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A FOURIER PSEUDOSPECTRAL METHOD FOR SOLVING COUPLED VISCOUS BURGERS EQUATIONS

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Abstract — The Fourier pseudo-spectral method has been studied for a one-dimensional coupled system of viscous Burgers equations. Two test problems with known exact solutions have been selected for this study. In this paper, the rate of convergence in time and error analysis of the solution of the first problem has been studied, while the numerical results of the second problem obtained by the present method are compared to those obtained by using the Chebyshev spectral collocation method. The numerical results show that the proposed method outperforms the conventional one in terms of accuracy and convergence rate.

2000 Mathematics Subject Classification: 35Q53.

Keywords: Coupled system of viscous Burgers equations, Fourier pseudospectral method.

1. Introduction

The coupled viscous Burgers equations were derived by S.E. Esipov [9] to study the model of polydispersive sedimentation. The coupled Burgers equations are described by the following nonlinear partial differential equations:

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} + \eta u \frac{\partial u}{\partial x} + \alpha \frac{\partial}{\partial x}(uv) = 0, \quad x \in \Omega, \quad t \in [0,T], \tag{1.1}$$

$$\frac{\partial v}{\partial t} - \frac{\partial^2 v}{\partial x^2} + \xi v \frac{\partial v}{\partial x} + \beta \frac{\partial}{\partial x}(uv) = 0, \quad x \in \Omega, \quad t \in [0,T], \tag{1.2}$$

with the initial conditions

$$u(x,0) = f(x), \quad v(x,0) = g(x), \quad x \in \Omega, \tag{1.3}$$

and the boundary conditions

$$u(-L,t) = u(L,t), \quad v(-L,t) = v(L,t), \quad t \in [0,T], \tag{1.4}$$

where $\Omega = [-L, L]$, η and ξ are real constants, α and β are arbitrary constants depending on the system parameters such as the Péclet number, the Stokes velocity of particles due to gravity, and the Brownian diffusivity [15]. This coupled system is a simple model of sedimentation or evolution of scaled volume concentrations of two kinds of particles in fluid suspensions or colloids under the effect of gravity. J.M. Burgers [4] and J.D. Cole [6] have

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found that Burgers equations describe various kinds of phenomena such as a mathematical model of turbulence and the approximate theory of flow through a shock wave traveling in viscous fluid.

This equation is of interest from the numerical point of views, because in general, analytical solutions are not available. S.E. Esipov [9] presented numerical simulations for (1.1)–(1.3) and compared the results with experimental data. D. Kaya [8] used the decomposition method and obtained the solution of the homogenous and inhomogeneous coupled viscous Burgers equations in the form of convergent power series. A.H. Khater et al. [13] used the Chebyshev spectral collocation method for solving the coupled Burgers equations and obtained approximate solutions. M. Dehghan, A. Hamidi, and M. Shakourifar [7] obtained numerical results of coupled viscous Burgers equations by using the Adomian-Pade technique.

Several authors studied mainly the exact solutions of nonlinear equations by using various methods. For example, A. A. Soliman [17] obtained an exact solution of coupled viscous Burgers equations by the modified extended tanh-function method. The variational iteration method was used in [1] to solve one-dimensional (1D) Burgers and coupled Burgers equations; the solution was obtained under a series of initial conditions and transformed into a closed form.

Spectral methods are becoming increasingly popular in applied mathematics and scientific computing for solving of partial differential equations. The main advantage of these methods in their accuracy for a given number of unknowns. For problems whose solutions are sufficiently smooth, they exhibit an exponential rate of convergence/spectral accuracy. There are three most commonly used spectral versions, namely the Galerkin, tau and collocation methods. Among them, the spectral collocation/ pseudospectral method is particularly attractive owing to its economy. Comprehensive discussions on spectral methods can be found in review articles and monographs (see, for example, [2, 3, 11, 14, 16]).

In solving time-dependent partial differential equations numerically by spectral methods, spectral differentiation is used in space, while the finite difference method is used in the time direction. In principle, we have to sacrifice the spectral accuracy in time, but in practice a small time step with a finite difference formula of order one or higher often results in a satisfactory global accuracy. Small time steps are much more affordable than small space steps, i.e., they affect the computation time but not the storage. Details are given in [5,10,18].

