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Switching Control for a Class of Non-linear Systems with an Application to Post-harvest Food Storage

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For a class of nonlinear systems with switching input, a controller is designed which achieves tracking to a desired state. The stability of the open- and closed-loop system is studied under the assumption of a common Lyapunov function. The results are motivated by and applied to an experimentally validated model of a bulk storage room for food products. It is shown that for this model a controller with excellent robustness and performance properties can be designed.

Keywords: Controller constraints, Food processing, Nonlinear dynamics, Switching control, Tracking

1. Introduction

Many systems are described by a non-linear model of the type

$$\frac{dx}{dt}(t) = A(u(t))x(t) + B(u(t)), \qquad t \geq 0. \qquad (1)$$

where x denotes the state, and u denotes the control. A and B are matrix- and vector-valued functions, respectively. As an example of such a system one may think of a bi-linear system, i.e., a system in which the control is multiplied by (one of) the state variables. Motivated by our application to the storage of food, we assume that the control u can only take two values. Hence depending on the value of u, the system (1) becomes an (affine) linear system, and by changing u we change the system dynamics. In other words, we switch between two (affine) linear systems. The control objective is to find a switching sequence such that $x(t)$ converges to a small interval around the desired state x_{opt}. As we switch between two systems, perfect tracking will not be possible in general. To satisfy our control objective, we first split the time axis into intervals of the length τ_f, where τ_f is chosen such that if $x(0) = x_{\text{opt}}$, then $x(t)$ lies within a small interval around x_{opt} for all $t \in [0, \tau_f]$ and for both values of the input. Next, in every discrete time interval, we switch from u_1 to u_2 to steer the state to x_{opt}. Hence, we switch at most once in each discrete time interval, in contrast to for example [4, 6, 15].

We characterize the set of reachable states, S, which forms a one-dimensional curve in the state space. Under the assumption that the system has a common Lyapunov function, we show that a switching sequence can be found such that the state converges to x_{opt}. In switching control literature, the assumption of a common Lyapunov function is considered to be a strong assumption, see e.g. [7]. For our class of systems this assumption is very natural. As can be seen from (1) we have one system operating with different sets of physical parameters. Because this study is motivated by a physical system, energy can be chosen as a natural Lyapunov function. Changing u will only imply that the rate of the energy decay becomes differently, but it will still decay. For our application of storage of food, the total heat serves as a common Lyapunov function.

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Climate control is essential in post-harvest food storage. For maintaining optimal product quality, the most important controlled variables are temperature, humidity, CO_2 concentration, and ethylene concentration inside the storage room. The most common control inputs are ventilation, cooling, heating, and (de)humidification. The storage room can be ventilated in two ways: ventilation with outside air, or via recirculation. Forced ventilation is realized by fans. Cooling and heating is effectuated by outside air ventilation or by a heat exchanger. The corresponding mathematical models have complex dynamics due to the airflow in combination with heat and moisture exchange. Some control inputs are of a discrete nature. Forced air ventilation, for example, is usually realized by a fan that is switched on and off, which motivates our study on models as in (1). Typical (model-based) control strategies that have been developed for food storage applications are model predictive control (MPC) and fuzzy control. In [5] and [14], MPC algorithms were used for the temperature and humidity control of a bulk storage room with outside air ventilation. In [1, 2, 9] fuzzy controllers have been designed and tested on either a mathematical model or on experimental data. In [10] a fuzzy controller was developed for fruit storage, using neural networks. Further, in [8] a PI controller was designed for CO_2 and O_2 concentrations, and was tested experimentally. In [3] a sensor-based control law for a bulk storage room that was ventilated with outside air was proposed. In general, the advantages of MPC are that the control algorithm is based on a mathematical model and objective function with or without constraints, and that the applicability extends to rather complicated models. A major drawback is that controller dynamics have to be solved by demanding on-line numerical computations. Fuzzy and sensor-based controllers are practically easy implementable, but have little mathematical background, and hence controller performance is hard to guarantee. Another design approach is by switching adaptive control. Stabilizing adaptive controllers are designed in [15, 4] for a large class of nonlinear multi-input multi-output systems and for a larger class of multi-input single-output systems in [6], with less restrictive assumptions. In these studies, and in contrast to this article, the control input is switched between two functions that depend continuously on the system states. The objective of our article is to demonstrate the analysis and synthesis of a switching controller on fixed intervals and to show its performance, in simulation, on an experimentally validated model of the climate in a food storage room.

