A New Design Framework for Heterogeneous Uncoded Storage Elastic Computing

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Abstract—Elasticity is one important feature in modern cloud computing systems and can result in computation failure or significantly increase computing time. Such elasticity means that virtual machines over the cloud can be preempted under a short notice (e.g., hours or minutes) if a high-priority job appears; on the other hand, new virtual machines may become available over time to compensate the computing resources. Coded Storage Elastic Computing (CSEC) introduced by Yang et al. in 2018 is an effective and efficient approach to overcome the elasticity and it costs relatively less storage and computation load. However, one of the limitations of the CSEC is that it may only be applied to certain types of computations (e.g., linear) and may be challenging to be applied to more involved computations because the coded data storage and approximation are often needed. Hence, it may be preferred to use uncoded storage by directly copying data into the virtual machines. In addition, based on our own measurement, virtual machines on Amazon EC2 clusters often have heterogeneous computation speed even if they have exactly the same configurations (e.g., CPU, RAM, I/O cost). In this paper, we introduce a new optimization framework on Uncoded Storage Elastic Computing (USEC) systems with heterogeneous computing speed to minimize the overall computation time. Under this framework, we propose optimal solutions of USEC systems with or without straggler tolerance using different storage placements. Our proposed algorithms are evaluated using power iteration applications on Amazon EC2.

I. INTRODUCTION

Coded Storage Elastic Computing (CSEC) system introduced by Yang et al. in [1] is an effective approach to overcome the elasticity of modern cloud computing system, where elasticity means that Virtual Machines (VMs) on the cloud systems, e.g., instances on Amazon EC2, can be preempted under a short notice (e.g., hours or minutes) if a high-priority job appears; on the other hand, new VMs may become available over time to compensate the computing resources. Such elasticity can result in computation failure or significantly increase computing time. In [1], using a Maximum Distance Separable (MDS) coded storage placement, the authors proposed a cyclic computation assignment scheme such that no redundant computation is needed when the number of available VMs N_t is between L and N where N is the maximum number of VMs in the systems and L is the smallest number of VMs in the system. In [2], the authors introduced a new metric, called transition waste, which is defined as the difference between the total number of changes and the number of necessary changes of the computation assignment if some VMs become preempted during one computation or time step.

This problem is combinatorial and is challenging to be solved in general. The authors proposed new algorithms using shifted cyclic task allocation to reduce the transition waste and showed it is optimal under some parameter settings. In [3], the authors proposed two hierarchical schemes that can further speed up the USEC system by effectively allocating tasks among available nodes while the encoding and decoding complexity may be increased. Some important limitations of [1]-[3] include the assumption that all available VMs have the same computing speed or the proposed schemes do not consider the heterogeneous computing speed among machines, and all VMs have the homogeneous storage constraint. In practice, based on our own measurement [4], the computing speed among VMs can be significantly different even if they have exactly the same configurations, e.g., same CUP, RAM and I/O cost. In [5], the authors considered the elastic computing systems with heterogeneous computing speed and homogeneous storage constraint, and formulated a new CSEC framework, that is to minimize the overall computation time, using a combinatorial optimization approach. In addition, one exact optimal solution is provided and can be achieved using the filling algorithm, which is a low-complexity iterative algorithm that can complete within N_t iterations, where N_t is the number of available VMs at time step t. Later, in [6], the authors considered the CSEC system with both heterogeneous computing speed and heterogeneous storage constraint, and formulated a new combinatorial optimization framework based on the result in [5] and designed algorithms to achieve the optimal computation time. Under the assumption of heterogeneous computing speed, in [4], the authors made preliminary attempts to study the scenario where both elasticity and stragglers are present and proposed new algorithms using the idea of the filling algorithm. An achievable trade-off between computation time and straggler tolerance was established. In addition, the authors in [4] implemented the proposed algorithms for heterogeneous CSEC systems using real applications on Amazon EC2 and demonstrated that large gain in terms of the computation time can be achieved by the proposed algorithms.

