Weighted Voting Method Using Atanassov's Intuitionistic Fuzzy Preference Relations and Ignorance Functions

D. Paternain[1] A. Jurio[1] E. Barrenechea[1] H. Bustince[1] B. Bedregal[2] E. Szmidt[3]

[1]Universidad Publica de Navarra. Pamplona, Spain
{daniel.paternain, aranzazu.jurio, edurne.barrenechea, bustince}@unavarra.es
[2]Universidade Federal do Rio Grande do Norte. Lagoa Nova-Natal, Brazil
bedregal@dimap.ufrn.br
[3]Polish Academy of Sciences. Warsaw, Poland
szmidt@ibspan.waw.pl

Abstract

In this work we present a method for constructing an Atanassov's intuitionistic fuzzy preference relation from a fuzzy preference relation that takes into account the ignorance of the expert when evaluating the preferences in the latter. Moreover, we present two algorithms that make use of this method for extending the weighted voting method to the Atanassov's intuitionistic fuzzy setting in decision making problems.

Keywords: Atanassov's Intuitionistic Fuzzy Sets, Ignorance Functions, Decision-making, Weighted Voting, Score functions.

1. Introduction

The decision making problem consists in finding the best alternative between several of them. Evaluation of alternatives is a procedure strongly linked to uncertainty. For instance, the expert suffers from this uncertainty when he or she needs to assign a numerical representation to the preference of one alternative over another. In this work we model this uncertainty or lack of knowledge by means of Atanassov's intuitionistic fuzzy sets which are built from ignorance functions.

The objectives of this work are the following:

- to present a construction method for Atanassov's intuitionistic fuzzy preference relations (AIFPR) starting from fuzzy preference relations and that takes into account the ignorance or lack of knowledge of the expert in the building of the original FPR;
- to develop a new decision making method that extends the voting method to the AIFPR setting.

Last years, the use of Atanassov's intuitionistic fuzzy sets in decision making has been widely studied [1, 2, 3, 4, 5, 6, 7]. In these works, the evaluations provided by experts are given in terms of intuitionistic values. But in this way, and from our point of view, the original idea of Atanassov's fuzzy sets is lost, namely, the representation by means of Atanassov's intuitionistic fuzzy index of the ignorance or lack of knowledge of the expert when providing his/her preference evaluations. For this reason, our method starts from a FPR which is provided by an expert and build a new AIFPR by quantifying the ignorance of the expert when he or she has provided the original FPR.

From the new AIFPR that we have built, we present two new decision making algorithms that extend the voting method for Atanassov's intuitionistic fuzzy sets. The first algorithm is a direct adaptation, in which we add the preferences for each alternative by means of the addition operator for Atanassov's intuitionistic fuzzy sets. We analyze the difficulties linked to this approach and we present a second algorithm that solves them and it is based in *score* functions.

Moreover, we have considered some cases in which the usual voting algorithm does not give a unique alternative as solution of the problem. We see that, in most of these cases, our decision making algorithm based on intuitionistic fuzzy sets and score functions is able to provide an ordering for the alternatives, which allows to choose as solution the best-valued alternative.

The structure of this is work is as follows. In Section 2 we recall some preliminary definitions. Next, in Section 3 we present a construction method of AIFPRs from FPRs. In Section 4 we propose two algorithms for solving decision-making problems, and in Section 5 we illustrate them with some examples. Finally, conclusions are presented in Section 6.

2. Preliminaries

We start by recalling some basic concepts and definitions used in this work.

2.1. Fuzzy Sets and Atanassov's Intuitionistic Fuzzy Sets

Definition 1 *[8] A fuzzy set A defined on a finite and non-empty universe $U = \{u_1, \ldots, u_n\}$ is given by*

$$A = \{(u_i, \mu_A(u_i))|u_i \in U\}.$$

We denote by $\mathcal{FS}(U)$ the set of all fuzzy sets in U. We know that a binary fuzzy relation R is a fuzzy subset of the cartesian product $U \times U$, that is $R : U \times U \to [0, 1]$.

In fuzzy theory, a function $n : [0, 1] \to [0, 1]$ is a strict negation if n is strictly decreasing, continuous, $n(0) = 1$ and $n(1) = 0$. If n is also involutive (i.e, $n(n(x)) = x$ for all $x \in [0, 1]$), then n is called a strong negation.

