_{\dot{y}}(t_0)}{r_x(t_0) + (t_i - t_0)r_{\dot{x}}(t_0)} \quad (11)$$

Let $Y = [r_x(t_0), r_{\dot{x}}(t_0), r_y(t_0)]^T$. Let us define by μ the parameter: $\mu = r_{\dot{y}}(t_0)$. The whole 4-dimensional state vector $[Y, \mu]^T$ in the xy plane is characterized by the three azimuths. Indeed, equation (11) gives

$$A_\Theta Y = \mu \begin{bmatrix} t_1 - t_0 \\ t_2 - t_0 \\ t_3 - t_0 \end{bmatrix} = \mu \mathbf{t} \quad (12)$$

where A_Θ is defined in (9) and \mathbf{t} in (7). Therefore, the set of the ambiguous trajectories in the xy plane is characterized by the initial relative following state vector

$$[r_x(t_0), r_{\dot{x}}(t_0), r_y(t_0), r_{\dot{y}}(t_0)]^T = [\mu A_\Theta^{-1} \mathbf{t}, \mu]^T \quad (13)$$

These 4-dimensional trajectories are the projections of the ambiguous trajectories in the 6-dimensional space. Thus, it remains now to determine the initial relative height $r_z(t_0)$ and the initial relative vertical velocity $r_{\dot{z}}(t_0)$ which are compatible with the exact elevation angles $\phi(t)$. It is sufficient to consider the two elevation angles $\phi(t'_1)$ and $\phi(t'_2)$ (7). We have for $i = 1, 2$

$$\tan \phi(t'_i) = \frac{r_z(t_0) + (t'_i - t_0)r_{\dot{z}}(t_0)}{[r_x(t'_i)^2 + r_y(t'_i)^2]^{1/2}} \quad (14)$$

Using the definition of B (7), the equation (14) yields

$$[r_z(t_0), r_{\dot{z}}(t_0)]^T = B^{-1} \begin{bmatrix} (r_x(t'_1)^2 + r_y(t'_1)^2)^{1/2} \tan \phi(t'_1) \\ (r_x(t'_2)^2 + r_y(t'_2)^2)^{1/2} \tan \phi(t'_2) \end{bmatrix}$$

The relative positions $r_x(t'_1), r_y(t'_1), r_x(t'_2)$ and $r_y(t'_2)$ can be deduced from the initial relative state vector (13) in the xy plane and from (6). After some calculations, we get

$$[r_z(t_0), r_{\dot{z}}(t_0)]^T = \mu B^{-1} \mathbf{s}_{\Theta, \Phi} \quad (15)$$

where $\mathbf{s}_{\Theta, \Phi}$ is defined in (10). Finally, by combining equations (13) and (15), we get the following family of 6-dimensional initial state vectors of the target $X_T(t_0, \mu)$

$$X_T(t_0, \mu) = X_o(t_0) + \mu \begin{bmatrix} A_\Theta^{-1} \mathbf{t} \\ 1 \\ B^{-1} \mathbf{s}_{\Theta, \Phi} \end{bmatrix}$$

For any $\mu \neq 0$, $X_T(t_0, \mu)$ generates an ambiguous trajectory which produces the set of azimuths and elevation angles $(\theta(t), \phi(t))$ characterized by (Θ, Φ) (7). Note that in the case where $\mu = 0$, the ambiguous trajectory match that of the observer trajectory: $X_T(t_0, 0) = X_o(t_0)$. The function tangent involved in (9) and (10) being of period π , it is sufficient to consider positive values of μ . Negative values produce point symmetrical trajectories.

It remains now to estimate the vector (Θ, Φ) when the angle measurements are noisy. This vector is of course always observable.

IV. ESTIMATION OF THE AMBIGUOUS TRAJECTORIES

We present a quick method to estimate the ambiguous trajectories using only the angle measurements. The observer and the target motions still have a constant velocity model. For that purpose, we have to estimate the vector (Θ, Φ) (7) which determine all the ambiguous trajectories by varying the parameter μ (see proposition 2). A useful method, based on the Gauss quadrature, allows us to estimate quickly and accurately these angles providing that the variation amplitude of the angles is not too large. The 5 angles (Θ, Φ) are evaluated by means of a linear combination of the measurements.