Despite clear advantages of the CSEC systems such as less storage overhead, it can only be applied to certain types of computations (e.g., linear) and may be challenging to be

¹Stragglers are often referred to as the machines with abnormally slower speed

applied to more involved computations (e.g., deep learning) due to the coded data storage. In this case, approximation is often needed. Hence, it may be preferred to use uncoded storage by just copying the data into the virtual machines since computations can be operated directly over the original data in this case. We refer to such systems as Uncoded Storage Elastic Computing (USEC) systems. In this paper, we introduce a new optimization framework on USEC with heterogeneous computing speed to minimize the overall computation time. We propose solutions to USEC systems with or without straggler tolerance using different storage placements.

Our contributions are summarized as follows:

- 1) When there is no straggler tolerance requirement, given the storage placement and the heterogeneous computing speed of VMs, we formulate a new USEC framework as a convex optimization problem which can be solved using typical convex optimization solvers. Further, we investigate the performance in terms of computation time using different uncoded storage placements.
- 2) We incorporate straggler tolerance into the above problem formulation and formulate it as a combinatorial optimization problem. In addition, we design a lowcomplexity algorithm to achieve the optimal solution of the proposed optimization problem given the uncoded storage placement.
- 3) We perform experiments using the proposed USEC framework with heterogeneous computing speed, and using the power iteration application under a simple setup. We demonstrate that about 20% gain in terms of computation time can be achieved using the proposed algorithms by taking the advantage of heterogeneous computing speed.

Notation Convention: We use $|\cdot|$ to represent the cardinality of a set or the length of a vector and $[n] \stackrel{\Delta}{=} \{1, 2, \dots, n\}$. A bold symbol such as a indicates a vector and a[i] denotes the *i*-th element of a. Calligraphic symbols such as A presents a set with numbers as its elements. Bold calligraphic symbols such as A represents a set whose elements are sets (e.g., A).

II. NETWORK MODEL AND PROBLEM FORMULATION

We consider a set of N VMs jointly store an uncoded data matrix X with dimension $q \times r$, which is row-wise partitioned in $\mathbf{X} = [\mathbf{X}_1; \mathbf{X}_2; \cdots; \mathbf{X}_G]$. With a slight abuse of notation, $\mathbf{X}_g, g \in [G]$ denotes both the row sets and sub-matrices of **X**. In particular, the number of rows in each $X_g, g \in [G]$ is q/G and we index them as [q/G]. Each \mathbf{X}_g is placed into J machines. Let $\mathcal{N}_g = \{n: \mathbf{X}_g \in \mathcal{Z}_n\}$ denote the set of VMs that stores \mathbf{X}_g and \mathcal{Z}_n be the storage placement for machine n. The set of the storage placements for all VMs is denoted by $\mathcal{Z} = \{\mathcal{Z}_n, n \in [N]\}$. Similar to [1], the machines collectively perform matrix-vector computations over multiple computation steps. In a given time step only a subset of the Nmachines are available to perform matrix computations. More specifically, in computation step t, a set of available machines

 $\mathcal{N}_t \subseteq [N]$ with $|\mathcal{N}_t| = N_t$ aims to compute

$$y_t = Xw_t, (1)$$

where w_t is some vector of length r. The machines of $[N] \setminus \mathcal{N}_t$ are preempted.

The VMs in \mathcal{N}_t do not compute \boldsymbol{y}_t directly. Instead, each machine $n \in \mathcal{N}_t$ computes $\mathbf{X}_{\mathcal{S}_n} \boldsymbol{w}_t$, where $\mathcal{S}_n \subset \mathbf{X}_g, \mathbf{X}_g \in$ \mathcal{Z}_n denotes a row set in the sub-matrix $\mathbf{X}_g \in \mathcal{Z}_n$. Then the results from VMs will be sent to the master machine to obtain \boldsymbol{y}_t . Let $\mathcal{T}_{g,n}$ denote the row set of sub-matrix \mathbf{X}_g computed at machine $n \in \mathcal{N}_t$.