A triangular norm (t-norm for short) $T : [0, 1]^2 \to [0, 1]$ is an associative, commutative, non-decreasing function such that $T(1, x) = x$ for all $x \in [0, 1]$. Examples of t-norms are the minimum t-norm ($T_M(x, y) = \min(x, y)$) and the product t-norm ($T_P(x, y) = x \cdot y$). A triangular conorm (t-conorm for short) $S : [0, 1]^2 \to [0, 1]$ is an associative, commutative, non-decreasing function such that $S(0, x) = x$ for all $x \in [0, 1]$. Examples of t-conorms are the maximum t-conorm ($S_M(x, y) = \max(x, y)$) and the probabilist sum t-conorm ($S_P(x, y) = x + y - x \cdot y$).

Definition 2 *[9] An Atanassov's Intuitionistic Fuzzy Set (A-IFS) in U is an expression A given by*

$$A = \{(u_i, \mu_A(u_i), \nu_A(u_i))|u_i \in U\}$$

where $\mu_A : U \to [0, 1]$, $\nu_A : U \to [0, 1]$ satisfy the condition $0 \leq \mu_A(u_i) + \nu_A(u_i) \leq 1$ for all $u_i \in U$.

The numbers $\mu_A(u_i)$ and $\nu_A(u_i)$ denote respectively the degree of membership and the degree of non-membership of the element u_i to the set A. We represent as $\mathcal{A} - \mathcal{IFS}(U)$ the set of all Atanassov's Intuitionistic Fuzzy Sets in U.

Let $A, B \in \mathcal{A} - \mathcal{IFS}(U)$. The following arithmetic operation is given in [10].

$$A + B = \{(u_i, \mu_A(u_i) + \mu_B(u_i) - \mu_A(u_i) \cdot \mu_B(u_i), \nu_A(u_i) \cdot \nu_B(u_i))|u_i \in U\} \quad (1)$$

Observe that $A + B$ can be rewritten in terms of t-norms and t-conorms as follows: $\{(u_i, S_P(\mu_A(u_i), \mu_B(u_i)), T_P(\nu_A(u_i), \nu_B(u_i)))|u_i \in U\}$.

Given $A \in \mathcal{A} - \mathcal{IFS}(U)$ we will call intuitionistic fuzzy index of the element u_i in the set A to the expression

$$\pi_A(u_i) = 1 - \mu_A(u_i) - \nu_A(u_i).$$

It is obvious that $0 \leq \pi_A(u_i) \leq 1$ for all $u_i \in U$. Moreover, if the set A considered is fuzzy, then $\pi_A(u_i) = 1 - \mu_A(u_i) - \nu_A(u_i) = 1 - \mu_A(u_i) - 1 + \mu_A(u_i) = 0$.

In this work we denote by L^* the set:

$$L^* = \{(x, y)|(x, y) \in [0, 1] \times [0, 1] \text{ and } x + y \leq 1\}.$$

We use the following total order relation between elements of L^*. Let $a = (a_1, a_2), b = (b_1, b_2) \in L^*$. $a < b$ if and only if $score(a) < score(b)$ or $score(a) = score(b)$ and $accuracy(a) < accuracy(b)$ where $score(a) = a_1 - a_2$ and $accuracy(a) = a_1 + a_2$.

An Atanassov's intuitionistic fuzzy binary relation R' [11, 12] is defined as an intuitionistic fuzzy subset of $U \times U$, that is, $R' : U \times U \to L^*$.

2.2. Implication operators

In this work we consider a fuzzy implication operator as an implication in the sense of Fodor and Roubens [13].

Definition 3 *An implication is a function $I : [0, 1]^2 \to [0, 1]$ that satisfies the following properties:*

(I1) $x \leq z$ *implies* $I(x, y) \geq I(z, y)$ *for all* $y \in [0, 1]$.
(I2) $y \leq t$ *implies* $I(x, y) \leq I(x, t)$ *for all* $x \in [0, 1]$.
(I3) $I(0, x) = 1$ *for all* $x \in [0, 1]$ *(dominance of falsity).*
(I4) $I(x, 1) = 1$ *for all* $x \in [0, 1]$.
(I5) $I(1, 0) = 0$.

Remark: Properties *(I3),(I4)* and *(I5)* imply that I is an extension of the standard Boolean implication. Indeed, it holds that $I(0, 0) = I(0, 1) = I(1, 1) = 1$ and $I(1, 0) = 0$.