This paper presents a Fourier pseudospectral method for solving the coupled viscous Burgers equations with a set of initial and periodic boundary conditions. Two test problems with known exact solutions have been selected for this study. It is worth mentioning that the numerical solutions of the first problem are not available in the literature, while the numerical solutions of the second problem are available. The rate of convergence in time and the error analysis of the first problem have been studied by the Fourier pseudospectral method. The results obtained by the Fourier pseudospectral method for the second problem are compared to those obtained by using the Chebyshev spectral collocation method. The numerical results for the second problem demonstrate that the Fourier pseudospectral method gives better results than those obtained by the Chebyshev spectral collocation method.

This paper is organized as follows: In Section 2, we describe the proposed pseudospectral method. The numerical results are presented and discussed in Section 3. Finally the conclusion is given in Section 4.

2. Fourier pseudospectral discretization

In this Section, we will apply the Fourier pseudospectral method to (1.1)–(1.4). For simplicity, we will consider the spatial domain $[-L, L]$. The Fourier pseudospectral method involves two basic steps. First, we construct a discrete representation of the solution by using a trigonometric polynomial to interpolate the solution at collocation points. Second, the equations for the discrete values of the solution are obtained from the original equations. This second step involves finding an approximation for the differential operator in terms of the discrete values of the solution at collocation points. (For details, see [3, 5, 16]).

We approximate the exact solutions $u(x,t)$ and $v(x,t)$ by $u_N(x,t)$ and $v_N(x,t)$, respectively, which interpolate $u(x,t)$ and $v(x,t)$ at the following set of collocation points:

$$-L = x_0 < x_1 < x_2 < \ldots < x_N = L \quad \text{with} \quad x_j = L\left(\frac{2j}{N} - 1\right), \quad j = 0, 1, \ldots, N,$$

where N is an even number.

The approximation $u_N(x,t)$ and $v_N(x,t)$ have the form

$$u_N(x,t) = \sum_{j=0}^{N} u_j g_j(x), \quad v_N(x,t) = \sum_{j=0}^{N} v_j g_j(x), \tag{2.1}$$

where $u_j = u(x_j, t)$, $v_j = v(x_j, t)$ and $g_j(x_k) = \delta_j^k (= 1$, if $j = k$, otherwise $0)$.

Therefore, we have $u_N(x_j, t) = u_j$, $v_N(x_j, t) = v_j$, $j = 0, 1, \ldots, N$. In fact, $g_j(x)$ can be chosen as

$$g_j(x) = \frac{1}{N+1} \sum_{\ell=-N/2}^{N/2} \frac{1}{c_\ell} e^{i\ell\mu(x - x_j)}, \tag{2.2}$$

where $c_\ell = 1(|\ell| \neq N/2)$, $c_{-N/2} = c_{N/2} = 2$, $\mu = \pi/L$. By direct computations, we can easily verify that $g_j(x_k) = \delta_j^k$. Substituting (2.2) into (2.1), we obtain

$$u_N(x,t) = \sum_{\ell=-N/2}^{N/2} \frac{1}{c_\ell} e^{i\ell\mu x} \frac{1}{N+1} \sum_{j=0}^{N} u_j e^{-i\ell\mu x_j}, \tag{2.3}$$

$$v_N(x,t) = \sum_{\ell=-N/2}^{N/2} \frac{1}{c_\ell} e^{i\ell\mu x} \frac{1}{N+1} \sum_{j=0}^{N} v_j e^{-i\ell\mu x_j}. \tag{2.4}$$

With the definition

$$\widehat{u}_\ell = \frac{1}{c_\ell(N+1)} \sum_{j=0}^{N} u_j e^{-i\ell\mu x_j}, \quad \widehat{v}_\ell = \frac{1}{c_\ell(N+1)} \sum_{j=0}^{N} v_j e^{-i\ell\mu x_j}, \tag{2.5}$$

(2.3), (2.4) becomes

$$u_N(x,t) = \sum_{\ell=-N/2}^{N/2} \widehat{u}_\ell e^{i\ell\mu x}, \quad v_N(x,t) = \sum_{\ell=-N/2}^{N/2} \widehat{v}_\ell e^{i\ell\mu x}. \tag{2.6}$$

