Discussion on ”A State Observer for a Class of Nonlinear Systems with Multiple Discrete and Distributed Time Delays

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1 Introduction - Main contribution

Although many works have been devoted to the analysis and control of time-delay systems during the last decades [1, 2, 3], much less have concerned the observer design problem while in practice the state variables may be not all measured. As illustration recent studies concern the observer design or analysis, as [4, 5, 6, 7], as well as [8, 9, 10] in the nonlinear case. Problems solved in the reported results include algebraic approaches to the study of observability properties for time-delay systems and the corresponding observer design (in the linear and nonlinear cases), eventually with unknown input.

In this paper by Alfredo Germani and Pierdomenico Pepe, a new design of state observers for nonlinear SISO systems with delays is proposed. It follows previous authors' contributions on this topic as [8, 9, 10].

Here, a specific class of nonlinear systems is considered, i.e. systems with multiple noncommensurate as well as distributed delays (which greatly improves the existing results), but such that the output and its $n-1$ derivatives depend on the present value of the system variables and do not depend on their past ones (which is a hard restriction).

A nonlinear observer is proposed, extending the results in [11]. Then, under Lipschitz hypotheses, the global or local convergence of the estimation error is proved, for any prescribed decay rate.

In this study some of the questions that can arise are the following:

- what system representations can be considered ?
- what kind of observability property is to be used ?
- how to ensure global or local convergence of the observer ?

2 About the system description

This paper tackles the case of nonlinear systems described by :

$$\left\{\begin{array}{lcl} \dot{x}(t) & = & f_1(x(t)) + g_1(x(t)).p(x_t, u(t)), \\ y(t) & = & h(x(t)), \end{array}\right. \tag{1}$$

where $x(t) \in \mathbb{R}^n$ and $x_t(\tau) = x(t+\tau)$ (for $\tau \in [-\Delta, 0]$ with Δ the maximal delay), p is a functional form such that $p : PC([-\Delta, 0]; \mathbb{R}^n) \times \mathbb{R} \to \mathbb{R}$.

In [9], the following class of systems was considered:

$$\left\{\begin{array}{lcl} \dot{x}(t) & = & f_2(x(t), x(t-\Delta)) + g_2(x(t), x(t-\Delta)).u(t), \\ y(t) & = & h(x(t)), \end{array}\right. \tag{2}$$

where Δ is the delay.
Clearly (1) extends (2) by allowing to include multiple noncommensurate delays and distributed ones, while (2) is restricted to the case of single discrete delay.

However, this is balanced by two main restrictions which are linked to the requirement that the output and its $n-1$ derivatives do not depend on the past system variables. Indeed the assumption used by A.Germani and P.Pepe to get the observability degree is given, using the Lie derivatives, for $\chi_0 \in \mathbb{R}^n$, as

$$L_{g_1} L_{f_1}^k h(\chi_0) = 0, \quad , k = 0, 1, \dots, n-2 \tag{3}$$

First, f_1 in (1) does not depend on the delay, which is strong. This was not necessary in [9].

As illustration, example 1 in the discussed paper, i.e.

$$\left\{\begin{array}{lcl} \dot{x}_1(t) & = & a x_1(t)(1 - x_1(t)/m) + b x_1(t) x_2(t) \\ \dot{x}_2(t) & = & c x_2(t) + d x_1(t) x_2(t-\Delta), \\ y(t) & = & x_1(t) \end{array}\right. \tag{4}$$

can be written according to (1):

$$f_1(x(t)) = \begin{pmatrix} ax_1(t)(1 - x_1(t)/m) + bx_1(t)x_2(t) \\ \\ cx_2(t), \end{pmatrix},$$

$g_1(x(t)) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ which are delay free and $p = dx_1(t)x_2(t - \Delta)$,
or according to (2) [9],

$$f_2(x(t), x_(t-\Delta)) = \begin{pmatrix} ax_1(t)(1 - x_1(t)/m) + bx_1(t)x_2(t) \\ \\ cx_2(t) + dx_1(t)x_2(t - \Delta), \end{pmatrix}$$

$g_2 \equiv 0$. This means, as we will see on the next section, that the observability matrix for the representation (1) is delay-free.

On the other hand, $p : PC([-\Delta, 0]; \mathbb{R}^n) \times \mathbb{R} \to \mathbb{R}$ is a functional which includes all the delay elements of the system, but spans in $\mathbb{R}$. As illustration the example given in [9], i.e.

$$\begin{cases} \dot{x}_1(t) &= -3x_2 + 0.5(t)x_1(t - \Delta)x_2(t - \Delta) \\ \\ \dot{x}_2(t) &= -(x_1(t))^2 x_2(t - \Delta) + u(t), \end{cases} \tag{5}$$

cannot be rewritten as the system representation (1). Indeed the "delay parts" (i.e. functionals) in the previous example are different in both state variable equations which makes impossible to find a functional p which should satisfy the representation (1).

As a conclusion, these restrictions on the class of considered systems is the price to pay for the extension of the results in [11] to the case of time-delay systems. Further interesting studies could concern the relaxation of these assumptions.

3 Observability property

Compared to previous authors' papers, the notion of observability is here not clearly described. However, in a similar way as in [9], assumption Hp requires that $\phi(\chi_0)$ is a diffeomorphism in $\mathbb{R}^n$, which allows $Q(\chi_0) = \partial\phi(\chi_0)/\partial\chi_0$ to be invertible.

Even if $\phi(\chi_0, \chi_{1,n-1})$ in [9] may include delayed elements while $\phi(\chi_0)$ here cannot, the "diffeomorphism" assumption concerns, in both cases, only the present (and not past or delayed) state variables. In other words observability is required for the delay - free part of the system.

As a comparison for linear time-delay systems of the form :

$$\dot{x}(t) = A_0 x(t) + A_h x(t - h); \quad y(t) = Cx(t)$$

this would consider to assume that the pair (C, A_0) is observable. Note that this is a necessary assumption for stability independent of delay.

4 Convergence of the estimation error

The proof of convergence extends the one in [11], assuming some (global or local) Lipschitz conditions on $L_f^n h(\phi^{-1}(\nu))$, ϕ, as well as on ϕ^{-1}. These are used to ensure the exponential convergence of the state estimation error. Thus the gain K of the observer is linked to eigenvalues of the estimated error system and to the given decay rate, but no design is actually proposed.

In the example section, it is shown that local Lipschitz conditions are achieved but it could have been interesting to illustrate the case of global Lipschitz conditions since, as mentioned in Remark 3.4, the Lipschitz constants can be very large (sufficiently to ensure that these conditions are true ?).

5 Conclusion

A new state observer for non linear systems with (non-commensurate and distributed) delays has been proposed which is an interesting extension of current solutions. Even if some restrictions are used, we can foresee further extensions (relaxations) of this work.

Let me conclude by a "robustness" analysis of such solutions, even if this is not the scope of the paper. Indeed the proposed observer needs the exact knowledge of the delay value to be implemented which, in practice, is quite difficult. In this case some results are provided in [12]. Here, the author could use an estimated delay in the observer description, and try to prove that the estimation error is ultimately bounded.

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