Depending on the application, the following properties can also be demanded:

(I6) $I(1, x) = x$ (neutrality of truth).
(I7) $I(x, I(y, z)) = I(y, I(x, z))$ (exchange property).
(I9) $I(x, 0) = n(x)$ where n is a strong negation.
(I13) I is a continuous function (continuity).

A deep study of these properties can be seen in [14].

2.3. Weak ignorance functions

The weak ignorance concept is defined in [15] as a measure of the lack of knowledge that the expert suffers when assigning a numerical value to the membership of an element to a fuzzy set.

Definition 4 *A function $g : [0, 1] \to [0, 1]$ is called a weak ignorance function if it satisfies the following conditions:*

(g1) $g(x) = g(1 - x)$ *for all* $x \in [0, 1]$*;*
(g2) *If* $x = 0.5$ *then* $g(x) = 1$*;*
(g3) $g(x) = 0$ *if and only if* $x = 0$ *or* $x = 1$.

There exist several construction methods of weak ignorance functions. In Theorem 1 we show one of them, proposed in [15].

Theorem 1 *Let T be a continuous t-norm such that $T(x,y) = 0$ if and only if $x \cdot y = 0$. Under these conditions, the function*

$$g(x) = \begin{cases} \frac{T(x,1-x)}{T(0.5,0.5)} & \text{if } T(x,1-x) \leq T(0.5,0.5) \\ \frac{T(0.5,0.5)}{T(x,1-x)} & \text{otherwise} \end{cases}$$

is a weak ignorance function.

Example:

- If we take $T = T_M$, then

$$g(x) = 2 \cdot \min(x, 1-x) \text{ for all } x \in [0,1].$$

- If we take $T = T_P$, then

$$g(x) = 4 \cdot x(1-x) \text{ for all } x \in [0,1].$$

3. Construction of Atanassov's Intuitionistic Fuzzy Preference Relations from Fuzzy Preference Relations

In this section we present a construction method of Atanassov's Intuitionistic Fuzzy Preference Relations from Fuzzy Preference Relations. For this purpose, we propose a construction method of elements of L^* from two elements in $[0,1]$ such that

- the first one represents the degree of preference of an alternative over another one in the fuzzy relation;
- the second one represents the ignorance of the expert when assigning the first value.

For the construction of these elements we consider functions $F : [0,1]^2 \to L^*$ built from functions F_μ that we present in Theorem 2.

Theorem 2 *Let $F_\mu : [0,1]^2 \to [0,1]$ such that*

(F1) $F_\mu(x,0) = x$ *for all* $x \in [0,1]$;
(F2) $F_\mu(x,y)$ *is increasing in the first argument;*
(F3) $F_\mu(x,y)$ *is decreasing in the second argument;*
(F4) $F_\mu(1,x) = 1 - x$ *for all* $x \in [0,1]$.

In these conditions the following items hold:

(1) $x \geq F_\mu(x,y)$ *for all* $x, y \in [0,1]$;
(2) $F_\mu(x,1) = 0$ *for all* $x \in [0,1]$;
(3) $F_\mu(0,x) = 0$ *for all* $x \in [0,1]$;
(4) The function $F : [0,1]^2 \to L^*$ *given by*

$$F(x,y) = (F_\mu(x,y), 1 - F_\mu(x,y) - y)$$

is such that $\pi(F(x,y)) = y$ *for all* $x, y \in [0,1]$.

Proof: (1) By *(F1),(F3)* $x = F_\mu(x,0) \geq F_\mu(x,y)$. (2) By *(F2),(F4)* $F_\mu(x,1) \leq F_\mu(1,1) = 0$. (3) By *(F1),(F3)* $F_\mu(0,x) \leq F_\mu(0,0) = 0$. (4) By *(F2),(F4)* it is clear that $0 \leq 1 - F_\mu(x,y) - y \leq 1$. Besides, $F_\mu(x,y) + 1 - F_\mu(x,y) - y = 1 - y \leq 1$ so F is well defined. By construction we have that $\pi(F(x,y)) = 1 - F_\mu(x,y) - 1 + F_\mu(x,y) + y = y$ $\square$

We study the construction of functions F_μ from implication operators, because these operators have been widely studied.

