Using Conditional Autoregressive Range Model to Forecast Volatility of the Stock Indices

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Abstract

This paper compares the forecasting performance of the conditional autoregressive range (CARR) model with the commonly adopted GARCH model. Two major stock indices, FTSE 100 and Nikkei 225, are studies using the daily range data and daily close price data over the period 1990 to 2000. Our results suggest that improvements of the overall estimation are achieved when the CARR models are used. Moreover, we find that the CARR model gives better volatility forecasts than GARCH, as it can catch the extra informational contents of the intra-daily price variations. Finally, we also find that the inclusion of the lagged return and the lagged trading volume can significantly improve the forecasting ability of the CARR models. Our empirical results also significantly suggest the existence of a leverage effect in the U.K. and Japanese stock markets.

Keywords: CARR, GARCH, Range, Volatility, Leverage Effect.

1. Introduction

Volatilities play a very important role in finance. Accurate forecasting of volatilities is key to risk management and derivatives pricing. The empirical finance literature reflects well that concern, nesting many different tools for volatility estimation and forecasting purposes. It is well known that many financial time series exhibit volatility clustering whereby volatility is likely to be high when it has recently been high and volatility is likely to be low when it has recently been low. These findings have been uncovered in three ways: By estimating parametric time series models like GARCH and Stochastic Volatility, from option price implied volatilities, and from direct measures, such as the realized volatility. Among them, The GARCH model is most-adopted for modeling the time-varying conditional volatility. GARCH models the time varying variance as a function of lagged squared residuals and lagged conditional variance. The strength of the GARCH model lies in its flexible adaptation of the dynamics of volatilities and its ease of estimation when compared to the other models.

Essentially, the GARCH model is return-based model, which is constructed with the data of closing prices. Hence, though the GARCH model is a useful tool to model changing variance in time series, and provides acceptable forecasting performance, it might neglect the important intraday information of the price movement. For example, when today's closing price equals to last day's closing price, the price return will be zero, but the price variation during the today might be turbulent. However, the return-based GARCH model cannot catch it. Using the intra-day GARCH, some studies try to remedy the limit of the traditional GARCH. An alternative way to model the intra-day price variation is adopting the price range data instead. The price range, the difference between the daily high and daily low of log-prices, has been used in the academic literature to measure volatility. Financial economists have long known that the daily range of the log price series contains extra information about the course of volatility over the day. Despite the elegant theory and the support of simulation results, the price range as a proxy of volatility has performed poorly in empirical studies. Chou (2005) conjectures that the fundamental reason for the poor empirical performance of price range is that it cannot well capture the dynamics of volatilities. By properly modeling the dynamic process, price range would retain its superiority in forecasting volatility. Therefore, Chou (2005) proposes an alternative range-based volatility model, the Conditional Autoregressive Range model (CARR) to forecast volatilities. The CARR model is very different from Alizadeh, Brandt, and Diebold's (2002) Range-based Stochastic Volatility model in several aspects. First, The CARR model involves the range data instead of the log-range data. Second, the CARR model describes the dynamics of the conditional mean of the range, while Range-based Stochastic Volatility model describes the dynamics of the conditional return volatility. Finally, Range-based Stochastic Volatility model focuses on estimation and in-sample fitting, whereas the CARR model's interest lies primarily in model specification and out-of-sample forecasting.

By applying to the weekly S&P 500 index data, Chou (2005) shows that the CARR model does provide sharper volatility estimates compared with a standard GARCH model. Application of CARR to other frequency of range intervals, say every day, will provide further understanding of the usefulness and limitation of the range model. Analyses using more stock index data will also be helpful. In order to induce a more general conclusion of CARR's superiority in forecasting the volatilities of stock markets, in this paper the CARR model is applied to the daily datasets of two major stock indices: the FTSE 100 and the Nikkei 225. Several performance measurements are employed to compare the results. Besides, several stylized features of stock markets, such as the "leverage effect" in the volatility-return relation and the positive volatility-volume relation have recently become the focus of detailed empirical study. Therefore, in the present paper we indicate a way of extending the CARR model to reflect these features. We examine whether the inclusion of lagged return and lagged trading volume can significantly improve the forecasting ability of the CARR model. Firstly, by incorporating the lagged return, we can catch the "leverage effect" in the stock markets. The leverage effect or volatility asymmetry is negative return sequences are associated with increases in the volatility of stock returns. The leverage effect was studied in some early work by Black (1976), while it motivated the introduction of the EGARCH model of Nelson (1991) and the threshold ARCH model of Glosten, Jagannathan, and Runkle (1993). An economic theory behind such effects is discussed by Campbell and Kyle (1993). Secondly, by incorporating the lagged trading volume into the CARR model, we re-examine the relationship between volatility and trading volume in the stock markets. Karpoff (1987) provides a detailed survey and concludes that volume is positively related to the volatility in equity markets.

The structure of this paper is as follows. Section 2 describes the sampling datasets. Section 3 presents the specification of the CARR model. Section 4 discusses the empirical results. Section 5 concludes this article.

