

Modelling and Controller Design for a UV Disinfection Plant

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A mathematical model describing fluid flow and concentration dynamics of microorganisms inside a UV photoreactor is developed. Using physical arguments and techniques from system theory, we approximate this model by a first-order linear one. For this reduced model, we design a controller. The controller is tested on the original model as well as on the reduced model by numerical simulation. This showed only small differences in dynamics, which indicates that for the original model a classic controller with excellent properties can be designed.

Keywords: Model reduction, UV disinfection, systems with flow

1. Introduction

For the disinfection of fluids, UV-treatment becomes increasingly popular. This method replaces more conventional options. For example, in drinking water treatment, chlorination is still the most used disinfection method. However, as the residuals are toxic to aquatic life [21], and some by-products of chlorination have been proven to be mutagenic, the UV disinfection method has been stimulated as an alternative [1, 20]. In [3]–[23], design and modelling of a photoreactor for disinfecting drinking water is proposed. In [12]–[24], experimental results are presented that link the UV dosage to the inactivation of microorganisms

in wastewater treatment in agriculture and horticulture. Air disinfection by UV is investigated in [7] and [14]. In [16], the required UV dosage for apple cider pasteurization is examined, and in [5] the possibility of UV treatment of process water in the food and beverage industries is discussed.

In order to improve the operation of UV disinfection, dynamic control needs to be incorporated. However, surprisingly little literature is available on control design for UV disinfection. Only one approach is known to the authors. In [10], a basic model was developed, and a controller was designed. The fluid mechanics was modelled by a plug flow, and the attenuation of the microorganisms due to the UV irradiation was modelled as a first order reaction. For effective control design, it is desirable to have a low order linear model that contains the essential system dynamics. This allows for standard linear control, which is very well documented and gives generally good performance, see for example [2, 11]. However, models describing a UV reactor are generally not of this type. Disinfection is a complex process with often higher order reaction kinetics [4]. Moreover, a detailed description of the fluid flow through the reactor is in general described by the full Navier–Stokes equations [8]. Altogether, a full reactor model leads to a nonlinear model, and hence it is not possible to design a controller in the traditional sense, employing standard techniques.

In this paper, a first order linear model-based controller is designed for a nonlinear partial differential

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equation model of a reactor, using various approximation techniques. In section 2, we start with describing the full transport model for the fluid and microorganism concentration, under the influence of UV irradiation. The reaction kinetics are described in a basic way, following [10]. Subsequently, we adopt the assumption of laminar flow and invoke basic symmetry properties of the solution. This yields a simplified model of which the fluid-mechanics part can be solved analytically. By specifying the model to conditions relevant to disinfection of *Escherichia coli* 0157:H7 bacteria in cider [6], we obtain the reference ‘nominal’ model. In section 3 the nonlinear nominal model is linearized around its reference state. This linear p.d.e. model is further approximated by using a Padé approximation for its transfer function, and using balancing [25]. The resulting reduced model is of first order. In section 4 a feedback controller is designed for this model. The controller is tested by a simulation study and found to perform equally well on the reduced model as on the nominal model. This indicates that under the reduction steps, the essential dynamics of the nominal model is maintained. Hence, for the rather complex original model it is shown that a classic controller with excellent properties can be designed. All modeling and reduction steps are carefully examined. This approach therefore gives a clear image of the conditions under which this type of controller would perform well.

2. Basic Modelling of the Disinfection Plant

In this section, we first sketch the general physical model. Then, we proceed to analyze laminar flow in the reactor and subsequently consider the dynamics of the active microorganism concentration in case diffusive transport can be neglected. Finally, the parameter values for the nominal model are chosen, corresponding to the particular case of UV-disinfection in a cider plant.

2.1. Physical Model

We propose a general model that describes the fluid flow and the reaction kinetics in a cylindrical reactor. This model is composed of the Navier–Stokes equations for an incompressible fluid, to describe conservation of mass and momentum, and a convection-diffusion-reaction equation which quantifies the concentration of active microorganisms in the reactor. Fig. 1 shows the reactor with a UV lamp in the center

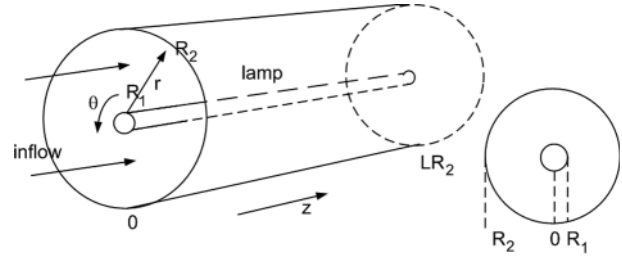


Fig. 1. Schematic overview of a UV reactor.

along the longitudinal direction. Here, R_2 is the outer radius, R_1 the radius of the lamp, and LR_2 the length of the reactor. It is convenient to work with dimensionless quantities and for that purpose all variables are scaled. As reference length-scale, we adopt R_2 . The reference velocity is denoted by u_0 for which we select the mass-average velocity, i.e., $u_0 A = Q$, where the area $A = \pi(R_2^2 - R_1^2)$, and Q is the volume-flux of fluid through the reactor in $m^3 s^{-1}$. These reference scales also define a time-scale R_2/u_0 . To finalize the scaling of the fluid flow part, we adopt the kinematic viscosity ν to quantify the so-called viscous fluxes in the Navier–Stokes equations [9].

