Index Codes for Interlinked Cycle Structures with Outer Cycles

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***Abstract*—For side-information graphs called Interlinked Cycle (IC) structures, which generalize cycles and cliques, Thapa, Ong and Johnson ("Interlinked Cycles for Index Coding: Generalizing Cycles and Cliques", *IEEE Trans. Inf. Theory, vol. 63, no. 6, Jun. 2017* and "Interlinked Cycles for Index Coding: Generalizing Cycles and Cliques", in arxiv (arxiv:1603.00092v2 [cs.IT] 25 Feb 2018)) have given an index code construction and a decoding algorithm, for the case where the IC structure does not have any cycles consisting only of non-inner vertices (called outer cycles). In this paper, for IC structures with outer cycles, we give a set of necessary and sufficient conditions for the code construction and the decoding algorithm of Thapa, Ong and Johnson to be valid.**

1. Introduction

The problem of index coding was introduced by Birk and Kol in [1]. The index coding problem consists of a single sender with a set of N independent messages $\mathcal{X} = \{x_1, x_2, \ldots, x_N\}$, and a set of M users $\mathcal{D} = \{D_1, D_2, \ldots, D_M\}$, connected to the sender by a single shared error-free link, with the k^{th} user identified as $D_k = (\mathcal{X}_k, \mathcal{A}_k)$, where $\mathcal{X}_k \subseteq \mathcal{X}$ is the set of messages desired by D_k, the set $\mathcal{A}_k \subset \mathcal{X}$ is comprised of the messages available to user D_k as side information. The set $\mathcal{A}_k$ satisfies $\mathcal{X}_k \cap \mathcal{A}_k = \phi$. In a scalar linear index coding scheme, each message $x_i \in \mathcal{F}_q$, $i \in [M]$ and $\mathcal{F}_q$ is a finite field. Sender encodes M messages to n symbols in $\mathcal{F}_q$ using a linear mapping and n is called the length of the index code. Each user will decode their desired messages using linear combinations of transmitted n symbols and side information available to them. An index coding problem is said to be unicast [2] if $\mathcal{X}_k \cap \mathcal{X}_j = \phi$ for $k \neq j$ and $k, j \in \{1, 2, \ldots, M\}$, ie., no message is desired by more than one user. The problem is said to be single unicast if the problem is unicast and $|\mathcal{X}_k| = 1$ for all $k \in \{1, 2, \ldots, M\}$. A unicast index coding problem can be reduced into single unicast index coding problem, by splitting the user demanding more than one message into several users, each demanding one message and with the same side information as the original user. Single unicast index coding problems can be described by a directed graph called as a side information graph [3], in which the vertices in the graph represent the indices of messages $\{x_1, x_2, \ldots, x_N\}$ and there is a directed edge from vertex i to vertex j if and only if the user requesting x_i has x_j as side information.

The set of vertices in a directed graph $\mathcal{G}$ is denoted by $V(\mathcal{G})$ and the set of vertices in the out-neighbourhood of a vertex q in $\mathcal{G}$ is denoted by $N^+_{\mathcal{G}}(q)$.

Interlinked Cycle Cover (ICC) scheme is proposed as a scalar linear index coding scheme to solve single unicast index coding problems by Thapa et al. [4], by defining a graph structure called an Interlinked Cycle (IC) structure. After a correction announced in [5] by Thapa et al., the definition of an IC structure is as given below. The correction is only in the definition of IC structure and the code construction and decoding algorithm continue to be the same as given in the original version [4].

***Definition* 1 (IC Structure** [5]**).** A side information graph $\mathcal{G}$ is called a K-IC structure with inner vertex set $V_I \subseteq V(\mathcal{G})$, such that $|V_I| = K$ if $\mathcal{G}$ satisfies the following four conditions.

1) There is no I-Cycle in $\mathcal{G}$, where I-Cycle is defined as a cycle which contains only one inner vertex.
2) There is a unique I-Path between any two different inner vertices in $\mathcal{G}$, where an I-path is defined as a path from one inner vertex to another inner vertex without passing through any other inner vertex (as a result, K rooted trees can be drawn where each rooted tree is rooted at an inner vertex and has the remaining inner vertices as the leaves).
3) $\mathcal{G}$ is the union of the K rooted trees.
4) There are no cycles in $\mathcal{G}$ containing only non-inner vertices (called as outer cycles).