Lemma 1 *Let $F_\mu : [0,1]^2 \to [0,1]$ satisfy (F1)-(F4). Then, the function*

$$I : [0,1]^2 \to [0,1] \text{ given by}$$

$$I(x,y) = n(F_\mu(x,y))$$

is an implication operator for all negation n.

Proof: *(I1)* and *(I2)* follow from *(F2)* and *(F3)*. *(I3)* $I(0,x) = n(F_\mu(0,x)) = n(0) = 1$. *(I4)* $I(x,1) = n(F_\mu(x,1)) = n(0) = 1$. *(I5)* $I(1,0) = n(F_\mu(1,0)) = n(1) = 0$ from *(F4)* $\square$

Theorem 3 *A function $F_\mu : [0,1]^2 \to [0,1]$ satisfies properties (F1)-(F4) if and only if there exists an implication operator satisfying (I6) and (I9) with respect to the standard negation such that*

$$F_\mu(x,y) = 1 - I(x,y). \quad (2)$$

Proof: (Sufficiency) *(I1),(I2)* by *(F2),(F3)*. *(I3)* $I(0,x) = 1 - F_\mu(0,x) = 1$. *(I4)* $I(x,1) = 1 - F_\mu(x,1) = 1$. *(I5)* $I(1,0) = 1 - F_\mu(1,0) = 0$ by *(F4)*. (Necessity) $F_\mu(x,0) = 1 - I(x,0) = x$ by *(I9)* and n standard negation. *(F2),(F3)* by *(I1),(I2)*. $F_\mu(1,x) = 1 - I(1,x) = 0$ by *(I6)* $\square$

Remark: By Theorem 3, the function $F : [0,1]^2 \to L^*$ is given by

$$F(x,y) = (1 - I(x,y), I(x,y) - y).$$

Example:

- If we take Reichenbach's implication $I(x,y) = 1 - x + xy$ then

$$F(x,y) = (x(1-y), 1 - x(1-y) - y).$$

- If we take Lukasiewicz's implication $I(x,y) = \min(1, 1-x+y)$ then

$$F(x,y) = (\max(0, x-y), \min(1, 1-x+y) - y).$$

- If we take Kleene-Dienes' implication $I(x,y) = \max(1-x, y)$ then

$$F(x,y) = (\min(x, 1-y), \max(1-x, y) - y).$$

4. Decision making

In this section we present two algorithms that extend weighted voting strategy for decision-making to the Atanassov's intuitionistic fuzzy sets setting. Given a finite set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$, the starting point is a reciprocal FPR given by an expert.

Recall that a fuzzy preference relation R on X is defined as a fuzzy subset of $X \times X$, that is, $R : X \times X \to [0,1]$. The value $R(x_i, x_j) = R_{ij}$ denotes the degree to which alternative x_i is preferred to alternative x_j. We denote by $\mathcal{FR}(X \times X)$ the set of all fuzzy relations on $X \times X$. We say that a FPR R

satisfies the property of reciprocity if $R_{ij} + R_{ji} = 1$ for all $i, j \in \{1, \ldots, n\}$ with $i \neq j$.

An Atanassov's intuitionistic fuzzy preference relation R' on X is defined as $R' : X \times X \to L^*$. The intuitionistic value $R'(x_i, x_j) = (\mu_{R'_{ij}}, \nu_{R'_{ij}})$ represents the preference degree and the non preference degree (respectively) of alternative x_i over alternative x_j. We denote by $\mathcal{AIFR}(X \times X)$ the set of all Atanassov's intuitionistic fuzzy relations on $X \times X$.

In both proposed algorithms we start from a FPR and construct an AIFPR taking into account the ignorance of the expert in the construction of the FPR. In Theorem 4 we present the construction method of AIFPRs from F functions studied in Section 3 and from weak ignorance functions.

Theorem 4 *Let $R \in FR(X \times X)$ and let g be a weak ignorance function. The relation R' given by*

$$R'_{ij} = F(R_{ij}, g(R_{ij})) \text{ for all } R_{ij} \in R$$

is an Atanassov's Intuitionistic fuzzy preference relation such that $\pi(R'_{ij}) = g(R_{ij})$.