To scale the convection-diffusion-reaction equation for the active microorganism concentration C , we adopt the same time- and length-scales, and use the average concentration at the inlet of the reactor, $\bar{C}_0$, as reference scale for the concentration. Furthermore, D denotes the microorganisms diffusion coefficient. Finally, we consider the first-order reaction mechanism as proposed in [10], using results from [3,17], and [18]. This is described by the disinfection reaction rate, which is governed by the intensity, taken as the UV intensity at the surface of the lamp at R_1 , the microorganism susceptibility factor, the monochromatic absorbance, and the geometric factor $\alpha = R_1/R_2$. The intensity $I^*(W/m^2)$ is scaled as $I = I^*/\bar{I}^*$, using the time average value of the lamp intensity as reference scale. This yields a normalized disinfection reaction rate K that is given by

$$K = \frac{I}{r} \exp(-E(r - \alpha)) = If(r). \quad (1)$$

Here, r denotes the scaled radial coordinate.

Written out in full, the scaled model equations for the fluid flow in cylindrical coordinates are given by (see [9] pp. 59–60) the continuity equation:

$$\frac{1}{r} \frac{\partial}{\partial r}(rv_r) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0 \quad (2)$$

and by the conservation of momentum:

$$\begin{aligned}
\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta \partial v_r}{r \partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} &= -\frac{\partial p}{\partial r} + \frac{1}{\text{Re}} \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r v_r) \right) + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2} \right] \\
\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta \partial v_\theta}{r \partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z} &= -\frac{1}{r} \frac{\partial p}{\partial \theta} + \frac{1}{\text{Re}} \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r v_\theta) \right) + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2} \right] \\
\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta \partial v_z}{r \partial \theta} + v_z \frac{\partial v_z}{\partial z} &= -\frac{\partial p}{\partial z} + \frac{1}{\text{Re}} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \right].
\end{aligned} \tag{3}$$

Here, $\text{Re} = u_0 R_2 / \nu$ is the Reynolds number, which quantifies the ratio of the convective forces over the viscous forces in the flow. We use cylindrical coordinates (r, θ, z) (cf. Fig. 1) with velocity components (v_r, v_θ, v_z) and dimensionless pressure p . The convection-diffusion-reaction equation for the active micro-organism concentration can be written as:

$$\begin{aligned}
\frac{\partial C}{\partial t} = & - \left(v_r \frac{\partial C}{\partial r} + v_\theta \frac{1}{r} \frac{\partial C}{\partial \theta} + v_z \frac{\partial C}{\partial z} \right) \\
& + \frac{1}{\text{Pe}} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial C}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 C}{\partial \theta^2} + \frac{\partial^2 C}{\partial z^2} \right] - \text{Da}KC,
\end{aligned} \tag{4}$$

Here, $\text{Pe} = u_0 R_2 / D$ is the Peclet number, which measures the relative importance of the convective mass transfer over the diffusive mass transfer. The Damköhler number $\text{Da} = \varepsilon \bar{T}^* \alpha R_2 / u_0$ indicates the chemical reaction rate relative to the convective mass transfer rate. It is assumed that the susceptibility factor $\varepsilon(m^2/J)$ and the scaled monochromatic absorbance E (dimensionless) are constant.

The description of the fluid flow and concentration dynamics given in (2)–(4) does not allow further analytical treatment in its general form. Therefore, we proceed with analyzing the velocity and concentration profiles under simplifying assumptions. Most importantly, these limit the applicability of the model to laminar flow conditions, as apply, e.g., to UV treatment in cider plants [6]. Further, suitable boundary conditions are chosen, together with the controlled and measured variables.