Therefore

$$u_j = u_N(x_j,t) = \sum_{\ell=-N/2}^{N/2} \widehat{u}_\ell e^{i\ell\mu x_j}, \quad v_j = v_N(x_j,t) = \sum_{\ell=-N/2}^{N/2} \widehat{v}_\ell e^{i\ell\mu x_j}. \tag{2.7}$$

In order to obtain the equations for u_j and v_j, we substitute (2.6) into (1.1), (1.2) and require that (1.1), (1.2) are satisfied exactly at the collocation points, i.e.,

$$\left[\frac{\partial}{\partial t}u_N(x,t) - \frac{\partial^2}{\partial x^2}u_N(x,t) + \eta u_N(x,t)\frac{\partial}{\partial x}u_N(x,t) + \alpha\frac{\partial}{\partial x}\left(u_N(x,t)v_N(x,t)\right)\right]_{x=x_j} = 0,$$

$$j = 0,1,\ldots,N, \tag{2.8}$$

$$\left[\frac{\partial}{\partial t}v_N(x,t) - \frac{\partial^2}{\partial x^2}v_N(x,t) + \xi v_N(x,t)\frac{\partial}{\partial x}v_N(x,t) + \beta\frac{\partial}{\partial x}\left(u_N(x,t)v_N(x,t)\right)\right]_{x=x_j} = 0,$$

$$j = 0,1,\ldots,N. \tag{2.9}$$

The crucial step is to obtain the values for the kth order derivatives $\partial^k u_N(x,t)/\partial x^k$ and $\partial^k v_N(x,t)/\partial x^k$ at the collocation points x_j in terms of the value u_j and v_j, respectively. We can do this by differentiating (2.1) and evaluating the resulting expressions at the points x_j.

$$\frac{\partial^k}{\partial x^k}u_N(x_j,t) = \sum_{n=0}^{N} u_n \frac{d^k}{dx^k}g_n(x_j,t) = (D_k\mathbf{u})_j, \quad j = 0,1,\ldots,N, \tag{2.10}$$

$$\frac{\partial^k}{\partial x^k}v_N(x_j,t) = \sum_{n=0}^{N} v_n \frac{d^k}{dx^k}g_n(x_j,t) = (D_k\mathbf{v})_j, \quad j = 0,1,\ldots,N, \tag{2.11}$$

where (D_k) is an $(N+1)\times(N+1)$ matrix with elements

$$(D_k)_{j,n} = \frac{d^k}{dx^k}g_n(x_j)$$

and

$$\mathbf{u} = (u_0,u_1,\ldots,u_N)^\top, \qquad \mathbf{v} = (v_0,v_1,\ldots,v_N)^\top.$$

We call (D_k) the kth-order spectral differentiation matrix.

Define S_N as the space of trigonometric polynomials of degree up to N

$$S_N = \operatorname{span}\left\{\frac{1}{\sqrt{L}}\exp\left(\frac{i\pi x_j}{L}\right) : j = 0,1,\ldots,N\right\},$$

where $i = \sqrt{-1}$.

In order to approximate the nonlinear terms $u_N\partial u_N/\partial x$, $v_N\partial v_N/\partial x$ and $\partial(u_Nv_N)/\partial x$ in the system of equations (2.8), (2.9) reasonably, we follow [12]. We define the operator $B : S_N \times S_N \longrightarrow S_N$ and the circle convolution as

$$u_N * v_N = \sum_{j=0}^{N}\sum_{k=0}^{N} u_k v_{j-k} g_j(x).$$

Using the spectral differentiation matrix, we treat the nonlinear terms as follows:

$$\left[u_N \frac{\partial u_N}{\partial x}\right]_{x=x_j} = B(u_j, u_j) = \frac{1}{3} P_c \left((D_1 \mathbf{u})_j * u_j \right) + \frac{1}{3} \left(D_1 \left(P_c (\mathbf{u} * \mathbf{u}) \right) \right)_j , \tag{2.12}$$

$$\left[v_N \frac{\partial v_N}{\partial x}\right]_{x=x_j} = B(v_j, v_j) = \frac{1}{3} P_c \left((D_1 \mathbf{v})_j * v_j \right) + \frac{1}{3} \left(D_1 \left(P_c (\mathbf{v} * \mathbf{v}) \right) \right)_j , \tag{2.13}$$

$$\left[\frac{\partial}{\partial x}(u_N v_N)\right]_{x=x_j} = \left[u_N \frac{\partial v_N}{\partial x} + v_N \frac{\partial u_N}{\partial x}\right]_{x=x_j} = B(u_j, v_j) =$$

$$\frac{1}{3} P_c \left((D_1 \mathbf{v})_j * u_j \right) + \frac{1}{3} \left(D_1 \left(P_c (\mathbf{u} * \mathbf{v}) \right) \right)_j + \frac{1}{3} P_c \left((D_1 \mathbf{u})_j * v_j \right) + \frac{1}{3} \left(D_1 \left(P_c (\mathbf{v} * \mathbf{u}) \right) \right)_j , \tag{2.14}$$

where P_c is the interpolation operator.