Definition 1: (Computation load) Let the computation load matrix be M and each entry of M, $[M]_{g,n} = \mu[g,n]$, is the computation load of sub-matrix \mathbf{X}_q at machine n defined as

$$\mu[g,n] \stackrel{\Delta}{=} \frac{|\mathcal{T}_{g,n}|}{q/G}.$$
 (2)

If $\mathbf{X}_g \notin \mathcal{Z}_n$, $\mu[g,n] = 0$. The computation load vector for Nmachines, $\mu = [\mu[1], \dots, \mu[n]]$, is defined as

$$\mu[n] = \sum_{g \in [G]} \mu[g, n], \quad \forall n \in \mathcal{N}_t, \tag{3}$$

which is the sum of the fractions of rows of the corresponding stored sub-matrices computed by machine n at time step t. \Diamond Note that $\mathcal{T}_{g,n}$, M and μ may change with each time step, but reference to t is omitted for ease of disposition. Moreover, the machines have varying computation speed defined by the strictly positive vector, s, which is known for each time step and defined as follows.

Definition 2: (Computation Speed) The computation speed vector s is a length-N vector with elements $s[n], n \in [N],$ where s[n] is the speed of machine n measured as the inverse of the time it takes machine n to compute all rows of one of its assigned sub-matrix.

The computation time is dictated by the VM that takes the most time to perform its assigned computations, and defined as follows.

Definition 3: (Computation Time) The computation time in a particular time step is defined as

$$c(\mathbf{M}) = c(\boldsymbol{\mu}) \stackrel{\Delta}{=} \max_{n \in \mathcal{N}_t} \frac{\mu[n]}{s[n]} = \max_{n \in \mathcal{N}_t} \frac{\sum_{g \in [G]} \mu[g, n]}{s[n]}.$$
 (4)

 \Diamond

A. USEC without straggler tolerance

We first formulate the optimization framework for the USEC systems without straggler tolerance. For a fixed storage placement \mathcal{Z} , we can formulate the following optimization problem.

$$\underset{\mathcal{T}_{q,n}}{\text{minimize}} \quad c\left(\boldsymbol{M}\right) \tag{5a}$$

It can be shown that the optimization problem (5) is equivalent to the following convex optimization problem.

minimize
$$c(\mathbf{M}) = \max_{n \in \mathcal{N}_t} \frac{\sum_{g \in [G]} \mu[g, n]}{s[n]}$$
 (6a) subject to: $\sum_{n \in \mathcal{N}_t : \mathbf{X}_g \in \mathcal{Z}_n} \mu[g, n] = 1, \forall g \in [G],$ (6b)

subject to:
$$\sum_{n \in \mathcal{N}_t : \mathbf{X}_q \in \mathcal{Z}_n} \mu[g, n] = 1, \forall g \in [G], \quad (6b)$$

$$\mu[g, n] = 0, \forall \mathbf{X}_g \notin \mathcal{Z}_n, n \in \mathcal{N}_t,$$
 (6c)

$$0 \le \mu[g, n] \le 1, \forall n \in \mathcal{N}_t. \tag{6d}$$

It can be seen that by solving (6), we can obtain the optimal computation assignment M^* , which can be used to find the corresponding $\mathcal{T}_{g,n}$ straightforwardly since each row in \mathbf{X}_g is computed only once (see Section III for examples).

B. USEC with straggler tolerance

When straggler tolerance is incorporated into the USEC framework, we use the redundant task assignment approach, meaning that each row in X can be computed 1+S times in order to tolerate at most S stragglers. This implies that the computation can be recovered when any S machines, denoted by S, of the available machines N_t become stragglers and S is not known a priori. Hence, this problem becomes a combinatorial optimization problem. In particular, a computation assignment within X_g is defined by F_g disjoint sets of rows in X_g , i.e., whith \mathbf{X}_g is defined by \mathbf{Y}_g disjoint sets of the \mathbf{X}_g and \mathbf{Y}_g , \mathbf{Y}_g , \mathbf{Y}_g such that $\bigcup_{f \in [F_g]} \mathcal{M}_{g,f} = \left[\frac{q}{G}\right]$. Then, F_g sets of machines, $\mathbf{P}_g = \{\mathcal{P}_{g,1}, \dots, \mathcal{P}_{g,F_g}\}$, which store and perform computation over \mathbf{X}_g , are defined such that The sets and perform computation over \mathcal{H}_g , the defined stein that $\mathcal{P}_{g,f} \subseteq \{n \in \mathcal{N}_t : \mathbf{X}_g \in \mathcal{Z}_n\}, |\mathcal{P}_{g,f}| = 1 + S, \forall f \in [F_g] \text{ and machines in } \mathcal{P}_{g,f} \text{ computes the row set } \mathcal{M}_{g,f} \text{ in } \mathbf{X}_g. \text{ Note that } \mathcal{T}_{g,n} = \bigcup_{f \in [F_g]: n \in \mathcal{P}_{g,f}} \mathcal{M}_{g,f}. \text{ The sets } \mathcal{M}_g, \mathcal{P}_g \text{ and } F_g \text{ may vary with each time step based on machines' availability.}$