2.2. Laminar Velocity Field Model

The laminar flow assumption and the assumption that the reactor is very long imply that the fluid motion is in the z direction only, i.e., $v_\theta = v_r = 0$. This requires that the Reynolds number is sufficiently small and the flow has become fully developed before the reactor inlet. In other words, there are no entry effects, and as a consequence the velocity profile depends on r and t , and not on z nor on θ . As a result we have $\mathbf{v} = (0, 0, v_z(r, t))$, which satisfies (2) and also implies

that $\frac{\partial^2 v_z}{\partial \theta^2} = \frac{\partial^2 v_z}{\partial z^2} = 0$. Therefore, the Navier–Stokes system (2) contains only one non-trivial equation. Only the longitudinal momentum equation needs to be retained, yielding

$$\frac{\partial v_z}{\partial t} = -\frac{\partial p}{\partial z} + \frac{1}{\text{Re}} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_z}{\partial r} \right) \right]. \tag{5}$$

Taking the derivative of this equation with respect to z , and using the fact that v_z does not depend on z , yields that $\partial p / \partial z$ is independent of z . As the pressure does not depend on r or θ , the pressure derivative depends on t only. We set

$$-\frac{\partial p}{\partial z}(t) = \beta(t). \tag{6}$$

Further, we assume no-slip boundary conditions at the walls, which is the most common choice for incompressible fluid flow,

$$v_z(\alpha) = v_z(1) = 0. \tag{7}$$

This completes the laminar fluid-mechanics description of the reactor flow.

2.3. Convection-Diffusion-Reaction Equation

As the velocity field, the UV-radiation field, the initial condition, and the geometry are assumed to be independent of θ , the concentration of active micro-organisms will also not depend on θ . Hence, (3) becomes

$$\frac{\partial C}{\partial t} = -v_z \frac{\partial C}{\partial z} + \frac{1}{\text{Pe}} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial C}{\partial r} \right) + \frac{\partial^2 C}{\partial z^2} \right] - \text{Da}KC. \tag{8}$$

To complete the formulation for C , we introduce boundary conditions in the z and r -direction. First, we assume that the concentration at the inlet is well-mixed, making it a function of t , but not of r . Moreover, we assume that there is no concentration

gradient in z at the inlet. This corresponds with the absence of UV radiation for. Thus

$$\begin{aligned} C(0, r, t) &= C_0(t) \text{ for } \alpha < r < 1, \\ \frac{\partial C}{\partial z}(0, r, t) &= 0 \text{ for } z \leq 0. \end{aligned} \quad (9)$$

At the walls the velocity is zero. As mass cannot pass through the wall the logical boundary conditions for $r = \alpha$ and $r = 1$ are

$$\frac{\partial C}{\partial r}(z, r, t) = 0 \text{ at } r = \alpha \text{ and } r = 1. \quad (10)$$

As the velocity is zero at the wall, the concentration is zero at the walls as well. This completes the formulation for the dynamics of the microorganism concentration. Recapitulating, the model describing the flow and concentration inside the reactor is

$$\begin{aligned} \frac{\partial v_z(r, t)}{\partial t} &= \beta(t) + \frac{1}{\text{Re}} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_z(r, t)}{\partial r} \right) \right] \\ v_z(\alpha, t) &= v_z(1, t) = 0 \\ \frac{\partial C(z, r, t)}{\partial t} &= -v_z(r, t) \frac{\partial C(z, r, t)}{\partial z} \\ &\quad + \frac{1}{\text{Pe}} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial C(z, r, t)}{\partial r} \right) + \frac{\partial^2 C(z, r, t)}{\partial z^2} \right] \\ &\quad - \text{Da} K(r, t) C(z, r, t) \\ C(0, r, t) &= C_0(t), \frac{\partial C}{\partial z}(0, r, t) = 0, \frac{\partial C}{\partial r}(z, \alpha, t) \\ &= \frac{\partial C}{\partial r}(z, 1, t) = 0 \end{aligned} \quad (11)$$

where v_z is a function of r and t , and C a function of z , r and t . In practical UV-treatment, control is exerted with $I(t)$, the intensity of the lamp, see (1). The average concentration at the outflow is a natural monitoring parameter that needs to be controlled. Here, we define the average concentration in terms of the total outflow of microorganisms relative to the average velocity of the liquid. As in [19] it is defined by

$$\bar{C}(L, t) = \frac{\int_{\alpha}^1 C(L, r, t) v_z(r) r dr}{\int_{\alpha}^1 v_z(r) r dr}. \quad (12)$$

The control will be aimed at reducing $\bar{C}(L, t)$ below a pre-set acceptance level, by adjusting the input I . We turn to this in the next section.

2.4. Analytical solutions for special cases

To get some insight into the disinfection process, the laminar flow and corresponding concentration pro-

files are analyzed next. For a reactor with a constant flow rate, i.e., $\partial v_z / \partial t = 0$ and β constant, the solution of the differential equation for the velocity in (8) is given by

$$v_z(r) = -\frac{\beta \text{Re}}{4} \left[r^2 + \frac{1 - \alpha^2}{\ln(\alpha)} \ln(r) - 1 \right]. \quad (13)$$

This shows a characteristic quadratic profile, reminiscent of the Poiseuille profile, with logarithmic corrections arising from the UV lamp along the center of the cylinder.