2. Stability Analysis and Control

As mentioned above, we assume the following for the system (1). The continuous time axis $[0, \infty)$ is divided into discrete time intervals with length τ_f, and the input u can take two values u_1 and u_2. The control problem is to determine the duration of both inputs. We assume, without loss of generality, that at the start of each time interval $u = u_1$. In the time-interval $[n\tau_f, (n+1)\tau_f]$ the input is switched from u_1 to u_2 at time $n\tau_f + \tau_n$, with $0 \le \tau_n \le \tau_f$. This gives the following piecewise linear system

$$\frac{dx}{dt}(t) = A(u_1)x(t) + B(u_1), \quad t \in [n\tau_f, n\tau_f + \tau_n), \tag{2}$$

$$\frac{dx}{dt}(t) = A(u_2)x(t) + B(u_2), \quad t \in [n\tau_f + \tau_n, (n+1)\tau_f) \tag{3}$$

with $x(n\tau_f + \tau_n^-) = x(n\tau_f + \tau_n^+)$, and $x(n\tau_f^-) = x(n\tau_f^+)$. From now on, the notation $A(u_1) = A_1$, $B_1 = B(u_1)$, etc. is used, i.e., the subscript denotes the relation with the input. The goal is to design a controller that steers x to the desired state x_{opt} by adjusting the switching time in each time interval, i.e., by choosing the sequence τ_n. Although we want to steer $x(t)$ to x_{opt}, it is easy to see that our switching system will never have a steady state. Therefore, we will steer $x(n\tau_f)$ to the desired state. As the original aim is to steer $x(t)$ to x_{opt}, we assume that τ_f is chosen such that $x(t)$ differs only a little bit from $x(n\tau_f)$ for $t \in [n\tau_f, (n+1)\tau_f]$. We remark that this assumption plays no role in our analysis. However, in applications it plays a role in the choice of τ_f. Given the systems (2) and (3) with their time constants, the calculation of the τ_f can be done off-line. Furthermore, the error between $x(t)$ and $x(n\tau_f)$ can be set off-line.

Throughout this article, we assume that A_1 and A_2 have their eigenvalues in the open left-half plane, i.e., $\sigma(A_1), \sigma(A_2) \subset \mathbb{C}^-$. Furthermore, we assume that there is a common (quadratic) Lyapunov function, i.e., there exists a $P > 0$ such that

$$A_1^T P + PA_1 < 0 \quad \text{and} \quad A_2^T P + PA_2 < 0. \tag{4}$$

For this P define the Lyapunov function $V(x) = x^T Px$. Then, from the Lyapunov inequalities (4) we have for all $t > 0$ and $x_0 \neq 0$ that

$$V(e^{A_1 t}x_0) < V(x_0) \quad \text{and} \quad V(e^{A_2 t}x_0) < V(x_0). \tag{5}$$

First, we obtain the "discrete" steady states of (2) and (3). That is, we characterize those states for which with a constant switching sequence, τ_n, we have that $x(n\tau_f) = x_0$ for all $n \in \mathbb{N}$. Because if $x(\tau_f) = x(0)$, then with $\tau_2 = \tau_1$ we have that $x(2\tau_f) = x(\tau_f) = x(0)$. This we can repeat for $n = 3, 4$, etc., and hence we can solve this problem by only looking at the first time interval.