A. Description of the method

Let $t_1 < \dots < t_n$ be n positive reals. Let us denote by l^2 the set of real valued functions f defined on \mathbb{R} such that

$$\sum_{t=t_1}^{t_n} f^2(t) < \infty .$$

l^2 is endowed with the inner product $\langle \cdot, \cdot \rangle$ defined by

$$\langle f, g \rangle = \sum_{t=t_1}^{t_n} f(t) g(t) , \quad (16)$$

for any $f, g \in l^2$. For any $1 \leq m \leq n$, let us denote by Ψ_m , the Legendre Polynomial of degree m such that

$$\langle \Psi_m, t^j \rangle = 0 , \quad (17)$$

for any integer $0 \leq j \leq m-1$. Ψ_m can be computed recursively using the Graham-Schmidt formula [8]. Let us denote by $T_1 \leq \dots \leq T_m$ the roots of Ψ_m . One can prove that for any $i \neq j$ with $1 \leq i, j \leq m$, T_i is a real such that

$$t_1 \leq T_i \leq t_n \quad \text{and} \quad T_i \neq T_j .$$

If $m = n$, then $T_i = t_i$, for any $1 \leq i \leq m = n$.

The roots of Ψ_m can be computed approximately by the Laguerre algorithm [8].

For any $1 \leq i \leq m$, let us introduce Φ_i the Lagrange polynomial of degree $m-1$ such that

$$\Phi_i(t) = \begin{cases} 1 & \text{if } t = T_i \\ 0 & \text{if } t = T_j \neq T_i \end{cases} \quad (18)$$

We consider now the following measurement model,

$$z(t) = \beta(t) + \varepsilon(t) \quad (19)$$

where $\varepsilon(t)$ are *i.i.d.* centered Gaussian noises with standard deviation (std) σ . $\beta(t)$ plays the role of the elevation angles or the role of the azimuths (3). Let n be the number of observations. We aim to estimate m values of $\beta(t)$ for $t = T_1, T_2, \dots, T_m$ with $1 < m < n$. The following proposition exhibits an estimator of the angles $\beta_{T_1}, \dots, \beta_{T_m}$ by means of a linear combination of the measurements $z(t)$ (19).

Proposition 3. *The following assertions holds.*

1) For $i = 1, 2, \dots, m$, the estimators

$$\hat{\beta}_{T_i} = \frac{1}{\|\Phi_i\|^2} \sum_{t=T_1}^{t_n} \Phi_i(t) z(t) \quad (20)$$

are nearly unbiased

$$\mathbb{E}(\hat{\beta}_{T_i}) \approx \beta_{T_i} \quad (21)$$

where Φ_i is the Lagrange polynomial of degree $m-1$ defined in (18).

2) The covariance matrix of the estimators is diagonal.

For $i, j = 1, 2, \dots, m$, we have

$$\text{cov}(\hat{\beta}_{T_i}, \hat{\beta}_{T_j}) = \delta_{i=j} \frac{\sigma^2}{\|\Phi_i\|^2} \quad (22)$$

$$\text{with } \|\Phi_i\|^2 = \sum_{t=T_1}^{t_n} \Phi_i^2(t)$$

These suitable properties (21) and (22) are obtained using the orthogonality of the Lagrange polynomials (18) which interpolate the roots of the Legendre polynomial [9], [10], [11]. One can prove that $\|\Phi_i\|^2 \sim \text{constant} \times n$, so that the std of $\hat{\beta}_{T_i}$ is of order $\frac{\sigma}{\sqrt{n}}$ when n is large.

The linear estimator (20) is very accurate providing that the signal has a dynamics order less than $2m$, namely, the signal must behave as a polynomial with a degree lower or equal to $2m-1$.

B. Application to the estimation of the ambiguous trajectories

We consider we have n noisy azimuths and elevation angles (3) measurements according to the model

$$\begin{cases} z_\theta(t) = \theta(t) + \varepsilon(t) \\ z_\phi(t) = \phi(t) + \nu(t) \end{cases} \quad (23)$$

where $\varepsilon(t)$ and $\nu(t)$ are *i.i.d.* centered Gaussian noises with std σ_θ and σ_ϕ respectively. $\theta(t)$ and $\phi(t)$ are defined in (3). Here, we do not take into account the relation between these angles and the whole state $X_T(t)$ (1). We aim to estimate the azimuths and elevation angles at times (t_1, t_2, t_3) and (t'_1, t'_2) respectively using only the angle measurements (23). For this purpose we choose $(t_1, t_2, t_3) = (T_1, T_2, T_3)$ and $(t'_1, t'_2) = (T'_1, T'_2)$ where (T_1, T_2, T_3) are the roots of the Legendre polynomial Ψ_m (17) with $m = 3$ and where (T'_1, T'_2) are the roots of the Legendre polynomial Ψ_m with $m = 2$.