Substituting the values of (2.12), (2.13) and (2.14) into (2.8), (2.9) and using the spectral differentiation matrices, we obtain

$$\frac{d}{dt} u_j - (D_2 \mathbf{u})_j + \eta B(u_j, u_j) + \alpha B(u_j, v_j) = 0, \tag{2.15}$$

$$\frac{d}{dt} v_j - (D_2 \mathbf{v})_j + \xi B(v_j, v_j) + \beta B(u_j, v_j) = 0, \tag{2.16}$$

where $j = 0, 1, \ldots, N$. The system of equations (2.15), (2.16) is the standard Fourier pseudospectral discretization for coupled Burgers equations. The above system of equations forms a system of first-order ordinary differential equations in time. Therefore, to advance the solution in time, we use an accurate ordinary differential equations solver such as the classical fourth-order Runge — Kutta method. The classical Runge — Kutta method of order four is given by

$$k_1 = f(u^n, t_n), \quad k_2 = f\left(u^n + \frac{\Delta t}{2} k_1, t_n + \frac{1}{2}\Delta t\right), \quad k_3 = f\left(u^n + \frac{\Delta t}{2} k_2, t_n + \frac{1}{2}\Delta t\right),$$

$$k_4 = f(u^n + \Delta t k_3, t_n), \quad u^{n+1} = u^n + \frac{\Delta t}{6}(k_1 + 2k_2 + 2k_3 + k_4),$$

where Δt is the mesh spacing of the variable t. The time interval $[0, T]$ is divided into M subintervals. The temporal grid points are given by $t_n = nT/M$, $n = 0, 1, \ldots, M$.

Remark 2.1. We can also evaluate the derivatives by using the Fast Fourier Transform (FFT) algorithm instead of the spectral differential matrix in $O(N \log N)$ operations rather than in $O(N^2)$ operations. However, it is more convenient to investigate the Fourier pseudospectral discretizations of coupled Burgers equations by using the spectral differential matrices.

3. Numerical results

In this section, we present some numerical results of our scheme (2.15), (2.16) for the coupled Burgers equations. All computations were carried out in Matlab 6.5 on a personal computer. For describing the error, we define maximum error and the relative discrete L_2-normed error for u as follows:

$$\|E(u)\|_\infty = \max_{0 \leqslant j \leqslant N} |u(x_j, t) - u_N(x_j, t)|,$$

and

$$\|E(u)\|_2 = \Big(\sum_{j=0}^{N} |u(x_j,t) - u_N(x_j,t)|^2 \Big)^{1/2} \Big/ \Big(\sum_{j=0}^{N} |u(x_j,t)|^2 \Big)^{1/2},$$

where $u_N(x_j,t)$ is the solution of the numerical scheme (2.15), (2.16), whereas $u(x_j,t)$ is the exact solution of (1.1), (1.2). Similarly we can define the maximum error and the relative discrete L_2-normed error for the variable v.

Problem (a): To examine the performance of the Fourier pseudospectral method for solving viscous Burgers equations, we set the parameters $\eta = -2$, $\xi = -2$, $\alpha = 1$, $\beta = 1$ and $L = \pi$. The system of equations (1.1), (1.2) takes the following form

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} - 2u\frac{\partial u}{\partial x} + \frac{\partial}{\partial x}(uv) = 0, \quad -\pi \leqslant x \leqslant \pi, \quad t > 0, \tag{3.1}$$

$$\frac{\partial v}{\partial t} - \frac{\partial^2 v}{\partial x^2} - 2v\frac{\partial v}{\partial x} + \frac{\partial}{\partial x}(uv) = 0, \quad -\pi \leqslant x \leqslant \pi, \quad t > 0, \tag{3.2}$$

subject to the initial conditions

$$u(x,0) = \sin(x), \quad v(x,0) = \sin(x), \quad -\pi \leqslant x \leqslant \pi, \tag{3.3}$$

and the exact solutions are taken from [8]

$$u(x,t) = e^{-t}\sin(x), \quad v(x,t) = e^{-t}\sin(x), \quad -\pi \leqslant x \leqslant \pi, \quad t > 0. \tag{3.4}$$