In a given time step t, our goal is to design the task assignments, $\mathcal{M}_g, \mathcal{P}_g, g \in [G]$, such that the computation $oldsymbol{y}_t = oldsymbol{X} oldsymbol{w}_t$ can be recovered when some VMs are stragglers that do not provide their assigned computations to the master machine.

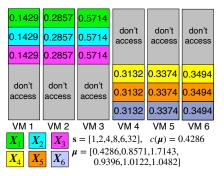
Then, we aim to design the computation assignment that minimizes the computation time of (4) resulting from the computation load matrix defined in (2). In time step t, given \mathbb{Z} , \mathcal{N}_t and s, the optimal computation time, c^* , is the minimum of computation times defined by all possible task assignments, such that S stragglers can be tolerated and the computation can be recovered. In particular c^* is the optimal value of the following combinatorial optimization problem.

$$\begin{array}{ll}
\text{minimize} & c\left(\boldsymbol{M}\right) \\
\boldsymbol{\mathcal{M}_g}, \boldsymbol{\mathcal{P}_g}
\end{array} \tag{7a}$$

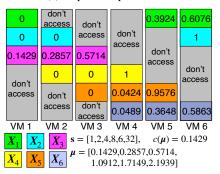
s.t.
$$\bigcup_{f \in [F_g]} \mathcal{M}_{g,f} = \left[\frac{q}{G}\right], \forall g \in [G], \tag{7b}$$

$$|\mathcal{P}_{g,f} \setminus \mathcal{S}| \ge 1, \forall g \in [G], \mathcal{P}_{g,f} \in \mathcal{P}_g, \forall \mathcal{S} \subset \mathcal{N}_t, |\mathcal{S}| = S.$$
(7c)

The optimization problem (7) is combinatorial and the optimal solution is challenging. In the following, we will propose a novel low-complexity algorithm to achieve the optimal



(a) Repetition placement.



(b) Cyclic placement.

Fig. 1: Illustration of the proposed USEC framework.

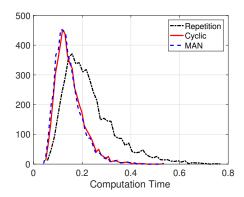


Fig. 2: Comparison of histograms of C(M) for repetition, cyclic and MAN storage placements over 5000 realizations of the computing speed vector.

solution for this combinatorial optimization problem. Interestingly, the *filling algorithm* introduced in the CSEC framework with heterogeneous computing speed [6] or the heterogeneous storage-constrained private information retrieval problem [7] can be applied here to obtain the proposed optimal solution for (7).

III. EXAMPLES

In this section, we will illustrate two examples of the proposed USEC framework with and without straggler tolerance, respectively, under the homogeneous storage constraint. We consider two commonly used uncoded storage schemes, which

TABLE I: Comparison between MAN, cyclic and repetition placements.

computation time	cyclic	repetition	MAN
mean	0.1492	0.2296	0.1442
variance	0.0033	0.0114	0.0032