Consequently, the estimations of (Θ, Φ) (7) are done using the linear estimator (20).

Estimation of (Θ, Φ)

1) For $i = 1, 2, 3$

$$\hat{\theta}_{T_i} = \frac{1}{\|\Phi_i\|^2} \sum_{t=T_1}^{t_n} \Phi_i(t) z_\theta(t) \quad (24)$$

2) For $i = 1, 2$

$$\hat{\phi}_{T'_i} = \frac{1}{\|\Phi'_i\|^2} \sum_{t=T_1}^{t_n} \Phi'_i(t) z_\phi(t) \quad (25)$$

The Lagrange polynomials Φ_i and Φ'_i (18) interpolate (T_1, T_2, T_3) and (T'_1, T'_2) respectively.

In the case of the Legendre polynomial with degree $m = 3$ or $m = 2$ and when $t_k = k$ for $1 \leq k \leq n$, the roots can be computed exactly with the following formula.

Roots of the Legendre polynomials

1) Case $m=3$

$$\begin{aligned} T_1 &= \frac{n+1}{2} - \tau, \quad T_2 = \frac{n+1}{2}, \quad T_3 = \frac{n+1}{2} + \tau, \\ \|\Phi_1\|^2 &= \|\Phi_3\|^2 = \frac{5n(n^2-1)}{6(3n^2-7)}, \quad \|\Phi_2\|^2 = \frac{4n(n^2-4)}{3n^2-7} \\ \text{with } \tau &= \sqrt{\frac{3n^2-7}{20}} \end{aligned}$$

2) Case $m=2$

$$\begin{aligned} T'_1 &= \frac{n+1}{2} - \tau, \quad T'_2 = \frac{n+1}{2} + \tau, \\ \|\Phi'_1\|^2 &= \|\Phi'_2\|^2 = \frac{n}{2}, \\ \text{with } \tau &= \sqrt{\frac{n^2-1}{12}} \end{aligned} \quad (26)$$

The estimators (24) (25) are nearly unbiased providing that the variation amplitude of the angles is not too large. For $i = 1, 2, 3$ and $j = 1, 2$, the estimators have the following variances

$$\text{var}(\hat{\theta}_{T_i}) = \frac{\sigma_\theta^2}{\|\Phi_i\|^2}, \quad \text{var}(\hat{\phi}_{T'_j}) = \frac{\sigma_\phi^2}{\|\Phi'_j\|^2} \quad (27)$$

More generally, in the case of any sample time t_k , the roots are easily computed offline with the Laguerre algorithm [8].

V. NUMERICAL RESULTS

Some simulation results are presented below which illustrate the ambiguous trajectories and their estimations. Both the target and the observer have a constant velocity model.

Scenario parameters

Positions are expressed in meters and velocities in meters/second

- Observer initial state

$$\begin{aligned} X_o(t_0) &= [x_o(t_0), \dot{x}_o, y_o(t_0), \dot{y}_o, z_o(t_0), \dot{z}_o]^T \\ &= [0, 30, 0, 0, 0, 5]^T \end{aligned}$$

- Target initial state

$$\begin{aligned} X_T(t_0) &= [x_T(t_0), \dot{x}_T, y_T(t_0), \dot{y}_T, z_T(t_0), \dot{z}_T]^T \\ &= [5000, -10, 5000, 10, 100, 3]^T \end{aligned}$$

- Measurements

Sampling period = $\Delta T = 1$ s

Number of measurements = $n = 100$

A. Illustration of the ambiguous trajectories

We present in the following figures the target, the observer, the ambiguous trajectories projected on different planes (Fig. 3-5) and two 3D representation (Fig. 6-7). These trajectories are computed using (8) and set by $\mu \in [0, 20]$. They produce the same set of azimuths and elevation angles (here noise-free) (Fig. 2).