4.1. First generalization of the weighted voting strategy

The weighted voting strategy is one of simplest and most widely methods for solving decision-making problems. In this algorithm, the preference value R_{ij} is considered as a weighted vote for the alternative x_i. Assuming that R satisfies the reciprocity property, the value $R_{ji} = 1 - R_{ij}$ is considered as a weighted vote for the alternative x_j. In this way, the final evaluation of each alternative x_i is calculated as a sum of votes:

$$\sum_{1 \leq i \neq j \leq n} R_{ij}.$$

The alternative that obtains the highest amount of votes the one chosen as solution.

In this subsection we present a first generalization of the weighted voting strategy using AIFPRs. Based on Expression 1, the scheme of the algorithm is:

1 Take a function F_μ and a weak ignorance function g.
2 Build the Atanassov's intuitionistic fuzzy preference relation R' from the fuzzy preference relation R following Theorem 4.
3 FOR each alternative $x_i \in X$ DO
  3.1 Calculate the intuitionistic value
  $$(S_P(\mu_{R'_{i1}}, \ldots, \mu_{R'_{in}}), T_P(\nu_{R'_{i1}}, \ldots, \nu_{R'_{in}}))$$
  END FOR
4 Order the alternatives in a decreasing way by the *score* and *accuracy* functions of the intuitionistic values calculated in *Step 3.1*

Algorithm 1

Although Algorithm 1 seems to be the logical adaptation of the weighted voting strategy, it is not a good solution for decision-making problems. Consider the following FPR:

$$R = \begin{pmatrix} - & 0.9 & 0.9 & 0.9 \\ 0.1 & - & 0.4 & 1 \\ 0.1 & 0.6 & - & 0.3 \\ 0.1 & 0 & 0.7 & - \end{pmatrix}$$

Step 1 We take $F_\mu(x, y) = x(1 - y)$ and $g(x) = 2 \cdot \min(x, 1 - x)$.
Step 2 We build the AIFPR (see Table 1).
Step 3 We calculate the intuitionistic value associated to each alternative

$$\begin{pmatrix} (0.9780, 0.0005) \\ (1, 0) \\ (0.2876, 0.0161) \\ (0.3376, 0.0864) \end{pmatrix}$$

Step 4 We calculate the *score* and we order the alternatives

- $score(0.9780, 0.0005) = 0.9775$
- $score(1, 0) = 1$
- $score(0.2876, 0.0161) = 0.2715$
- $score(0.3376, 0.0864) = 0.2512$

The order of the alternatives is $x_2 > x_1 > x_3 > x_4$. Observe that, attending to the preference of the expert given in the FPR, x_1 is preferred over the rest of the alternatives. However, x_1 is not the solution that we obtain. This is due to the fact that if the expert expresses the preference of an alternative with a high value, as in the case $R_{24} = 1$, the final evaluation of this alternative is maximum, without taking into account the rest of preferences of this alternative. This behaviour is due to the use of t-conorms and t-norms in the calculation of the sum of votes for each alternative.

This problem has led us to propose a new decision-making algorithm.

4.2. Second generalization of the weighted voting strategy based on score functions

In Algorithm 1 we have checked that the use of t-conorms and t-norms is not a good approximation. In this subsection we present a new adaptation of the weighted voting strategy based on the calculus of *score* functions.

Following the original idea of the weighted voting strategy, we consider each element of the AIFPR as a weighted vote. The preference degree $\mu_{R'_{ij}}$ is considered as a positive vote while the non preference degree $\nu_{R'_{ij}}$ is considered as a negative vote. In this way, each element of the relation R' is considered as a vote given by $score(\mu_{R'_{ij}}, \nu_{R'_{ij}}) = \mu_{R'_{ij}} - \nu_{R'_{ij}}$. The final evaluation of the alternative x_i is given by

$$\sum_{1 \leq j \neq i \leq n} score(\mu_{R'_{ij}}, \nu_{R'_{ij}}) = \sum_{1 \leq j \neq i \leq n} \mu_{R'_{ij}} - \nu_{R'_{ij}}.$$

	x_1	x_2	x_3	x_4
x_1	-	(0.72,0.08)	(0.72,0.08)	(0.72,0.08)
x_2	(0.08,0.72)	-	(0.08,0.12)	(1,0)
x_3	(0.08,0.72)	(0.12,0.08)	-	(0.12,0.28)
x_4	(0.08,0.72)	(0,1)	(0.28,0.12)	-

Table 1: Intuitionistic fuzzy binary relation R' constructed by Algorithm 1.