The set $V(\mathcal{G}) \backslash V_I$ is called the set of non-inner vertices, denoted by V_{NI}. Let $V_{NI}(i)$ be the set of non-inner vertices that are present in the rooted tree T_i of the inner vertex i.

For IC structures, Thapa et al. proposed a method of construction of index code (presented below as *Construction* 1 in this paper) and a decoding algorithm to decode the obtained index code (presented as *Algorithm* 1 in this paper).

Let the K-IC structure be called $\mathcal{G}$ and let $|V(\mathcal{G})| = N$. Let $V(\mathcal{G}) = \{1, 2, \ldots, N\}$, $V_I = \{1, 2, \ldots, K)$ be the set of the K inner vertices and hence $V_{NI} = \{K+1, K+2, \ldots, N\}$. Let $x_n \in \mathbb{F}_q$ be the message corresponding to the vertex $n \in V(\mathcal{G})$ and where $\mathbb{F}_q$ is the finite field of characteristic 2 to which the all the N messages at the sender belong to (note that in single unicast setting, the number of messages will be equal to the number of users, i.e., $N = M$).

***Construction* 1** ([4])**.** Given the inner and non-inner vertices, the following coded symbols are transmitted.

1) An index code symbol W_I obtained by XOR of messages corresponding to inner vertices is transmitted,

where $W_I = \bigoplus_{i=1}^{K} x_i$.

2) An index code symbol corresponding to each non-inner vertex, obtained by XOR of message corresponding to the non-inner vertex with the messages corresponding to the vertices in the out-neighbourhood of the non-inner vertex is transmitted, i.e., for $j \in V_{NI}$, W_j is transmitted, where$W_j = x_j \bigoplus_{q \in N_{\mathcal{G}}^{+}(j)} x_q$,

where $\oplus$ denotes modulo addition over $\mathbb{F}_q$.

***Algorithm* 1.** It is the algorithm proposed by Thapa et al. [4] to decode the index code obtained by using *Construction* 1 on an IC structure, $\mathcal{G}$.

- The message x_j corresponding to a non-inner vertex j is decoded directly using the transmission W_j and
- the message x_i corresponding to an inner vertex i is decoded using $Z_i = W_I \bigoplus_{q \in V_{NI}(i)} W_q$.

***Definition* 2.** The class of side-information graphs which satisfy only the first three conditions of *Definition* 1, is called as "IC structures with outer cycles".

This paper deals with IC structures with outer cycles and the contributions may be summarized as follows:

- An important property of IC structures with outer cycles is presented in *Lemma* 2.1 which is a basic ingredient in the proof of Theorem 1.
- The class of IC structures with outer cycles for which index code obtained from *Construction* 1 is decodable by *Algorithm* 1 is characterized in *Theorem* 1. This is achieved by giving a set of necessary and sufficient conditions on the IC structures with outer cycles to have the property mentioned.
- An alternate proof using Theorem 1 for the optimality of *Construction* 1 for the case of IC structures without outer cycles is given in Theorem 2.

The proofs of all the claims including Lemma 2.1 and Theorems 1 and 2 can be found in [7] along with several illustrative examples which have been omitted here due to lack of space.

It is important to note that while the paper [6] also deals with IC structures with outer cycles, the class of IC structures with outer cycles considered in [6] is the one where there is only one outer cycle. Also, the code construction and the decoding algorithm presented therein are different from *Construction* 1 and *Algorithm* 1.

The rest of the paper is organized as follows. Section 2 contains the main results. Section 3 contains illustrative examples and Section 4 has concluding remarks and some directions for further research.