Using this velocity profile, we analyze the concentration of active microorganisms next. As mentioned above, we restrict to cases in which diffusive transport is negligible compared to convection and UV irradiation, or equivalently, $\text{Pe} \gg 1$ and $\text{Pe} \gg \text{Da}$. For the particular case of UV disinfection in a cider plant, this is the case [6], and this motivates neglecting mass diffusion. By doing so, the third equation in (8) becomes

$$\begin{aligned} \frac{\partial C}{\partial t}(z, r, t) &= -v_z(r) \frac{\partial C}{\partial z}(z, r, t) - \text{Da} K(r, t) C(z, r, t) \\ C(0, r, t) &= C_0(t), \end{aligned} \quad (14)$$

which is a standard convection-reaction process. Note that with $v_z(r)$ from (13) the boundary conditions in (11) are satisfied. In view of the control design that focuses on the output concentration C at $z = L$ we may readily derive

$$\begin{aligned} C(L, r, t) &= C_0 \left(t - \frac{L}{v_z(r)} \right) \\ &\quad \exp \left(-\text{Da} \int_{t - \frac{L}{v_z(r)}}^t K(r, \tau) d\tau \right). \end{aligned} \quad (15)$$

This expresses the instantaneous solution $C(L, r, t)$ in terms of the inflow-value at a previous time $t - t^*(r)$ where the residence time $t^* = L/v_z(r)$. In addition, the accumulated effect of the UV irradiation at distance r , acquired during a time-interval t^* is expressed by the exponential. To simplify the analysis, assume that C_0 and K do not depend on t . Expression (15) then becomes

$$C(L, r) = C_0 \exp \left(-\frac{\text{Da} K(r) L}{v_z(r)} \right), \quad (16)$$

This is plotted on a logarithmic scale for different values of α in Fig. 2. The physical parameters are the same as in the nominal model that will be introduced in section 2.5. We observe that close to the boundaries $r = \alpha$ and $r = 1$ the concentration drops strongly – this is

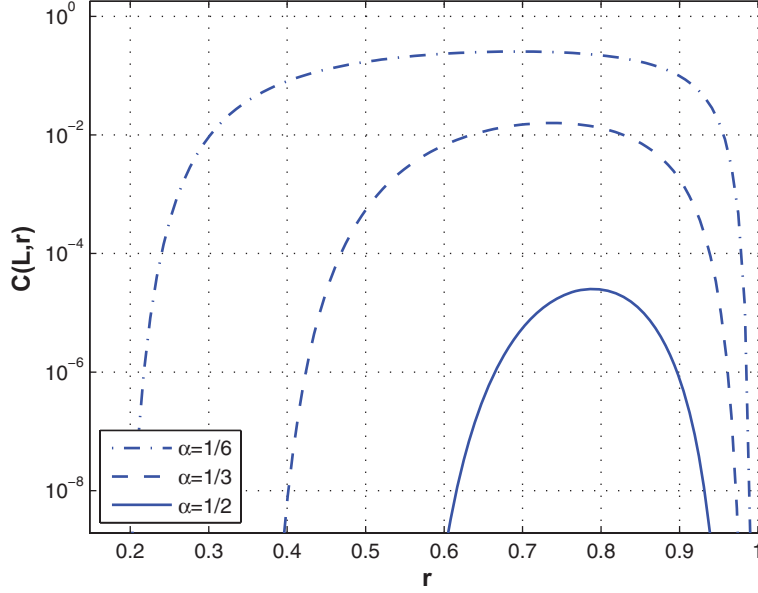


Fig. 2. Concentration profile of $C(L, r)$ for different values of α . The relevant parameter values are listed in equation (17).

associated with the very long residence time of microorganisms that enter the reactor close to a wall. Conversely, only the 'inner' region of the reactor contributes significantly to the outflow of still active microorganisms. Hence, especially for quite large values ($\alpha > 1/3$), the contribution of $C(L, r)$ to $\bar{C}(L)$ is significant only in a small part of the reactor. Expressions (9) and (12) illustrate that $\bar{C}(L)$ is influenced strongly by the smallest residence time L/v_z . A realistic value is $\alpha > 1/3$, see for example [17] where $\alpha = 0.4$ is used.