Lemma 2.1: *Consider the system (2)–(3), and assume that (4) holds. Given the state x_{opt}, there exists a switching* *time* $\tau_0 := \tau \in [0, \tau_f]$ *such that* $x(0) = x(\tau_f) = x_{\text{opt}}$ *if and only if* x_{opt} *and* τ *satisfy*

$$x_{\text{opt}} = \left[I - e^{A_2(\tau_f - \tau)} e^{A_1 \tau}\right]^{-1} \left[e^{A_2(\tau_f-\tau)} \times (e^{A_1\tau} - I)A_1^{-1}B_1 + (e^{A_2(\tau_f-\tau)} - I)A_2^{-1}B_2\right]. \tag{6}$$

Proof: It is easy to see that $x(\tau)$ is given by

$$x(\tau) = e^{A_1\tau}x_{\text{opt}} - \left[I - e^{A_1\tau}\right]A_1^{-1}B_1, \quad \text{and}$$

$$x(\tau_f) = e^{A_2(\tau_f-\tau)}x(\tau) - \left[I - e^{A_2(\tau_f-\tau)}\right]A_2^{-1}B_2.$$

If $x(\tau_f)$ equals x_{opt}, then

$$x_{\text{opt}} = e^{A_2(\tau_f-\tau)}e^{A_1\tau}x_{\text{opt}} - e^{A_2(\tau_f-\tau)}\left[I - e^{A_1\tau}\right]A_1^{-1}B_1 - \left[I - e^{A_2(\tau_f-\tau)}\right]A_2^{-1}B_2. \tag{7}$$

Hence, we obtain equation (6), provided one is not an eigenvalue of $e^{A_2(\tau_f-\tau)}e^{A_1\tau}$. If $z \neq 0$ is such that $z = e^{A_2(\tau_f-\tau)}e^{A_1\tau}z$, then we also have that $V(z) = V(e^{A_2(\tau_f-\tau)}e^{A_1\tau}z)$. However, from (5), we find

$$V(e^{A_2(\tau_f-\tau)}e^{A_1\tau}z) < V(e^{A_1\tau}z) < V(z).$$

Hence, we obtain a contradiction. So one is not an eigenvalue of the matrix $e^{A_2(\tau_f-\tau)}e^{A_1\tau}$, and thus $I - e^{A_2(\tau_f-\tau)}e^{A_1\tau}$ is invertible. We conclude from (7) that (6) holds. □

If the assumption of a joint Lyapunov function does not hold, then it is not hard to construct an example in which $I - e^{A_2(\tau_f-\tau)}e^{A_1\tau}$ is not invertible, and so (6) becomes meaningless.

The set of all steady states is denoted by S, i.e.,

$$S = \{x_{\text{opt}} \in \mathbb{R}^n \mid \text{there exists a } \tau \in [0, \tau_f] \text{ s.t. (6) holds}\}. \tag{8}$$

From (6) and (8) we see that S is the range of the compact interval $[0, \tau_f]$ under a continuous function, and so S is a compact subset of the state space. In Fig. 1, we show the set S for $A_1 = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $A_2 = \begin{bmatrix} -2 & -3 \\ 3 & -0.1 \end{bmatrix}$, $B_2 = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$, and $\tau_f = 1$.

In order to show that the state at discrete time instances $n\tau_f$ converges to a "ball" around S, we introduce the system dynamics at the discrete times $n\tau_f$. This (time-varying) discrete-time system is given by, see also (7)

$$x((n+1)\tau_f) = e^{A_2(\tau_f-\tau_n)}e^{A_1\tau_n}x(n\tau_f) - e^{A_2(\tau_f-\tau_n)} \times \left[I - e^{A_1\tau_n}\right]A_1^{-1}B_1 - \left[I - e^{A_2(\tau_f-\tau_n)}\right]A_2^{-1}B_2. \tag{9}$$

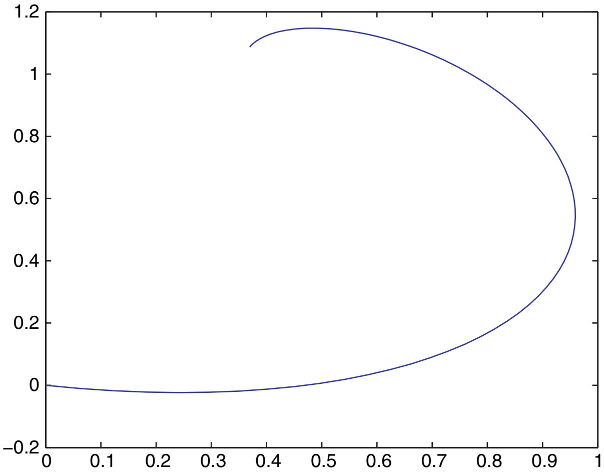

Fig. 1. The set S in $\mathbb{R}^2$.