2. Main Results

Let the K-IC structure, $\mathcal{G}$, have inner vertex set $V_I = \{1, 2, \ldots, K\}$ and non-inner vertices $V_{NI} = \{K+1, K+2, \ldots, N\}$. Let T_i be the rooted tree corresponding to an inner vertex i where $i \in \{1, 2, \ldots, K\}$. Let $V_{NI}(i)$ be the set of non-inner vertices in $\mathcal{G}$ which appear in the rooted tree T_i of an inner vertex i. For each $i \in \{1, 2, \ldots, K\}$ and for a non-inner vertex j which is at a depth $\geqslant 2$ in the rooted tree T_i, define $a_{i,j}$ as the number of vertices in $V_{NI}(i)$ for which j is in out-neighbourhood in $\mathcal{G}$, i.e., for each $i \in \{1, 2, \ldots, K\}$ and for $j \in V_{NI}(i)$,

$$a_{i,j} \triangleq |\{v : v \in V_{NI}(i), \quad j \in N_{\mathcal{G}}^{+}(v), \quad j \notin N_{T_i}^{+}(i)\}|.$$

Also, for each $i \in \{1, 2, \ldots, K\}$ and for a non-inner vertex j not in the rooted tree T_i, define $b_{i,j}$ as the number of vertices in $V_{NI}(i)$ for which j is in out-neighbourhood in $\mathcal{G}$, i.e., for each $i \in \{1, 2, \ldots, K\}$ and $j \in V(\mathcal{G}) \backslash V(T_i)$,

$$b_{i,j} \triangleq |\{v : v \in V_{NI}(i), \quad j \in N_{\mathcal{G}}^{+}(v)\}|.$$

The following lemma shows that $b_{i,j}$ can take one of the two values, either the value 0 or 1.

***Lemma* 2.1.** Given an IC structure with outer cycles $\mathcal{G}$ with inner vertex set $V_I = \{1, 2, \ldots, K\}$, we have $b_{i,j} \in \{0, 1\}$ for each $i \in \{1, 2, \ldots, K\}$ and $j \in V(\mathcal{G}) \backslash V(T_i)$.

The following theorem characterizes the IC structures with outer cycles for which the index code construction and the decoding algorithm of Thapa, Ong and Johnson [4] remain valid in spite of the presence of the outer cycles.

***Theorem* 1.** Given an IC structure $\mathcal{G}$ with outer cycles, the index code obtained from *Construction* 1 on $\mathcal{G}$ is decodable by *Algorithm* 1 if and only if $\mathcal{G}$ satisfies the following two conditions $c1$ and $c2$.

Condition 1 ($c1$). $a_{i,j}$ must be an odd number for each $i \in \{1, 2, \ldots, K\}$ and $j \in V_{NI}(i) \backslash N_{T_i}^{+}(i)$.

Condition 2 ($c2$). $b_{i,j}$ must be zero for each $i \in \{1, 2, \ldots, K\}$ and $j \in V(\mathcal{G}) \backslash V(T_i)$.

For an IC structure without outer cycles that the *Construction* 1 gives an optimal index code follows from the results in [4]. The following theorem gives an alternate proof using Theorem 1.

***Theorem* 2.** An IC structure which has no cycles containing only non-inner vertices satisfies both the conditions $c1$ and $c2$.

3. Illustrative Examples

***Example* 1.** Consider $\mathcal{G}_1$, a side-information graph which is an IC structure with outer cycles, shown in Fig. 1. $\mathcal{G}_1$ is an IC structure with outer cycles and with inner vertex set $V_I = \{1, 2, 3, 4, 5, 6\}$ because

1) there are no cycles with only one vertex from the set $\{1, 2, 3, 4, 5, 6\}$ in $\mathcal{G}_1$ (i.e., no I-cycles),
2) using the rooted trees for each vertex in the set, $\{1, 2, 3, 4, 5, 6\}$, which are given in Fig. 2a, 2b, 2c, 2d, 2e and 2f respectively, it is verified that there exists a unique path between any two different vertices in V_I in $\mathcal{G}_1$ and does not contain any other vertex in V_I (i.e., unique I-path between any pair of inner vertices),
3) $\mathcal{G}_1$ is the union of all the 6 rooted trees.