2.5. The nominal model

Using the results of the previous sections, we obtain a model for an apple cider plant. The dimensionless constants for this plant are

$$\text{Pe} = 3.5 \cdot 10^5, \text{Da} = 5.8, \text{Re} = 1.3 \cdot 10^3, \text{ and } \alpha = 0.5, \quad (17)$$

and they are determined by using parameter values specific for apple cider, [6]. In [19] all the physical constants are listed. From section 2.1 we have $\text{Da} = \varepsilon \bar{T}^* \alpha R_2 / u_0$, where $\bar{T}^*$ is the average lamp intensity. The lamp intensity is determined such that it gives a '5 log reduction' of the inlet concentration of the *Escherichia coli* 0157:H7 bacteria, i.e., $\bar{C}(L) = 10^{-5} C_0$. Furthermore, we assume that the flow rate through the reactor is constant. By section 2.4 this implies that v_z is given by (10). Combining (1), (13) and (14), we obtain the following nominal model

$$\begin{aligned} v_z(r) &= -\frac{\beta \text{Re}}{4} \left[r^2 + \frac{1 - \alpha^2}{\ln(\alpha)} \ln(r) - 1 \right] \\ \frac{\partial C(z, r, t)}{\partial t} &= -v_z(r) \frac{\partial C(z, r, t)}{\partial z} - \text{Da } I(t) f(r) C(z, r, t) \\ C(0, r, t) &= C_0(t). \end{aligned} \quad (18)$$

In this model, we regard $I(t)$ as the control input, and $C_0(t)$ as the disturbance. The measurement $\bar{C}(L, t)$ is given by (12).

3. Model Reduction for the Nominal Model

In the nominal model we see that our control input $I(t)$ gets multiplied with the state $C(z, r, t)$, and so it is a non-linear model. Furthermore, it is a distributed parameter model. Using linearization and reduction via balancing [25], we will obtain a first order linear model for which the control design is easy.

Since the nominal model is given in the scaled variables, we have as reference values $C_0^{\text{ref}} = 1$, $I^{\text{ref}} = 1$. For these reference values, the solution of (18) gives the reference concentration profile

$$\begin{aligned} C^{\text{ref}}(z, r) &= C_0^{\text{ref}} \exp\left(-\frac{\text{Da} f(r) z}{v_z(r)}\right) \\ &= \exp\left(-\frac{\text{Da} f(r) z}{v_z(r)}\right). \end{aligned} \quad (19)$$

Defining $I(t) = I^{\text{ref}} + I^{\text{var}}(t)$, $C_0(t) = C_0^{\text{ref}} + C_0^{\text{var}}$, and $C(z, r, t) = C^{\text{ref}}(z, r) + C^{\text{var}}(z, r, t)$, and linearizing (18)

around the reference concentration, gives the following p.d.e. for C^{var}

$$\begin{aligned} \frac{\partial C^{var}(z, r, t)}{\partial t} &= -v_z(r) \frac{\partial C^{var}(z, r, t)}{\partial z} \\ &\quad - \text{Da} I^{var}(t) f(r) C^{ref}(z, r) \\ &\quad - \text{Da} f(r) C^{var}(z, r, t) \end{aligned} \quad (20)$$

$$C^{var}(0, r, t) = C_0^{var}(t).$$

We use this to obtain the transfer functions from input and disturbances to the state. Let denote the Laplace transform of the variable x . Then after Laplace transformation with respect to time, equation (14) becomes

$$\begin{aligned} s \hat{C}^{var}(z, r, s) &= -v_z(r) \frac{\partial \hat{C}^{var}(z, r, s)}{\partial z} \\ &\quad - \text{Da} \hat{I}^{var}(s) f(r) C^{ref}(z, r) \\ &\quad - \text{Da} f(r) \hat{C}^{var}(z, r, s) \end{aligned} \quad (21)$$

$$\hat{C}^{var}(0, r, s) = \hat{C}_0^{var}(s).$$

The output depends on the concentration at $z = L$, see (12), and we find after inserting (19)

$$\begin{aligned} \hat{C}^{var}(L, r, s) &= \frac{\text{Da} f(r)}{s} \left[\exp\left(-\frac{s + \text{Da} f(r)}{v_z(r)} L\right) \right. \\ &\quad \left. - \exp\left(-\frac{\text{Da} f(r) L}{v_z(r)}\right) \right] \hat{I}^{var}(s) \\ &\quad + \exp\left(-\frac{s + \text{Da} f(r)}{v_z(r)} L\right) \hat{C}_0^{var}(s) \\ &= \tilde{G}_1(r, s) \hat{I}^{var}(s) + \tilde{G}_2(r, s) \hat{C}_0^{var}(s). \end{aligned} \quad (22)$$

The transfer function $\tilde{G}_1(r, s)$ is a non-rational function in s . Hence it is not straightforward to find a balanced realization of it. Therefore, we approximate $\tilde{G}_1(r, s)$ by a rational function. For fixed r we use the first order Padé approximation, i.e.,

$$\begin{aligned} \tilde{G}_1(r, s) &\approx \frac{\tilde{G}_1(r, 0)^2}{-\frac{\partial \tilde{G}_1(r, 0)}{\partial s} s + \tilde{G}_1(r, 0)} \\ &= \frac{-2\text{Da} f(r) C^{ref}(L, r)}{s + \frac{2v_z(r)}{L}}. \end{aligned} \quad (23)$$