To see whether the proposed numerical scheme exhibits the expected convergence rates in time, we perform a numerical experiment for various values of Δt and a fixed value of N. In this experiment, we take $N = 64$ for the present method to ensure that the spatial error is negligible. The rate of convergence for the scheme is calculated by using the formula

$$\text{rate of convergence} \approx \frac{\ln(E(N_2)/E(N_1))}{\ln(N_1/N_2)}, \tag{3.5}$$

where $E(N_j)$ is the L_2-error in using N_j subintervals. The convergence rates are shown in Table 3.1. Because we have eliminated the spatial discretization errors, the errors in the table are due to the time discretizations solely. The computed convergence rates agree well with the expected rates when the fourth-order (classical Runge — Kutta) scheme is applied to the viscous coupled Burgers equations in the time direction. It is seen from Table 3.1 that the rate of convergence increases with a smaller time step.

Table 3.1. **Convergence rates in time calculated from relative L_2 errors of u and v for the proposed method, $t = 1$**

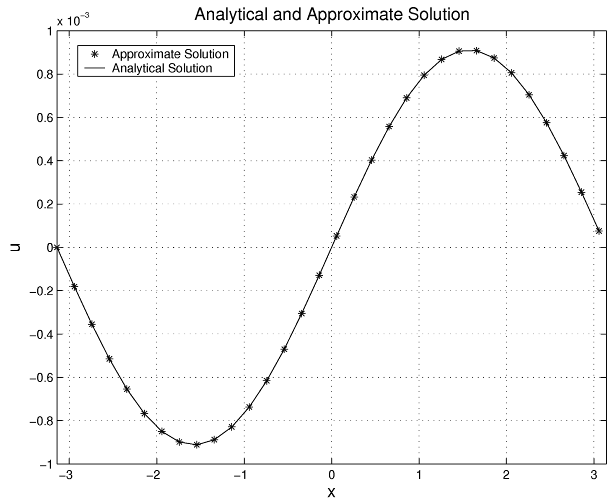
Δt	$\|E(u)\|_2$	Order	$\|E(v)\|_2$	Order
0.0200	4.5353E-01	—	4.5351E-01	—
0.0100	2.8904E-02	3.9719	2.8902E-02	3.9719
0.0050	1.8153E-03	3.9930	1.8151E-03	3.9930
0.0020	4.6555E-05	3.9980	4.6553E-05	3.9980
0.0010	2.9114E-06	3.9992	2.9112E-06	3.9992
0.0005	2.2042E-07	3.7234	2.2043E-07	3.7232

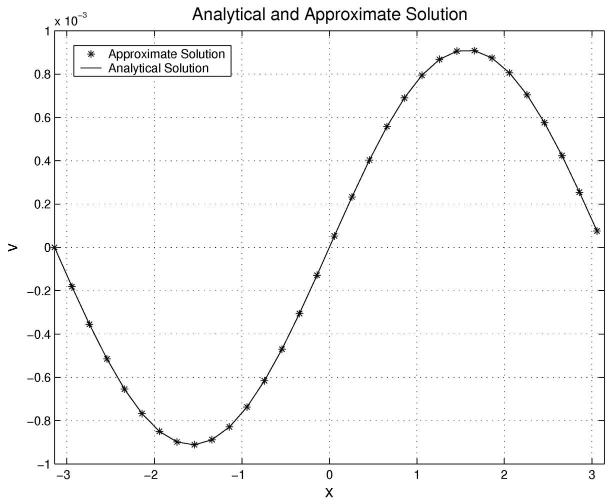
To see whether the pseudospectral scheme exhibits the expected convergence rate in space, we performed some further numerical experiments for various values of N. In these experiments, we take $\Delta t = 0.0001$ to minimize temporal errors. The results are shown in Table 3.2 for an increasing number of subintervals. We present the relative L_2-errors and Maximum errors for variables u and v. The pseudospectral scheme gives a good accuracy in space as would be expected by the present method. The numerical results for the variable v are similar to those for u, since problem (3.1), (3.2) with the initial conditions (3.3) is totally symmetrical with respect to the two components u and v of the solution. The numerical scheme (2.15), (2.16) preserves the same symmetry.