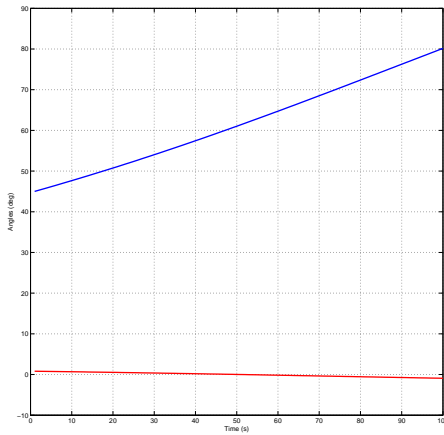


Fig. 2. Azimuths (—) and elevation angles (—)

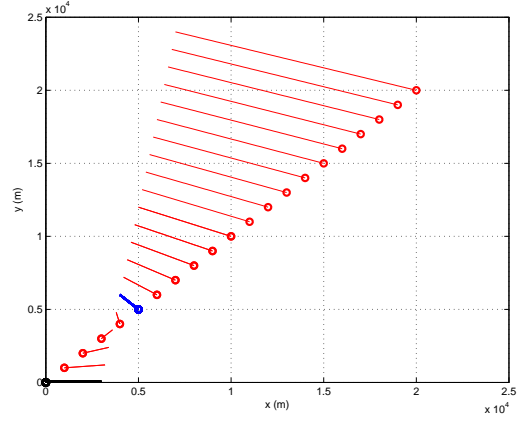


Fig. 3. Ambiguous trajectories. Projection on xy plane. —: ambiguous trajectories. •: initial point. —: target trajectory. —: observer trajectory

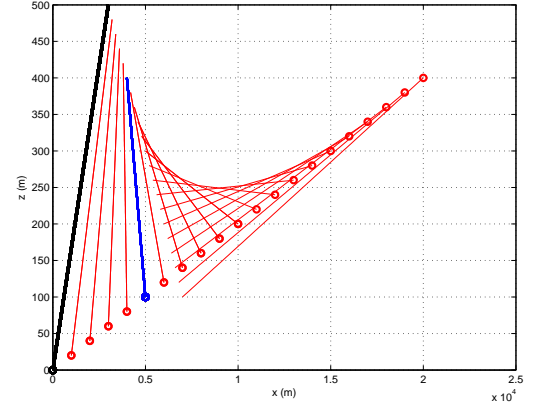


Fig. 4. Ambiguous trajectories. Projection on xz plane. —: ambiguous trajectories. •: initial point. —: target trajectory. —: observer trajectory

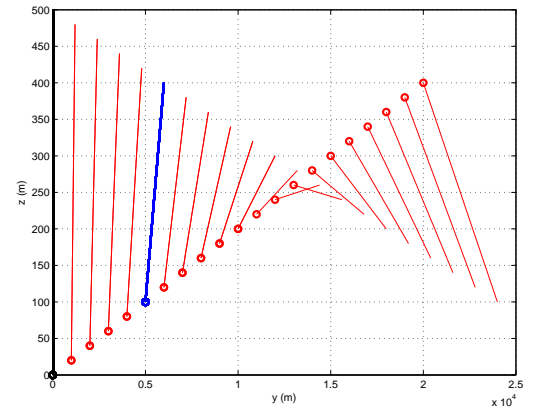


Fig. 5. Ambiguous trajectories. Projection on yz plane. —: ambiguous trajectories. •: initial point. —: target trajectory. —: observer trajectory

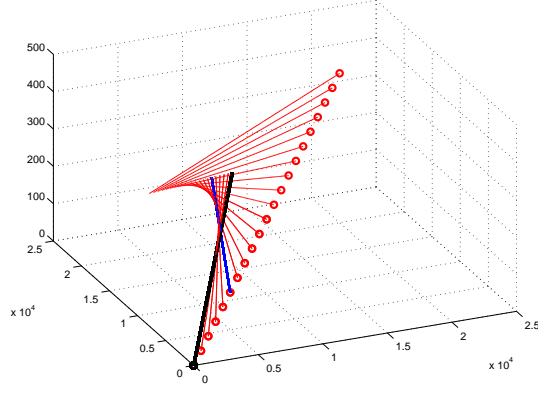


Fig. 6. 3D view of the ambiguous trajectories. —: ambiguous trajectories. o: initial point. —: target trajectory. —: observer trajectory