The transfer function $\bar{\tilde{G}}_1$ that connects $I^{var}(t)$ to $\bar{C}^{var}(L, t)$ is obtained by taking the mean concentration, see (12)

$$\bar{\tilde{G}}_1(s) = \frac{\left(\int_{\alpha}^1 \frac{\text{Da} f(r) C^{ref}(L, r)}{s + \frac{2v_z(r)}{L}} v_z(r) r dr \right)}{\left(\int_{\alpha}^1 v_z(r) r dr \right)}. \quad (24)$$

A state space realization of this is given by the parametrized o.d.e.

$$\begin{aligned} \frac{dx(r, t)}{dt} &= a(r)x(r, t) + b(r)I^{var}(t) \\ \bar{C}^{var}(L, t) &= \frac{1}{\bar{v}r} \int_{\alpha}^1 x(r, t) v_z(r) r dr, \end{aligned} \quad (25)$$

where

$$\begin{aligned} a(r) &= -2\frac{v_z(r)}{L}, b(r) = -2\text{Da} f(r) C^{ref}(L, r) \\ \text{and } \bar{v}r &= \int_{\alpha}^1 v_z(r) r dr. \end{aligned} \quad (26)$$

As the state x at time t is a function of r , this is an infinite-dimensional system. Next, we approximate this by a finite-dimensional one. Therefore we discretize $x(r, t)$ with respect to r . We introduce the uniform grid for $r, r_j = \alpha + j\Delta r$. The new state vector is $x(t) = [x(r_1, t) \dots x(r_n, t)]^T$, and the model becomes

$$\begin{aligned} \frac{dx}{dt}(t) &= \mathbf{A}x(t) + \mathbf{B}I^{var}(t) \\ \bar{C}^{var}(L, t) &= \mathbf{C}x(t) \end{aligned} \quad (27)$$

with

$$\begin{aligned} \mathbf{A} &= \text{diag}[a(r_1), \dots, a(r_n)] \\ \mathbf{B} &= [b(r_1), \dots, b(r_n)]^T \\ \mathbf{C} &= \frac{1}{\bar{v}r} \left[\frac{v(r_1)r_1}{2}, v(r_2)r_2, \dots, v(r_{n-1})r_{n-1}, \frac{v(r_n)r_n}{2} \right]. \end{aligned}$$

Matrix $\mathbf{C}$ is obtained by trapezoidal integration. For this model we calculate the Hankel singular values, see Fig. 3. When the first singular value dominates the others, the model may be truncated down to one state [25]. Since the first state is much larger than the second by a factor 10^3 , we may truncate (27) down to the first order differential equation

$$\frac{d\bar{C}^{var}(L, t)}{dt} = A_{red} \bar{C}^{var}(L, t) + B_{red} I^{var}(t). \quad (28)$$

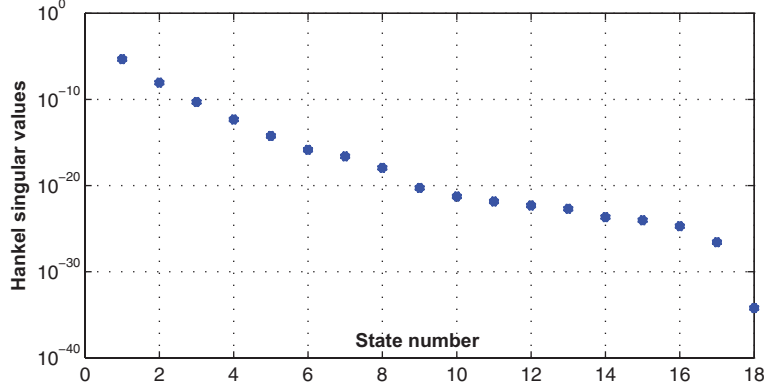


Fig. 3. The Hankel singular values of the laminar flow model without diffusion.