Table 3.2. **Maximum errors and relative L_2 errors for u and v at $t = 1$**

N	$\|E(u)\|_2$	$\|E(u)\|_\infty$	$\|E(v)\|_2$	$\|E(v)\|_\infty$
4	1.520E-03	1.542E-03	1.520E-03	1.542E-03
8	2.958E-05	1.165E-05	2.890E-05	1.165E-05
16	2.943E-05	1.164E-05	2.943E-05	1.164E-05
32	2.929E-05	1.163E-05	2.929E-05	1.163E-05
64	2.912E-05	1.160E-05	2.912E-05	1.161E-05
128	2.887E-05	1.159E-05	2.886E-05	1.159E-05

The exact and approximate solutions for u and v are shown in Figure. It is seen that u and v have similar behavior.





Graph of the approximate solution and the exact solution for the variable u and v at $N = 32$, $t = 1$

Problem (b): For this problem we consider the coupled Burgers equations with $\eta = 2$, $\xi = 2$ and $L = 10$. The system of equations (1.1), (1.2) takes the following form:

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} + 2u\frac{\partial u}{\partial x} + \alpha\frac{\partial}{\partial x}(uv) = 0, \quad -10 \leqslant x \leqslant 10, \quad t > 0, \tag{3.6}$$

$$\frac{\partial v}{\partial t} - \frac{\partial^2 v}{\partial x^2} + 2v\frac{\partial v}{\partial x} + \beta\frac{\partial}{\partial x}(uv) = 0, \quad -10 \leqslant x \leqslant 10, \quad t > 0, \tag{3.7}$$

subject to the initial conditions

$$u(x,0) = a_0 - 2A\left(\frac{2\alpha - 1}{4\alpha\beta - 1}\right)\tanh(Ax), \quad -10 \leqslant x \leqslant 10, \tag{3.8}$$

$$v(x,0) = a_0\left(\frac{2\beta-1}{2\alpha-1}\right) - 2A\left(\frac{2\alpha-1}{4\alpha\beta-1}\right)\tanh(Ax), \quad -10 \leqslant x \leqslant 10, \tag{3.9}$$

and the exact solutions are taken from A. Soliman [17]

$$u(x,t) = a_0 - 2A\left(\frac{2\alpha-1}{4\alpha\beta-1}\right)\tanh(A(x-2At)), \quad -10 \leqslant x \leqslant 10, \quad t>0, \tag{3.10}$$

$$v(x,t) = a_0\left(\frac{2\beta-1}{2\alpha-1}\right) - 2A\left(\frac{2\alpha-1}{4\alpha\beta-1}\right)\tanh(A(x-2At)), \quad -10 \leqslant x \leqslant 10, \quad t>0, \tag{3.11}$$

where $A = a_0(4\alpha\beta-1)/(4\alpha-2)$ and a_0, α, β are arbitrary constants. In Tables 3.3 and 3.4, we present a comparison between the numerical solutions of this problem obtained by the proposed pseudospectral method and those obtained by the Chebyshev spectral collocation method taken from [13]. Tables 3.3 and 3.4 give the maximum error and the relative L_2 error for various values of t, α, and β. As is seen from Tables 3.3 and 3.4, the proposed method is much more accurate than the the method presented in [13].

Table 3.3. **Comparison of numerical results of the problem (b) with the results obtained from Khater [13] for the variable u with $a_0 = 0.05$, $N = 16$**

t	α	β	Present Method		Khater (2008)	
			$\Vert E(u)\Vert_2$	$\Vert E(u)\Vert_\infty$	$\Vert E(u)\Vert_2$	$\Vert E(u)\Vert_\infty$
0.5	0.1	0.3	3.2453E-5	9.6185E-4	4.38E-5	1.44E-3
	0.3	0.03	2.7326E-5	4.3102E-4	4.58E-5	6.68E-4
1.0	0.1	0.3	2.4054E-5	1.1529E-3	8.66E-5	1.27E-3
	0.3	0.03	2.8316E-5	1.2684E-3	9.16E-5	1.30E-3