The scheme of Algorithm 2 is the following:

1 Take a function F_μ and a weak ignorance function g.

2 Build the Atanassov's intuitionistic fuzzy preference relation R' from the fuzzy preference relation R following Theorem 4.

3 FOR each element in R' DO

3.1 Calculate the *score* of the element.

$$\begin{pmatrix} - & \mu_{R'_{12}} - \nu_{R'_{12}} & \cdots & \mu_{R'_{1n}} - \nu_{R'_{1n}} \\ \mu_{R'_{21}} - \nu_{R'_{21}} & - & \cdots & \mu_{R'_{2n}} - \nu_{R'_{2n}} \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{R'_{n1}} - \nu_{R'_{n1}} & \cdots & \cdots & - \end{pmatrix}$$

END FOR

4 Apply the weighted voting strategy over the score matrix calculated in *Step 3* and give as solution the alternative x_i such that

$$\arg \max_{i=1,\cdots,n} \sum_{1\le j\ne i\le n} score(\mu_{R'_{ij}}, \nu_{R'_{ij}})$$

Algorithm 2

5. Illustrative examples

In this section we present two examples to show the effectiveness of Algorithm 2 in decision-making problems. In the first example we check that if the weighted voting strategy applied over a given FPR provides a good solution, Algorithm 2 gives the same solution. In the second example, we show that Algorithm 2 is able to obtain a solution when the weighted voting strategy fails.

In the two examples, the expressions used in *Step 1* are:

$$F_\mu(x, y) = x(1 - y)$$

$$g(x) = 2 \cdot \min(x, 1 - x).$$

5.1. First example

Given the reciprocal FPR:

$$R = \begin{pmatrix} - & 0.3360 & 0.4888 & 0.7123 \\ 0.6640 & - & 0.6124 & 0.5814 \\ 0.5112 & 0.3876 & - & 0.7009 \\ 0.2877 & 0.4186 & 0.2991 & - \end{pmatrix}$$

Step 2 We build the Atanassov's intuitionistic fuzzy preference relation R'

See Table 2.

Step 3 We calculate the *score* of each element in R'

$$\begin{pmatrix} - & -0.1076 & -0.0005 & 0.1803 \\ 0.1076 & - & 0.0505 & 0.0265 \\ 0.0005 & -0.0505 & - & 0.1614 \\ -0.1803 & -0.0265 & -0.1614 & - \end{pmatrix}$$

Step 4 We sum the *scores* of each alternative obtaining

- $x_1 \to 0.0722$.
- $x_2 \to 0.1846$.
- $x_3 \to 0.1114$.
- $x_4 \to -0.3682$.

We order the alternatives

$$x_2 > x_3 > x_1 > x_4$$

Observe that the result is in agreement with the one obtained by the weighted voting strategy.

5.2. Second example

There exist problems where the weighted voting strategy is not able to choose between two or more alternatives (two or more alternatives have the same sum of votes). We present an example where Algorithm 2 solves this problem. Given the following FPR:

$$R = \begin{pmatrix} - & 0.78 & 0.60 & 0.28 \\ 0.22 & - & 0.75 & 0.69 \\ 0.40 & 0.25 & - & 0.44 \\ 0.72 & 0.31 & 0.56 & - \end{pmatrix}$$

Step 2 We build the Atanassov's intuitionistic fuzzy preference relation R'

See Table 3.

Step 3 We calculate the *score* for each element

$$\begin{pmatrix} - & 0.3136 & 0.0400 & -0.1936 \\ -0.3136 & - & 0.2500 & 0.1444 \\ -0.0400 & -0.2500 & - & -0.0144 \\ 0.1936 & -0.1444 & 0.0144 & - \end{pmatrix}$$

Step 4 We sum the *scores* for each alterantive

- $x_1 \to 0.1600$.
- $x_2 \to 0.0808$.
- $x_3 \to -0.3044$.
- $x_4 \to 0.0636$.

	x_1	x_2	x_3	x_4
x_1	-	(0.1102,0.2178)	(0.0109,0.0115)	(0.3024,0.1222)
x_2	(0.2178,0.1102)	-	(0.1377,0.0871)	(0.0947,0.0681)
x_3	(0.0115,0.0109)	(0.0871,0.1377)	-	(0.2816,0.1202)
x_4	(0.1222,0.3024)	(0.0681,0.0947)	(0.1202,0.2816)	-

Table 2: Intuitionistic fuzzy binary relation R' from Example 5.1.

	x_1	x_2	x_3	x_4
x_1	-	(0.4368,0.1232)	(0.12,0.08)	(0.1232,0.3168)
x_2	(0.1232,0.4368)	-	(0.375,0.125)	(0.2322,0.1178)
x_3	(0.08,0.12)	(0.125,0.375)	-	(0.0528,0.0672)
x_4	(0.3168,0.1232)	(0.1178,0.2322)	(0.0672,0.0528)	-

Table 3: Intuitionistic fuzzy binary relation R' from Example 5.2.