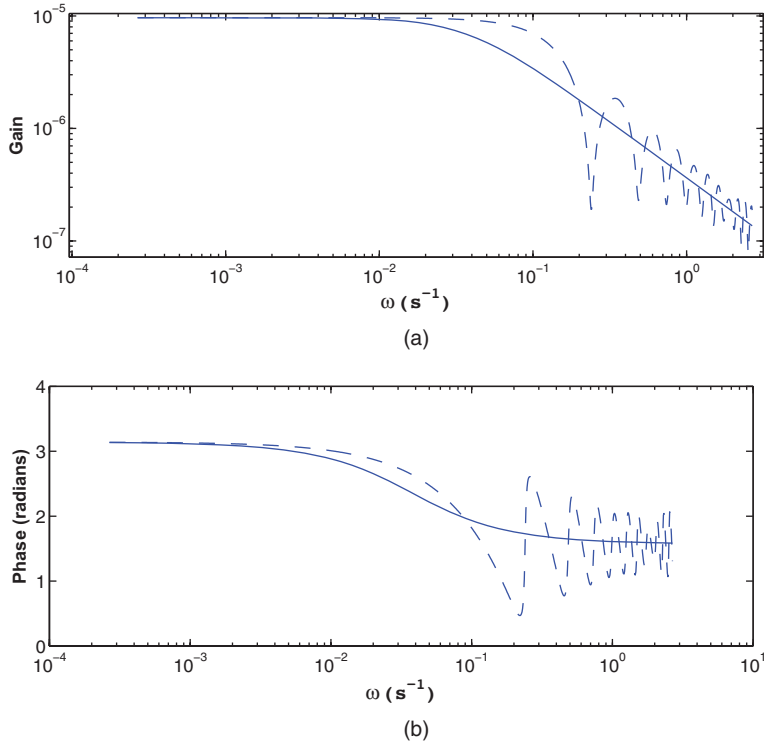


Fig. 4. Gain $|\tilde{G}_1(s)|$ and phase $\arg(\tilde{G}_1(s))$ (in radians) plot of $G_1(s)$ in model (22) (dashed line), and of model (28) (solid line). Here, $s = i\omega$.

Using the constants of our nominal model, steps (18)–(28) lead to $A_{red} = -1.2$, $B_{red} = -7.3 \cdot 10^{-3}$.

To see whether all approximations have not discarded any essential dynamics, the Bode plot of (28) is compared to that of system $\tilde{G}_1(s)$ in (22) with output (12), see Figure 4. It is clear that (28) is a crude approximation of (22) with output (12) for large ω . More importantly, it captures the first order dynamics of the model, characterized by the static gain and the time constant. Hence, we use (28) as our model for controller design. Using (22), (12), and (28) we have the following model in the s -domain

$$\begin{aligned} \overline{C^{var}}(L, s) &= \frac{B_{red}}{s - A_{red}} \hat{I}^{var}(s) + \frac{1}{v\bar{r}} \int_{\alpha}^1 \tilde{G}_2(r, s) v_z(r) r dr \hat{C}_0^{var}(s) \\ &:= G_1(s) \hat{I}^{var}(s) + \tilde{G}_2(s) \hat{C}_0^{var}(s). \end{aligned} \quad (29)$$

The amplitude Bode plot of $\tilde{G}_2(s)$ is flat. The approximation by a pure time delay

$$\tilde{G}_2(s) \approx G_2(s) = c \exp(-\tau s), \quad (30)$$

with $c = \tilde{G}_2(0)$, and $\tau = -\frac{1}{c} \frac{d\tilde{G}_2}{ds}(0)$ is a very accurate approximation, see [19] for details.

4. Controller Design

For the approximate model in the s -domain (see (29) and (30))

$$\overline{C}^{var}(L, s) = G_1(s) \hat{I}^{var}(s) + G_2(s) \hat{C}_0^{var}(s) \quad (31)$$

we design a controller. This controller will be tested on our nominal model (18) with output equation (12).

Since no dynamic properties of $C_0^{var}(t)$ are known, and since $G_2(s)$ is a pure time delay, we regard the term $G_2(s) \hat{C}_0^{var}(s)$ as delayed white noise. This is a general noise model. Since the dynamics of I^{var} to $\overline{C}^{var}$ is of first order, we design a simple PI controller. We choose

$$K(s) = \frac{A_{red} A_{red} - s}{B_{red} s}. \quad (32)$$

This controller has the property that the crossover frequency of the complementary sensitivity function

equals that of $G_1(s)$. Fig. 5 shows the control loop schematically. For more details on the design, we refer to [19].

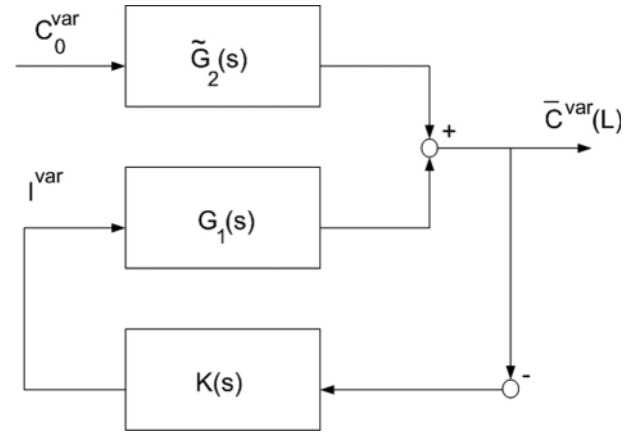


Fig. 5. Schematic representation of the controlled model.