2. Data

We analyze the daily data on the FTSE 100 (London) and Nikkei 225 (Tokyo). It covers eleven years period, from January 1990 to December 2000. The estimation process is run using eight years of data (1990-2000) while the remaining 3 years are used for forecasting. The data are available from CRSP. The daily closing prices are transformed into continuously compounded rates of returns as followed.

$$r_t = 100\left[\ln(P_t / P_{t-1})\right] \tag{1}$$

where P_t is the closing stock index on day t and the sample size runs from 1 to *T*. These returns will be used to construct a GARCH model for the comparison purpose. The range of the log-prices is defined as the difference between the daily log high stock index and the daily log low stock index.

$$R_t = 100(\ln P_t^H - \ln P_t^L) \tag{2}$$

where P_t^H and P_t^H respectively are the highest and lowest stock index on day *t*.

As is typical with financial time series, both daily returns and daily ranges of FTSE 100 and Nikkei 225 exhibit excess kurtosis. As a consequence, the Jarque-Bera test results in a rejection of normality at the 1% significance level for both indices. Besides, compared with the return, the price range catches higher variation of intraday price movement on average; but the standard deviation of the price range is approximately only one-fourth the standard deviation of the return. Hence, the superior efficiency of the price range measure, relative to the return, emerges clearly.

Augmented Dickey and Fuller (1979) (ADF) and Phillips and Perron (1988) (PP) unit root tests for non-stationarity in the price-range data of FTSE 100 and Nikkei 225 both indicate no evidence of non-stationarity. Each of the unit-root test statistics is calculated with an intercept in the test regression. For each of these tests, the null hypothesis is a non-stationary time series and the alternative hypothesis is a stationary time series. The lag length for the ADF test regression is set using the Schwarz information criteria, and the bandwidth for the PP test regression is set using a Bartlett kernel. As a basis of comparison, recall that the autocorrelations for a randomly distributed variable should be less than two standard errors. The first ten autocorrelations for the range of financial series report that the large and slowly decaying autocorrelations of the range of both series show strong volatility persistence.

3. Model

This section provides a brief overview of the CARR model used to forecast range-based volatility. With the time series data of daily price range R_t, Chou (2005) presents the CARR model of order (*p*,*q*), or CARR (*p*,*q*) is shown as

$$R_t = \lambda_t \varepsilon_t \tag{3}$$

$$\lambda_t = \omega + \sum_{i=1}^{p} \alpha_i R_{t-i} + \sum_{i=1}^{q} \beta_i \lambda_{t-i}$$

$$\varepsilon_t \sim f(.),$$

where λ_t is the conditional mean of the range based on all information up to time t, and the distribution of the disturbance term ε_t, or the normalized range, is assumed to have a density function $f(.)$ with a unit mean. Since ε_t is positively valued given that both the price range R_t and its expected value λ_t are positively valued, a natural choice for the distribution is the exponential distribution. Assuming that the distribution follows an exponential distribution with unit mean, Chou (2005) shows that the log likelihood function can be written as

$$L(\alpha_i, \beta_i; R_1, R_2, ..., R_T) = -\sum_{t=1}^{T} [\ln(\lambda_t) + \frac{R_t}{\lambda_t}]. \tag{4}$$

Chou (2005) also shows that the unconditional long-term mean of range $\overline{\omega}$ can be calculated as

$$\overline{\omega} = \omega / [1 - (\sum_{i=1}^{p} \alpha_i + \sum_{i=1}^{q} \beta_i)] \tag{5}$$

and for the model to be stationary and to ensure the nonnegative range, the coefficients ω, α_i and β_i must meet the following conditions:

$$\sum_{i=1}^{p} \alpha_i + \sum_{i=1}^{q} \beta_i < 1 \text{ and } \omega, \alpha_i, \beta_i > 0. \tag{6}$$

One of the important properties for the CARR model is the ease of estimation. Specifically, the Quasi-Maximum Likelihood Estimation (QMLE) of the parameters in the CARR model can be obtained by estimating a GARCH model with a particular specification: specifying a GARCH model for the square root of range without a constant term in the conditional mean equation. The intuition behind this property is that with some simple adjustments on the specification of the conditional mean, the likelihood function in the CARR model with an exponential density function is identical to the GARCH model with a normal density function. Furthermore, all asymptotic properties of the GARCH model are applicable to the CARR model. Given that the CARR model is a model for the conditional mean; its regularity conditions are less stringent than the GARCH model. The details of this and other related issues are beyond the scope of this paper, and the interested readers can be referred to Chou (2005).