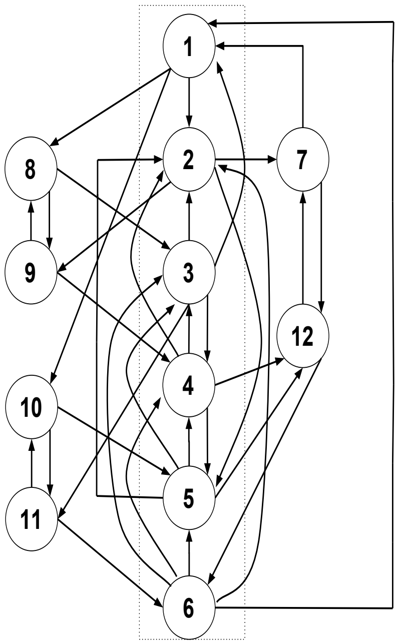


Fig. 1: IC structure $\mathcal{G}_1$ with $V_I = \{1,2,3,4,5,6\}$ and outer cycles $\{8,9\}$ $\{10,11\}$ and $\{7,12\}$.

Also, notice that there are three disjoint cycles each of them consisting of only the non-inner vertices, i.e., three outer cycles $\{8,9\}$ $\{10,11\}$ and $\{7,12\}$.

Verification of $c1$ *and* $c2$*.* From Table I, it is observed that $c1$ and $c2$ are satisfied by $\mathcal{G}_1$. As a result, *Algorithm* 1 can be used to decode the index code obtained by using *Construction* 1 on the IC structure $\mathcal{G}_1$.

The index code obtained is $W_I = x_1 \oplus x_2 \oplus x_3 \oplus x_4 \oplus x_5 \oplus x_6$, $W_7 = x_7 \oplus x_1 \oplus x_{12}$, $W_8 = x_8 \oplus x_3 \oplus x_9$, $W_9 = x_9 \oplus x_4 \oplus x_8$, $W_{10} = x_{10} \oplus x_5 \oplus x_{11}$, $W_{11} = x_{11} \oplus x_6 \oplus x_{10}$, $W_{12} = x_{12} \oplus x_6 \oplus x_7$. Messages x_7, x_8, x_9, x_{10}, x_{11} and x_{12} are decoded directly using W_7, W_8, W_9, W_{10}, W_{11} and W_{12} respectively. The computation of Z_i and the decoding of x_i, for $i = 1, 2, \ldots, 6$ by *Algorithm* 1 is shown in Table II. Thus, *Algorithm* 1 is used to decode the index code obtained by using *Construction* 1 on $\mathcal{G}_1$.

***Example* 2.** Consider $\mathcal{G}_2$, a side-information graph which is a 5-IC structure, shown in Fig. 3. $\mathcal{G}_2$ is a 6-IC structure with inner vertex set $V_I = \{1,2,3,4,5\}$ and an outer cycle $\{8,9\}$ since

1) there are no cycles with only one vertex from the set $\{1,2,3,4,5\}$ in $\mathcal{G}_2$ (i.e., no I-cycles),
2) using the rooted trees for each vertex in the set, $\{1,2,3,4,5\}$, which are given in Fig. 4a, 4b, 4c, 4d and 4e respectively, it is verified that there exists a unique path between any two different vertices in V_I in $\mathcal{G}_2$ and does not contain any other vertex in V_I (i.e., unique I-path between any pair of inner vertices),
3) $\mathcal{G}_2$ is the union of all the 5 rooted trees.

Verification of $c1$ *and* $c2$*.* From Table III, it is observed that $c1$ is not satisfied ($a_{1,8} = 2$, an even number) and $c2$ is satisfied by $\mathcal{G}_2$. As a result, *Algorithm* 1 fails to decode the index code obtained by using *Construction* 1 on the IC structure $\mathcal{G}_2$. It is verified as follows.

The index code obtained is $W_I = x_1 \oplus x_2 \oplus x_3 \oplus x_4 \oplus x_5$; $W_6 = x_6 \oplus x_7 \oplus x_8$; $W_7 = x_7 \oplus x_3$; $W_8 = x_8 \oplus x_4 \oplus x_9$; $W_9 = x_9 \oplus x_5 \oplus x_8$; $W_{10} = x_{10} \oplus x_3 \oplus x_{11}$; $W_{11} = x_{11} \oplus x_1$. Messages x_6, x_7, x_8, x_9, x_{10} and x_{11} are decoded directly using W_6, W_7, W_8, W_9, W_{10} and W_{11} respectively. The computation of Z_1 and the decoding of x_i, for $i = 1, 2, \ldots, 5$ by *Algorithm* 1 is shown in Table IV. The inability of the *Algorithm* 1 to decode x_1 can also be observed in Table IV. Thus, *Algorithm* 1 is fails to decode the index code obtained by using *Construction* 1 on $\mathcal{G}_2$ since user requesting message x_1 does not have x_8 in its side-information.