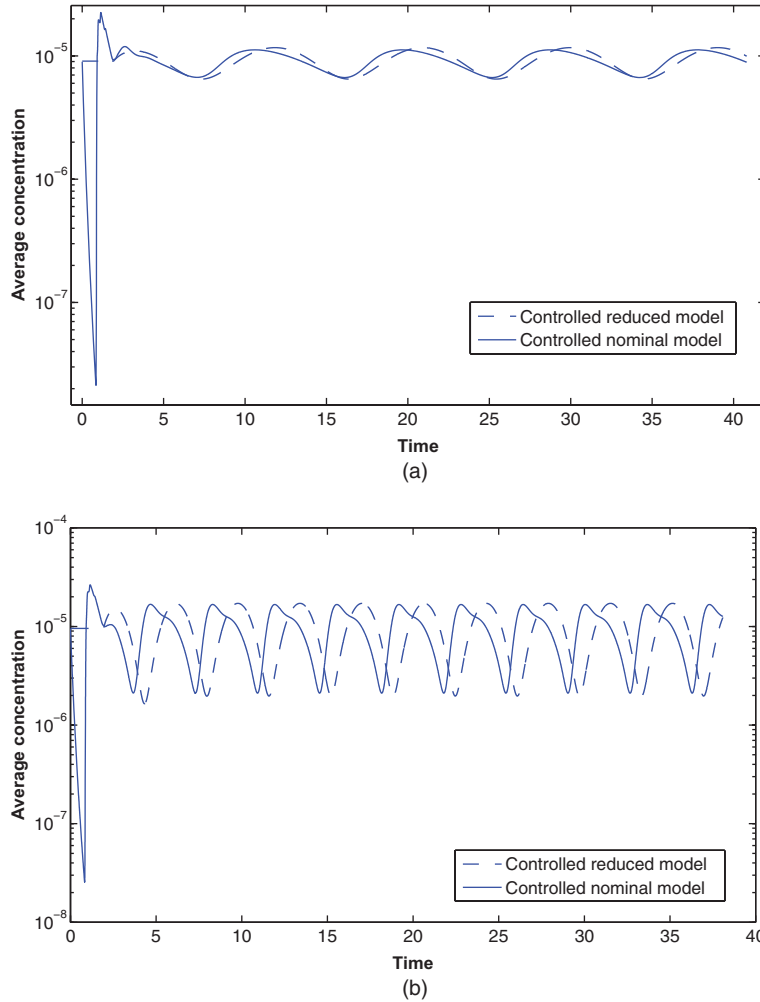


Fig. 6. The dynamics of the average concentration $\overline{C}(L, t)$ and the reduced model (28), and the nominal model (18). Left: $A = 0.5$ and $F = 0.8$. Right: $A = 0.9$ and $F = 2$.

4.1. Simulations

Simulation studies are conducted to see whether the reduced model (28) is a sufficiently accurate approximation of the nominal model (18). This is done by connecting the controller to both models. The controlled systems are solved numerically by a forward Euler method in time, and an upwind scheme in space, see [19] for details. To visualize the difference in output dynamics clearly, $C_0(t)$ is disturbed by $A \sin(Ft)$, with a significant amplitude of $A = 0.5$ and a frequency of $F = 0.8$. We assume that this is a large and fast disturbance for a cider plant. For this frequency, the error in gain in Figure 4 is relatively large. The left hand plot in Figure 6(a) shows the concentration $\bar{C}(L, t)$ for the reduced model (28) and the nominal model (18). Initially, the difference between the two models is large. This is caused by the transient behavior of the nominal model. The small initial concentration near the inlet is decreased further by the lamp, resulting in a smaller and smaller $\bar{C}(L)$. After the transient dynamics has gone, the outputs of the nominal and the reduced model match very well. Further increase of A and F leads to larger errors. To illustrate this, the amplitude and the frequency of the input disturbance are increased to 0.9 and 2 respectively. The dynamics is shown in Fig. 6b. The higher amplitude causes a larger approximation error in (14), and the high frequency causes a larger phase error in Fig. 4.

5. Conclusions

A basic model was developed, describing the fluid and concentrations dynamics inside an annular disinfection plant. As was shown by the analysis of the Hankel singular values, and the Bode plots, that the essential dynamics of an apple cider plant can be approximated accurately by a first order model. Consequently, a simple classical model-based controller with excellent properties can be designed. This was further confirmed by a simulation study with significant disturbances.

Up to the balancing, the formulas are analytical and contain all the physical properties of the nominal model. This allows a clear analysis, like in Section 2.4. For more complex models, for example with an irregular geometry, or without discarding mass diffusion as was done here, the reduction steps as well as the model simulations can become numerically more involved, since an analytical linearization may not be possible. These practical drawbacks leave the door open for alternative design methods, such as nonlinear

control. The next fundamental steps would be to validate the basic model experimentally, and to check the controller performance in a real-life situation.

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