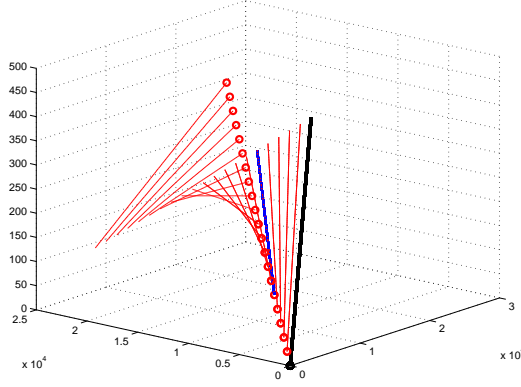


Fig. 7. 3D view of the ambiguous trajectories. —: ambiguous trajectories. o: initial point. —: target trajectory. —: observer trajectory

We can see on figure 3 that the target's heading K on the xy plane becomes constant as μ increases. Indeed, we have

$$K = \arctan\left(\frac{\dot{x}_T}{\dot{y}_T}\right) = \arctan\left(\frac{\dot{x}_o + \mu \alpha(\Theta)}{\dot{y}_o + \mu}\right) \\ \rightarrow \arctan(\alpha(\Theta)) \text{ when } \mu \rightarrow +\infty$$

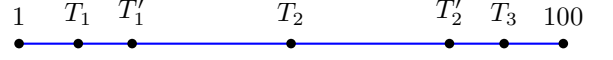
where $\alpha(\Theta)$ is deduced from (8).

B. Estimation of the ambiguous trajectories

We present below the performances of the linear estimators (24)-(25). The azimuths and elevation angles are measured every second ($\Delta T = 1s$), thus, we have $n = 100$ pairs of angle measurements. The std of the measurements (23) ($\sigma = \sigma_\theta = \sigma_\phi$) are set to 1 deg and 0.1 deg. One thousand ($N = 1000$) of Monte Carlo trials have been done in order to estimate the RMSE (root-mean-square error) of the five angles (Θ, Φ) :

$$E_r(\hat{\beta}) \triangleq RMSE = \sqrt{\mathbb{E}[\hat{\beta} - \beta]^2} \approx \sqrt{\frac{1}{N} \sum_{i=1}^N [\hat{\beta}^i - \beta]^2} \quad (28)$$

where $\hat{\beta}^i$ is a trial of one the angle estimations $\hat{\theta}_{T_1}^i, \hat{\theta}_{T_2}^i, \hat{\theta}_{T_3}^i, \hat{\phi}_{T'_1}^i, \hat{\phi}_{T'_2}^i$. The roots (T_1, T_2, T_3) and (T'_1, T'_2) are distributed as follows (26)



We can see in following table the accuracy of the linear estimators of (Θ, Φ) . Note that these good performances are valid if the variation amplitude of the angles is not too large. The computing time of these estimators is negligible.

TABLE I
RMSE OF $(\hat{\Theta}, \hat{\Phi})$ (deg)

σ	$E_r(\hat{\theta}_{T_1})$	$E_r(\hat{\theta}_{T_2})$	$E_r(\hat{\theta}_{T_3})$	$E_r(\hat{\phi}_{T'_1})$	$E_r(\hat{\phi}_{T'_2})$
1 deg	0.18	0.15	0.19	0.14	0.14
0.1 deg	0.02	0.02	0.02	0.01	0.01

We recall that the five angles (Θ, Φ) allows us to generate all the ambiguous trajectories by varying the parameter μ (see proposition 2). Suppose we have an a priori knowledge about some dynamics parameters of the target, like a speed range or a height range. Then it is possible without maneuver to estimate all the trajectories of the target satisfying the constraints using (8) by varying μ . In what follows, we calculate the target trajectories which are compatible with a vertical speed constraint. For example, if the modulus of vertical speed of the target is supposed to be small, precisely $|\dot{z}_T| < 0.5m/s$, than we get the restricted set of trajectories (in green colour) presented in figures (8-11). These trajectories are estimated thanks to the linear estimators (24)-(25) with $\sigma_\theta = \sigma_\phi = 0.1$ deg (23).