***Example* 3.** This examples illustrates *Theorem* 2. Consider $\mathcal{G}_3$, a side-information graph which is a 3-IC structure shown in Fig. 5. Notice that $\mathcal{G}_3$ does not have any cycles consisting of only non-inner vertices and that it is

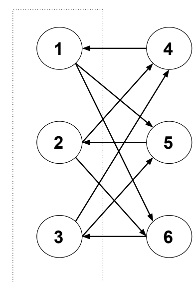
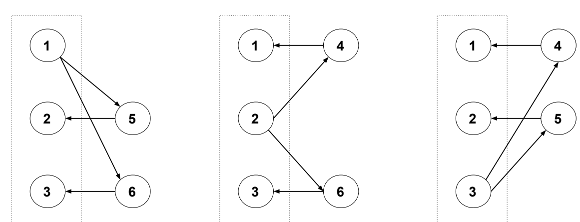


Fig. 5: The 3-IC structure $\mathcal{G}_3$ with $V_I = \{1,2,3\}$.

indeed a 3-IC structure with inner vertex set $V_I = \{1,2,3\}$ since

1) there are no cycles containing only one vertex from the set $\{1,2,3\}$ in $\mathcal{G}_3$ (i.e., no I-cycles),
2) using the rooted trees for each vertex in the set $\{1,2,3\}$, which are given in Fig. 6a, 6b and 6c respectively, it is verified that there exists a unique path between any two different vertices in V_I in $\mathcal{G}_3$ and does not contain any other vertex in V_I (i.e., unique I-path between any pair of inner vertices),
3) $\mathcal{G}_3$ is the union of all the 3 rooted trees.



(a) (b) (c)

Fig. 6: Figures showing rooted trees of inner vertices $1, 2, 3$ of $\mathcal{G}_3$, respectively.

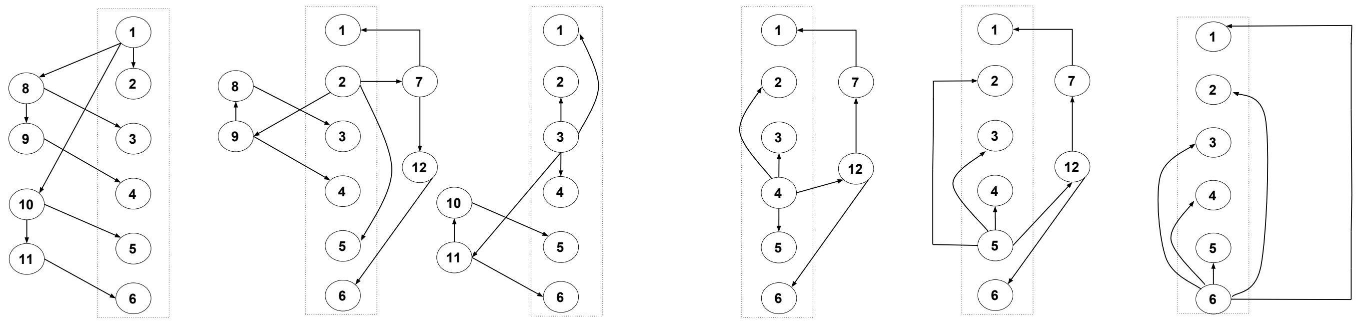


Fig. 2: Figures showing rooted trees of inner vertices $1, 2, 3, 4, 5, 6$ of $\mathcal{G}_1$, respectively.

T_i	$V_{NI}(i)$	$j \in V(T_i)\backslash\{V_I \cup N^+_{T_i}(i)\}$	$a_{i,j}$	$j \in V(\mathcal{G}_1)\backslash V(T_i)$	$b_{i,j}$
T_1	$\{8, 9, 10, 11\}$	$\{9, 11\}$	$1, 1$	$\{7, 12\}$	$0, 0$
T_2	$\{7, 8, 9, 12\}$	$\{8, 12\}$	$1, 1$	$\{10, 11\}$	$0, 0$
T_3	$\{10, 11\}$	$\{10\}$	1	$\{7, 8, 9, 12\}$	$0, 0, 0, 0$
T_4	$\{7, 12\}$	$\{7\}$	1	$\{8, 9, 10, 11\}$	$0, 0, 0, 0$
T_5	$\{7, 12\}$	$\{7\}$	1	$\{8, 9, 10, 11\}$	$0, 0, 0, 0$
T_6	ϕ	ϕ	$-$	$\{7, 8, 9, 10, 11, 12\}$	$-$

TABLE I: Table that verifies conditions $c1$ and $c2$ for $\mathcal{G}_1$.