We order the solutions and we obtain:

$$x_1 > x_2 > x_4 > x_3$$

Observe that if we apply the weighted voting strategy over R, the rank of alternatives is:

$$x_1 = x_2 > x_4 > x_3$$

In this example, the weighted voting cannot decide between alterantives x_1 and x_2. However, using F functions and weak ignorance function in the construction of the Atanassov's intuitionistic fuzzy preference relation, Algorithm 2 is able to order and decide the best alternative.

6. Conclusions

In this work we have present a construction method of AIFPRs from FPRs quantifying the ignorance of the expert when expressing his/her preferences. Moreover, we have studied and presented two adaptations of the well known weighted voting strategy that work with AIFPRs. We have proved that in problems where the weighted voting cannot distinguish some alternatives our second algorithm finds the best alternative.

Acknowledgements: This work has been partially supported by the National Science Foundation of Spain, reference TIN2010-15055 and by the Research Services of the Universidad Publica de Navarra.

References

[1] SM Chen and JM Tan. Handling multicriteria fuzzy decision-making problems based on vague set-theory. *Fuzzy Sets and Systems*, 67(2):163–172, 1994.

[2] E Szmidt and J Kacprzyk. Using intuitionistic fuzzy sets in group decision making. *CONTROL AND CYBERNETICS*, 31(4):1037–1053, 2002.

[3] Fatih Emre Boran, Serkan Genc, Mustafa Kurt, and Diyar Akay. A multi-criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method. *Expert Systems With Applications*, 36(8):11363–11368, 2009.

[4] Chunqiao Tan and Xiaohong Chen. Intuitionistic fuzzy Choquet integral operator for multicriteria decision making. *Expert Systems With Applications*, 37(1):149–157, 2010.

[5] Jian-Zhang Wu and Qiang Zhang. Multicriteria decision making method based on intuitionistic fuzzy weighted entropy. *Expert Systems With Applications*, 38(1):916–922, 2011.

[6] Jun Ye. Fuzzy decision-making method based on the weighted correlation coefficient under intuitionistic fuzzy environment. *European Journal of Operational Research*, 205(1):202–204, 2010.

[7] Jian-qiang Wang and Jing-jing Li. Multicriteria fuzzy decision-making method based on cross entropy and score functions. *Expert Systems With Applications*, 38(1):1032–1038, 2011.

[8] LA Zadeh. Fuzzy Sets. *Information and Control*, 8(3):338–353, 1965.

[9] KT Atanassov. Intuitionistic Fuzzy-Sets. *Fuzzy Sets and Systems*, 20(1):87–96, 1986.

[10] KT Atanassov. New operations defined over the intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 61:137–142, 1994.

[11] P Burillo and H Bustince. Intuitionistic fuzzy relations. (Part I). *Mathware and Soft Computing*, 2(1):5–38, 1994.

[12] P Burillo and H Bustince. Intuitionistic fuzzy relations. (Part II) Effect of Atanassov's operators on the properties of the intuitionistic fuzzy relations. *Mathware and Soft Computing*, 2(2):117–148, 1994.

[13] J Fodor and M Roubensm. *Fuzzy preference modelling and multicriteria decision support.* Kluwer, Dordrecht, 1994.

[14] H Bustince, P Burillo, and F Soria. Automorphisms, negations and implication operators. *Fuzzy Sets and Systems*, 134(2):209–229, 2003.

[15] H. Bustince, M. Pagola, E. Barrenechea, J. Fernandez, P. Melo-Pinto, P. Couto, H.R. Tizhoosh, and J. Montero. Ignorance functions. An application to the calculation of the threshold in prostate ultrasound images. *Fuzzy Sets and Systems*, 161(1):20–36, 2010.