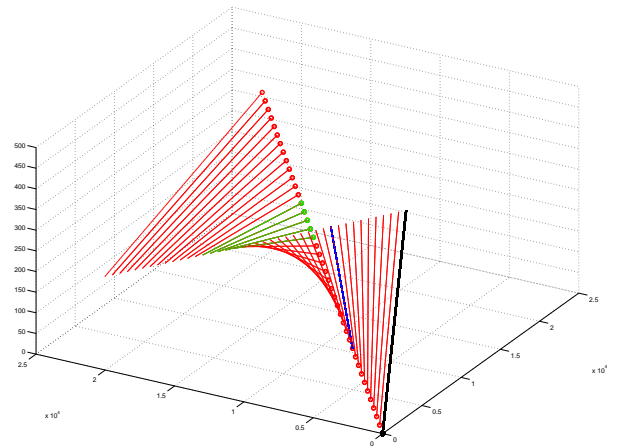


Fig. 8. 3D view of the ambiguous trajectories. —: ambiguous trajectories. o: initial point. —: target trajectory. —: observer trajectory. —: constrained trajectories

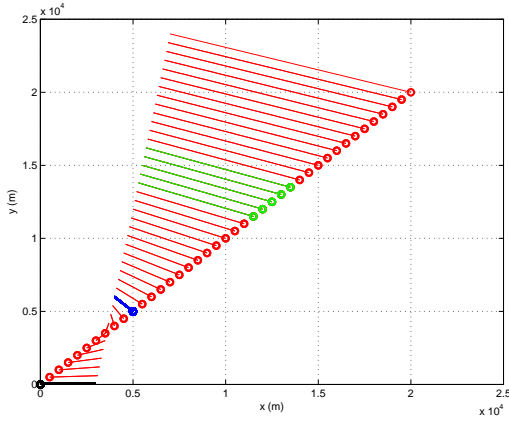


Fig. 9. Ambiguous trajectories. Projection on xy plane. —: ambiguous trajectories. o: initial point. —: target trajectory. —: observer trajectory. —: constrained trajectories

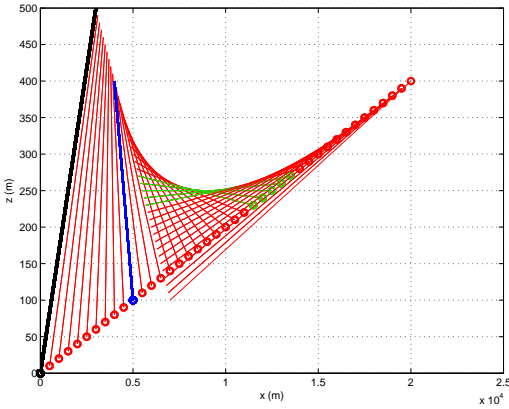


Fig. 10. Ambiguous trajectories. Projection on xz plane. —: ambiguous trajectories. o: initial point. —: target trajectory. —: observer trajectory. —: constrained trajectories

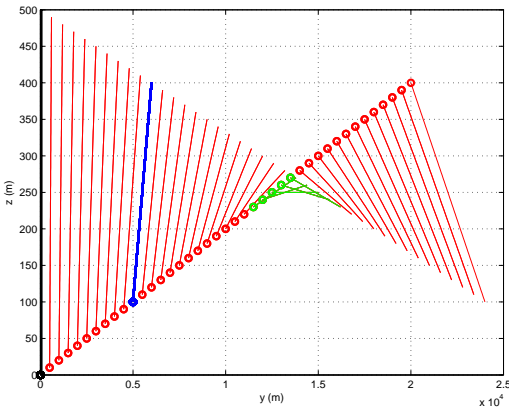


Fig. 11. Ambiguous trajectories. Projection on yz plane. —: ambiguous trajectories. o: initial point. —: target trajectory. —: observer trajectory. —: constrained trajectories

VI. CONCLUSION

The paper proposes a method to calculate the set of all the ambiguous target trajectories compatible with the angle measurements in the case of both the observer and the target follow a constant velocity model in a three-dimensional space. These trajectories depend on five observable angles. A quick linear estimator of these angles is presented. Any a priori information about the target state allows us to discriminate among ambiguous trajectories. In doing this, the observer can take a decision to maneuver (or not) in order to estimate accurately the whole 6-dimensional target state.

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