4. Results

To estimate and forecast the volatility of these indices, we first compare various CARR model specifications to determine the best form of the model for the price-range data of FTSE 100 and Nikkei 225. Specifically, we consider three forms of the CARR model: CARR(1,1), CARR(1,2) and CARR(2,1). Using the case of FTSE 100 as an example, the p-value indicates that both the α_2 coefficient in the CARR(2,1) model and the β_2 coefficient in the CARR(1,2) model are not significant at the 5% level. The value of the log likelihood function (LLF) further indicates that the CARR(1,1) model outperforms both the CARR(1,2) model and the CARR(2,1) model and the CARR(1,1) model is sufficient for both financial time series. This results consist with Chou (2005), which also finds that the CARR(1,1) model appears to work quite well in practice as a general-purpose model. On the other hand, among GARCH model specifications GARCH(1,1) is the best form of the model for the return data of FTSE 100 and the Nikkei 225. The range-based volatility models clearly outperform the return-based models, since the LLF strongly increases to 2.44 and 2.98 with CARR(1,1) versus 2.38 and 2.84 with GARCH(1,1), for the FTSE 100 and the Nikkei 225 respectively.

Based on the appropriate model specification for CARR and GARCH, we then perform out-of-sample forecasts to assess the forecasting ability of these two volatility models. No matter for FTSE 100 and Nikkei 225, both the RMSE and MAE measures indicate that the forecasting error of the CARR (1,1) model is lower than that of the GARCH (1,1). This means that CARR (1,1) model outperforms the GARCH (1,1) model. In other words, both measures provide support for Chou’s (2005) proposition that the range contain more information than the return and, as a result, the CARR (1,1) model can provide sharper volatility forecasts than the standard GARCH (1,1) model. Upon closer examination of the numbers across the forecast horizon h, we also find that as the forecast horizon h increases, the forecasting ability of the model deteriorates. This finding is consistent with West and Cho (1995) and Christoffersen and Diebold (2000). According to the MDM test, for the time series of FTSE 100 and Nikkei 225, the CARR(1,1) model outperforms the GARCH(1,1) model. This is encouraging because it means that the benchmark GARCH(1,1) is consistently beaten by the CARR(1,1) model. The results of the Mincer-Zarnowitz regression test are consistent with the methods using RMSE, MAE and MDM. The dominance of CARR over the GARCH model is clear. Once the CARR-predicted-volatility is included, the GARCH-predicted-volatility often becomes insignificant or with wrong signs.

The CARR model of order (*p*,*q*), or CARR (*p*,*q*), can be easily extended to incorporate exogenous variables X_{t-i} by modifying the conditional mean of the range λ_t:

$$\lambda_t = \omega + \sum_{i=1}^{p} \alpha_i R_{t-i} + \sum_{i=1}^{q} \beta_i \lambda_{t-i} + \sum_{i=1}^{l} \gamma_i X_{t-i} \tag{7}$$

This model is denoted by CARRX (*p*,*q*). In this article, we add two exogenous variables, the lagged return and trading volume, into the CARR model to catch the stylized futures of stock markets, and also to investigate whether the forecasting ability of the CARR model can be significantly improved.

By incorporating exogenous variables, the lagged return and trading volume, we consider two forms of the CARRX(1,1) model: CARRX(1,1)-a and CARRX(1,1)-b. The CARRX(1,1)-a model incorporates only the lagged return Y_{t-1}, and the CARRX(1,1)-b model incorporates only the trading volume V_{t-1}. The p-value indicates that the γ_1

coefficient for the lagged return Y_{t-1} and the γ_2 coefficient for the lagged trading volume V_{t-1} are both significant at the 5% level. The γ_1 coefficient suggests a negative relation between lagged return Y_{t-1} and volatility: as lagged return Y_{t-1} decreases, volatility would increase. The γ_2 coefficient suggests a positive relation between the lagged trading volume V_{t-1} and volatility: As the lagged trading volume V_{t-1} decreases, price volatility would also decrease. The γ_1 coefficient suggests the existence of a leverage effect, such that bad news would have a greater impact on future volatility than good news. Meanwhile, the γ_2 coefficient also suggests the positive volatility-volume relation, which means price volatility steadily declines with less trading volume.

Note the reduction of the Ljung-Box Q statistics of the CARRX (1,1) model when compared to the original CARR (1,1) model. The reduction of the Ljung-Box Q statistics indicates that the CARRX (1,1) model has better forecasting ability than the CARR (1,1) model. The increasing value of the log likelihood function also further indicates such.

5. Conclusions

This paper examines the empirical performance of the CARR model by analyzing daily data on the FTSE 100 and Nikkei 225 over the period 1990 to 2000. We find that the CARR model produces sharper volatility forecasts than the commonly adopted GARCH model. Furthermore, we find that the inclusion of the lagged return and trading volume can significantly improve the forecasting ability of the CARR model. Our empirical results also suggest the existence of a leverage effect in the U.K. and Japanese stock markets.

The CARR model provides a simple, yet effective framework for forecasting the volatility dynamics. It would be interesting to explore whether alternative choices of the range, such as the monthly and quarterly range, fit the class of the CARR models. Generally, the empirical results of this article provide strong support for the application of the CARR model in the stock markets that will be of great interest to academics and practitioners, particularly those involved in making international risk management decisions.

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