Table 3.4. **Comparison between the numerical results of problem (b) and the results obtained in [13] for the variable v with $a_0 = 0.05$, $N = 16$**

t	α	β	Proposed Method		Khater (2008)	
			$\Vert E(v)\Vert_2$	$\Vert E(v)\Vert_\infty$	$\Vert E(v)\Vert_2$	$\Vert E(v)\Vert_\infty$
0.5	0.1	0.3	2.7459E-5	3.3317E-4	4.99E-5	5.42E-4
	0.3	0.03	2.4541E-4	1.1485E-3	1.81E-4	1.20E-3
1.0	0.1	0.3	3.7450E-5	1.1620E-3	9.92E-5	1.29E-3
	0.3	0.03	4.5247E-4	1.6389E-3	3.62E-4	2.35E-3

4. Conclusions

In this paper, we have discussed the coupled Burgers equations. We proposed a numerical scheme for solving the system of nonlinear Burgers equations by the Fourier pseudospectral method. The numerical results given in the previous section demonstrate a good accuracy of this scheme. For the test problems, the Fourier pseudospectral method provides more accurate results than the Chebyshev spectral collocation method presented by A.H. Khater [13]. The method is also capable of solving Burgers-type equations with periodic boundary conditions.

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References

1. M. A. Abdou and A. A. Soliman, *Variational iteration method for solving Burgers and coupled Burgers equations*, J. Comput. Appl. Math., **181** (2005), no. 2, pp. 245–251.
2. C. Bernardi and Y. Maday, *Spectral methods, in:* P. G. Ciarlet, J. L. Lions (Eds.), *Handbook of Numerical Analysis*, Elsevier Science, North-Holland, **5** (1997), pp. 209–485.
3. J. P. Boyd, *Chebyshev and Fourier spectral methods*, Springer-Verlag, New York, 2002.
4. J. M. Burgers, *A mathematical model illustrating the theory of turbulence*, Adv. Appl. Mech., **I** (1948), pp. 171–199.
5. C. Canuto, M. Y. Hussaini, A. Quarteroni, and T. A. Zang, *Spectral methods (fundamental in single domains)* Springer-Verlag, Berlin, 2006.
6. J. D. Cole, *On a quasilinear parabolic equations occurring in aerodynamics*, Quart. Appl. Math., **9** (1951), pp. 225–236.
7. M. Dehghan, A. Hamidi, and M. Shakourifar, *The solution of coupled Burger's equations using Adomian-Pade technique*, Appl. Math. Comput., **189** (2007), pp. 1034–1047.
8. D. Kaya, *An explicit solution of coupled viscous Burgers equations by the decomposition method*, IJMMS, **27** (2001), no. 11, pp. 675–680.
9. S. E. Esipov, *Coupled Burgers equations: A model of polydispersive sedimentation*, Phys. Rev. E, **52** (1995), pp. 3711–3718.
10. B. Fornberg, *A Practical Guide to Pseudospectral Methods*, Cambridge University Press, Cambridge, 1998.
11. D. Gottlieb and S. A. Orszag, *Numerical Analysis of Spectral Methods: Theory and Applications*, SIAM, Philadelphia, 1977.
12. Guo Benyu and Heping Ma, *Fourier pseudo-spectral method for Burgers equation*, Northestern Math. J., **4** (1986), pp. 383–394.
13. A. H. Khater, R. S. Temsah, and M. M. Hassan, *A Chebyshev spectral collocation method for solving Burgers-type equations*, J. Comput. Appl. Math., **222** (2008), no. 2, pp. 333–350.
14. J. C. Mason and T. N. Phillips (Guest Editors), *Chebyshev Polynomials and Spectral Methods*, Numerical Algorithms, **38** (2005), no. 1, pp. 1–236.
15. J. Nee and J. Duan, *Limit set of trajectories of the coupled viscous Burgers' equations*, Appl. Math. Lett., **11** (1998), no. 1, pp. 57–61.
16. R. Peyret, *Spectral methods in incompressible viscous flow*, Springer-Verlag, New York, 2002.
17. A. A. Soliman, *The modified extended tanh-function method for solving Burgers-type equations*, Physica A, **361** (2006), pp. 394–404.
18. L. N. Trefethen, *Spectral Methods in MATLAB*, SIAM, Philadelphia, 2000.