are fractional repetition placement (referred to as repetition placement hereafter) and cyclic placement, which are widely used in the distributed storage and gradient coding literatures [8]-[10]. In particular, we consider a USEC system with N = 6 VMs and the speed vector is s = [1, 2, 4, 8, 16, 32]. The data matrix X is partitioned into G = 6 sub-matrices, each placed into J=3 machines. Fig. 1 shows this system with repetition placement (Fig. 1a) and with cyclic placement (Fig. 1b), respectively. Let $N = N_t$, all $\mu[g, n], g \in [6], n \in [N]$ are computed by solving the convex optimization problem (6). In Fig. 1, the colors represent the storage placement of each submatrix and the numbers inside represent the corresponding $\mu[g,n]$ for sub-matrix g and machine n. The computation time for the cyclic placement is $c(\mu) = 0.1429$, which is significantly better than that of the repetition placement $c(\mu) = 0.4286$. However, interestingly, the cyclic placement is not necessarily better than the repetition placement for any speed vector. For example, if machines 3 and 4 are much faster than other VMs, then the repetition placement can be better than the cyclic placement since machines 3 and 4 stores the entire data matrix under the repetition placement. In order to have a better understanding of this phenomenon, we ran an experiment by randomly generating s based on an exponential distribution. By solving the minimum computation time for each s using (5), we obtain the distribution of the computation time for these two storage placements shown in Fig. 2, where the cyclic placement (red) is much better than the repetition placement (yellow) in most realizations. In particular, there are only 68 cyclic placement realizations out of 5000 worse than repetition placement realizations. Although these results show the promising performance of the cyclic storage placement, it is not optimal in general. For example, using the Maddah-Ali Niesen coded caching (MAN) storage placement scheme [11] to repeat the same experiment, we can obtain slightly better results as shown in Fig. 2 (blue). In particular, out of 5000 realizations, there are only 9 MAN storage realizations worse than repetition placement realizations and 1621 MAN placement realizations worse than cyclic placement realizations. Moreover, the MAN placement indeed achieves the minimum computing time in terms of both mean and variance compared to cyclic and repetition placements (see Table I).

When the straggler tolerance is considered, we need to solve the optimization problem (7) to obtain the optimal M^{\star} and then find a feasible computation assignment that meets M^* . Consider an example of a USEC system with homogeneous computing speed. Here, we let $N=N_t=6$, $J=3,\,S=1,$ and the repetition placement is used. The optimal $\mu^*[g,n], g \in [6], n \in [6]$ are shown in Fig. 3 and the optimal $\mu^* = [2, 2, 2, 3, 3]$. The optimal computation time is

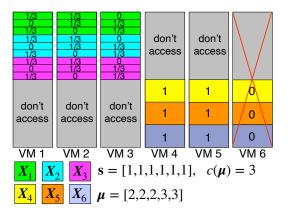


Fig. 3: Illustration of uncoded USEC with straggler tolerance for S = 1 using redundant task assignment.

$$c^{\star}(\boldsymbol{\mu}) = 3.$$

IV. PROPOSED USEC DESIGN

The proposed USEC design with straggler tolerance is given by Algorithm 1, which is obtained by solving the combinatorial optimization (7) in a similar fashion as in [5] (line 6 in Algorithm 1). The proposed design is adaptive by measuring (line 14 in Algorithm 1) and updating (line 4 in Algorithm 1) the speed vector at time step. Interestingly, this algorithm adapts the previous CSEC (not USEC) computation assignment [5] to assign computations to 1 + S machines.

Next we will explain the proposed design. Since the proposed design without straggler tolerance is a special case of the general design with straggler tolerance for the combinatorial optimization problem (7), then we will focus on designing algorithms to solve (7).

Similar to [5], we will solve the combinatorial optimization problem (7) exactly in two steps. In the first step, we solve the following relaxed convex optimization problem to obtain the optimal M^* without considering whether such a computation assignment exists or not.

minimize
$$c(\boldsymbol{M}) = \max_{n \in \mathcal{N}_t} \frac{\sum_{g \in [G]} \mu[g, n]}{s[n]}$$
 (8a) subject to: $\sum_{n \in \mathcal{N}_t : \mathbf{X}_g \in \mathcal{Z}_n} \mu[g, n] = 1 + S, \forall g \in [G],$ (8b)

subject to:
$$\sum_{n \in \mathcal{N}_t : \mathbf{X}_g \in \mathcal{Z}_n} \mu[g,n] = 1 + S, \forall g \in [G], \quad \text{(8b)}$$

$$\mu[g, n] = 0, \forall \mathbf{X}_g \notin \mathcal{Z}_n, n \in \mathcal{N}_t,$$
 (8c)

$$0 \le \mu[g, n] \le 1, \forall n \in \mathcal{N}_t. \tag{8d}$$

difference between (8) and (6) is to change (6b) from $\sum_{n \in \mathcal{N}_t : \mathbf{X}_g \in \mathcal{Z}_n} \mu[g, n] = 1, \forall g \in [G]$ to $\sum_{n \in \mathcal{N}_t : \mathbf{X}_g \in \mathcal{Z}_n} \mu[g, n] = 1 + S, \forall g \in [G]$ as in (8b). After obtaining the optimal M^* , we will apply the *filling* algorithm developed in [5] to assign computations for each $\mathbf{X}_q \in \mathcal{Z}_n, n \in \mathcal{N}_t$. Now we will describe the filling algorithm for USEC with homogeneous and heterogeneous computing speed, respectively.