Message x_i	Computation of Z_i	$N^+_{\mathcal{G}_1}(i)$
x_1	$W_I \oplus W_8 \oplus W_9 \oplus W_{10} \oplus W_{11} = x_1 \oplus x_2 \oplus x_8 \oplus x_{10}$	x_2, x_8, x_{10}
x_2	$W_I \oplus W_7 \oplus W_8 \oplus W_9 \oplus W_{12} = x_2 \oplus x_5 \oplus x_7 \oplus x_9$	x_5, x_7, x_9
x_3	$W_I \oplus W_{10} \oplus W_{11} = x_3 \oplus x_1 \oplus x_2 \oplus x_4 \oplus x_{11}$	x_1, x_2, x_4, x_{11}
x_4	$W_I \oplus W_7 \oplus W_{12} = x_4 \oplus x_2 \oplus x_3 \oplus x_5 \oplus x_{12}$	x_2, x_3, x_5, x_{12}
x_5	$W_I \oplus W_7 \oplus W_{12} = x_5 \oplus x_2 \oplus x_3 \oplus x_4 \oplus x_{12}$	x_2, x_3, x_4, x_{12}
x_6	$W_I = x_1 \oplus x_2 \oplus x_3 \oplus x_4 \oplus x_5 \oplus x_6$	x_1, x_2, x_3, x_4, x_5

TABLE II: Table that shows the working of Algorithm 1 on index code obtained from Construction 1 on $\mathcal{G}_1$.

T_i	$V_{NI}(i)$	$j \in V_{NI}(T_i)\backslash N^+_{T_i}(i)$	$a_{i,j}$	$j \in V(\mathcal{G}_2)\backslash V(T_i)$	$b_{i,j}$
T_1	$\{6, 7, 8, 9\}$	$\{7, 8, 9\}$	$1, 2, 1$	$\{10, 11\}$	$0, 0$
T_2	ϕ	ϕ	$-$	$\{6, 7, 8, 9, 10, 11\}$	$0, 0, 0, 0, 0, 0$
T_3	$\{8, 9, 11\}$	$\{8\}$	1	$\{6, 7, 10\}$	$0, 0, 0$
T_4	$\{10, 11\}$	$\{11\}$	1	$\{6, 7, 8, 9\}$	$0, 0, 0, 0$
T_5	$\{10, 11\}$	$\{11\}$	1	$\{6, 7, 8, 9\}$	$0, 0, 0, 0$

TABLE III: Table that verifies conditions $c1$ and $c2$ for $\mathcal{G}_2$.

Message x_i	Computation of Z_i	$N^+_{\mathcal{G}_2}(i)$
x_1	$W_I \oplus W_6 \oplus W_7 \oplus W_8 \oplus W_9 = x_1 \oplus x_2 \oplus x_6 \oplus x_8$	x_2, x_6
x_2	$W_I = x_1 \oplus x_2 \oplus x_3 \oplus x_4 \oplus x_5$	x_1, x_3, x_4, x_5
x_3	$W_I \oplus W_8 \oplus W_9 \oplus W_{11} = x_3 \oplus x_2 \oplus x_{11}$	x_2, x_9, x_{11}
x_4	$W_I \oplus W_{10} \oplus W_{11} = x_4 \oplus x_2 \oplus x_5 \oplus x_{10}$	x_2, x_5, x_{10}
x_5	$W_I \oplus W_{10} \oplus W_{11} = x_5 \oplus x_2 \oplus x_4 \oplus x_{10}$	x_2, x_4, x_{10}

TABLE IV: Table that shows the working of Algorithm 1 on index code obtained from Construction 1 on $\mathcal{G}_2$.