Proposed USEC with homogeneous computation assignment: Consider $\mathcal{N}_g = \{n : \mathbf{X}_g \in \mathcal{Z}_n\}$ with $|\mathcal{N}_g| = N_g$. Then we define a computation assignment with $F_g=N_g$ row sets of \mathbf{X}_g . There are N_g disjoint equally-sized row sets that collectively span all rows: $\mathcal{M}_{g,f}=\{1+(f-1)\frac{q}{N_gG},\ldots,f\frac{q}{N_gG}\}$ for $f\in[N_g]$. Then, define a cyclic assignment such that machine set $\mathcal{P}_{g,f}=\{f\% N_g,\ldots,(f+S)\% N_g\}$ for $f\in[N_g]$, where we define $a\% N_g\triangleq a-\left\lfloor\frac{a-1}{N_g}\right\rfloor N_g$ to facilitate the cyclic design. Proposed USEC with heterogeneous computation assign-

Proposed USEC with heterogeneous computation assignment: Given the computation load matrix M^* , we can obtain the computation assignment by applying the assignment algorithm in [5] to assign computations to 1 + S VMs for each \mathbf{X}_g (line 6 in Algorithm 1). The computation assignment algorithm for \mathbf{X}_g is given by Algorithm 2.

Remark 1: For both designs, we observe that the computation time $c(\boldsymbol{M})$ increases with the straggler tolerance, S. This demonstrates a trade-off between the computation time and straggler tolerance of the system.

Algorithm 1 Adaptive Straggler Tolerant Uncoded Storage Elastic Computing

```
Input: \hat{s}, \gamma, S, T, w_1
 1: \nu \leftarrow \hat{s}: same for all worker VMs
 2: for t \in [T] do
         At Master Machine:
 3:
             \hat{s} \leftarrow \gamma \boldsymbol{\nu} + (1 - \gamma) \hat{s} (update estimate of speed
 4:
     vector).
             \mathcal{N}_t \leftarrow \text{list of available machines}
 5:
             \{F_g, \mathcal{M}_g, \mathcal{P}_g : \forall g \in [G]\} \leftarrow \text{Results of computa-}
     tion assignment algorithm for X_q with straggler tolerance
     of S for available machines \mathcal{N}_t with speeds of \hat{s}
 7:
             Send w_t and \{F_g, \mathcal{M}_g, \mathcal{P}_g : \forall g \in [G]\} to worker
     VMs
          At Worker VMs:
 8:
 9:
             n \leftarrow \text{index of worker VM}
             \mu[n] \leftarrow \text{total computation load of worker VM } n
10:
             \tau_1 \leftarrow \text{current time}
11:
             Perform assigned
                                            computations
12:
                                                                   based
                                                                               on
     \{F_g, \mathcal{M}_g, \mathcal{P}_g : \forall g \in [G]\}
             \tau_2 \leftarrow \text{current time}
13.
             \nu[n] \leftarrow \mu[n]/(\tau_2 - \tau_1) (calculate speed based on
14:
     current time step)
             Send computations and \nu[n] to Master Machine
15:
          At Master Machine: after receiving results from at
16:
     most N_t - S workers.
             w_{t+1} \leftarrow \text{Combine worker results}
17:
18: end for
Output: w_T
```

V. EVALUATIONS ON AMAZON EC2

We evaluate the proposed algorithm using power iteration applications on Amazon EC2 instances. The goal is to compare the performance difference in terms of computation time between the homogeneous and heterogeneous task assignments.

Algorithm 2 Computation Assignment for \mathbf{X}_g for Heterogeneous Computing Speed

```
Input: \mu_g^{\star}, q, \mathcal{Z} and \mathcal{N}_g = \{1, \cdots, N_g\}.
 1: m{m} \leftarrow m{\mu}_g^\star
 2: f \leftarrow 0
 3: while m contains a non-zero element do
            \begin{array}{l} f \leftarrow f + 1 \\ L' \leftarrow \sum_{i=1}^{N_g} m[i] \end{array}
             N' \leftarrow number of non-zero elements in m
 6:
             \ell \leftarrow indices that sort the non-zero elements of m from
      smallest to largest<sup>5</sup>
             \mathcal{P}_{g,f} \leftarrow \{\ell[1], \ell[N'-L+2], \dots, \ell[N']\}
 8:
            if N' \geq L+1 then
\alpha_{g,f} \leftarrow \min\left(\frac{L'}{L} - m[\ell[N'-L+1]], m[\ell[1]]\right)^6
 9:
10:
11:
                  \alpha_{g,f} \leftarrow m[\ell[1]]
12:
13:
            for n \in \mathcal{P}_{g,f} do m[n] \leftarrow m[n] - \alpha_{g,f}
14:
15:
            end for
17: end while
18: F \leftarrow f
19: Partition rows \left[\frac{q}{G}\right] of \mathbf{X}_g into F disjoint row sets
      \mathcal{M}_{g,1},\ldots,\mathcal{M}_{g,F} of size \frac{\alpha_1 g}{G},\ldots,\frac{\alpha_F q}{G} rows, respectively
Output: F, \{\mathcal{M}_{g,1}, \ldots, \mathcal{M}_{g,F}\} and \{\mathcal{P}_{g,1}, \ldots, \mathcal{P}_{g,F}\}
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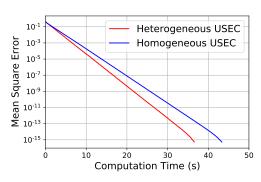


Fig. 4: **Power Iteration**: Results using USEC designs on Amazon EC2 without stragglers (top) and with 2 stragglers each iteration (bottom). The y-axis represents the normalized mean square error between the true dominant eigenvector and the estimated eigenvector.

Power Iteration: The power iteration algorithm computes the largest eigenvalue and the corresponding eigenvector of a large matrix \mathbf{X} . In particular, it starts with a vector \mathbf{b}_0 , which may be an approximation to the dominant eigenvector or a random vector. The method is described by the recursive relation, $\mathbf{b}_{k+1} = \frac{\mathbf{X}\mathbf{b}_k}{\|\mathbf{X}\mathbf{b}_k\|}$. The sequence \mathbf{b}_k converges to an eigenvector associated with the dominant eigenvalue. It can be seen that at each iteration, we can directly apply the proposed Algorithm 1. In particular, a dense 6,000-by-6,000 symmetric matrix is row-wise split into G=6 sub-matrices which will be stored at each machine. We apply the repetition placement.

 $^{{}^5\}ell$ is an N'-length vector and $0 < m[\ell[1]] \le m[\ell[2]] \le \cdots \le m[\ell[N']]$. 6 This is the condition obtained by using Lemma 1 in [6].

A vector of length 6,000 is updated by performing a matrixvector multiplication in a distributed manner on the available worker VMs. The master machine combines the results and normalizes the vector. This process is repeated such that the vector converges to the eigenvector associated with the largest eigenvalue.

The network has one t2.x2large master machine with 8 vCPUs and 32 GiB of memory. The worker VMs consist of 3t2.large instances, each with 2 vCPUs and 8 GiB of memory, and 3t2.xlarge instances, each with 4 vCPUs and 16 GiB of memory. Similar to [4], we observed that all VMs have very different computing speed. For simplicity, we let $N=N_t$ and S=0 in order to show the advantage of the heterogeneous task assignment over the homogeneous task assignment. The result is shown in Fig. 4, where the gain of Algorithm 1 is about 20% in terms of the computation time.

VI. CONCLUSIONS

In this paper, we introduce a new optimization framework on USEC with heterogeneous computing speed to minimize the overall computation time. In particular, we consider the USEC systems under different uncoded storage placements and with or without straggler tolerance. For both scenarios, we propose optimal algorithms given the storage placements. These algorithms are evaluated using real applications on Amazon EC2 to demonstrate their gains in terms of computation time compared to the designs using the homogeneous computing speed assumption.

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