Conditions $c1$ and $c2$ are illustrated for $\mathcal{G}_3$ as follows. The rooted trees T_1, T_2 and T_3 have no non-inner vertices at depth ≥ 2 and hence $c1$ need not be verified. From Table V, it is clear that $b_{i,j} = 0$ for each $i \in \{1, 2, 3\}$ and $j \in V(\mathcal{G})\backslash V(T_i)$. It is thus verified that $c1$ and $c2$ are satisfied by $\mathcal{G}_3$.

T_i	$V_{NI}(i)$	$j \in V(\mathcal{G}_3)\backslash V(T_i)$	$b_{i,j}$
T_1	$\{5, 6\}$	$\{4\}$	0
T_2	$\{4, 6\}$	$\{5\}$	0
T_3	$\{4, 5\}$	$\{6\}$	0

TABLE V: Tables that verify $c2$ for $\mathcal{G}_3$.

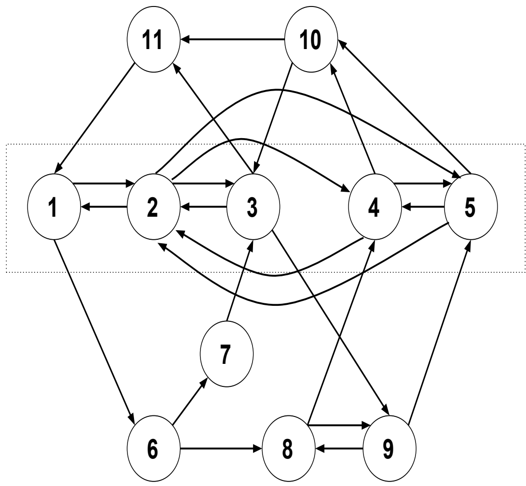
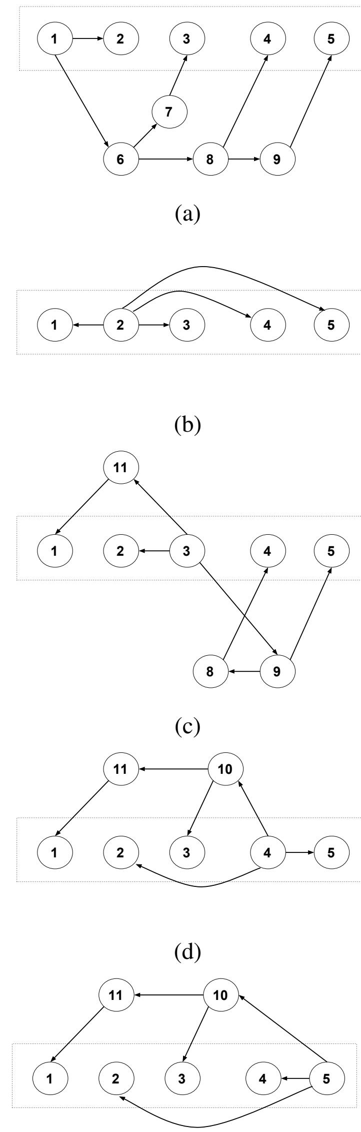


Fig. 3: IC structure $\mathcal{G}_2$ with outer cycles and with $V_I = \{1, 2, 3, 4, 5\}$.

4. CONCLUSION

IC structures with outer cycles for which the combination of index code construction and the decoding algorithm of Thapa, Ong and Johnson [4] is valid are characterized. In other words, this paper explores index code construction for IC structures with outer cycles, with any number of outer cycles whereas in [6] IC structures with outer cycles with only one outer cycle are considered. However, alternate index code constructions and decoding algorithms for IC structures with outer cycles that do not satisfy $c1$ and/or $c2$ remain to be found. It can be verified that the code obtained in Example 2 is not decodable using any linear decoding algorithm (shown explicitly in [7]). Characterizing the class of side-information graphs for which *Construction* 1 gives a valid index code (possibly with a different decoding algorithm) is an open problem. Also the optimality of the obtained index codes is a problem that remains to be solved.



(a)

(b)

(c)

(d)

(e)

Fig. 4: Figures showing rooted trees of inner vertices $1, 2, 3, 4, 5$ of $\mathcal{G}_2$, respectively.

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