We write this as

$$x((n+1)\tau_f) = \mathcal{A}(\tau_n)x(n\tau_f) + \mathcal{B}(\tau_n). \tag{10}$$

with

$$\mathcal{A}(\tau) = e^{A_2(\tau_f-\tau)}e^{A_1\tau},$$

$$\mathcal{B}(\tau) = e^{A_2(\tau_f-\tau)}\left[I - e^{A_1\tau}\right]A_1^{-1}B_1 - \left[I - e^{A_2(\tau_f-\tau)}\right]A_2^{-1}B_2.$$

Note that Lemma 2.1 gives the equilibrium points of the system (10).

Lemma 2.2: *Let P be the solution of (4). Then there exists a positive $\delta < 1$ such that for all $\tau \in [0, \tau_f]$ there holds*

$$\mathcal{A}(\tau)^T P \mathcal{A}(\tau) - \delta P \leq 0. \tag{11}$$

Proof: For $\tau \in [0, \tau_f]$, we have that, see (5) and (10)

$$x^T\mathcal{A}(\tau)^T P\mathcal{A}(\tau)x = V(\mathcal{A}(\tau)x) = V(e^{A_2(\tau_f-\tau)}e^{A_1\tau}x) \leq V(e^{A_1\tau}x) \leq V(x) = x^TPx,$$

where at least one of the inequalities must be strict. Combining this with the compactness of $[0, \tau_f]$ and of the unit ball in our state space, we have that there exists an $\varepsilon > 0$ such that

$$\max_{\tau\in[0,\tau_f], \|x\|=1} \left[x^T\mathcal{A}(\tau)^T P\mathcal{A}(\tau)x - x^TPx\right] = -\varepsilon.$$

This implies that (independent of τ)

$$\mathcal{A}(\tau)^T P\mathcal{A}(\tau) - P \leq -\varepsilon I. \tag{12}$$

Now we choose a $\delta < 1$ such that $(1-\delta)P \leq \varepsilon I$. Substituting this in (12) gives (11). □

Now we define a "ball" around S. Let $\|x\|_P$ be defined as

$$\|x\|_P = \sqrt{x^T P x} = \sqrt{V(x)}. \tag{13}$$

Because P is strictly positive, this defines a norm. With respect to this norm we define the ball around S. Let $r \geq 0$, then

$$D(r) = \left\{x \in \mathbb{R}^n \mid \min_{x_{\rm opt} \in S} \|x - x_{\rm opt}\|_P \leq r\right\}. \tag{14}$$

Hence, we have that $D(0) = S$. Because S is compact, we have that $\sup_{x,y\in S} \|x - y\|_P = \max_{x,y\in S} \|x - y\|_P < \infty$.

Theorem 2.3: *Let $\alpha = \max_{x,y\in S} \|x - y\|_P$, and let $\rho = \frac{\sqrt{\delta}\alpha}{1-\sqrt{\delta}}$. For any $r > \rho$, for any initial condition and for any switching sequence $\{\tau_n, n \in \mathbb{N}\}$, the solution of (10) will lie within $D(r)$ after finitely many steps. Furthermore, the set $D(\rho)$ is an invariant set.*

For any switching sequence $\{\tau_n, n \in \mathbb{N}\}$ and any initial condition the solution of (2)–(3) is bounded. If τ_n is kept constant to τ, then $x(n\tau_f)$ converges to the state $x_{\rm opt}$ given in (6).

Proof: Let x_0 be an initial condition, and assume that

$$\min_{y\in S} \|x_0 - y\|_P = r_0. \tag{15}$$

Let τ_1 be the first switching time. By the definition of S, we know that there exists an element in S such that (6) holds for $\tau = \tau_1$. We denote this element of S by $x_{{\rm opt},\tau_1}$. By definition we have that $\mathcal{A}(\tau_1)x_{{\rm opt},\tau_1} + \mathcal{B}(\tau_1) = x_{{\rm opt},\tau_1}$. So for $x(\tau_f)$ we have that, see (10),

$$x(\tau_f) - x_{{\rm opt},\tau_1} = \mathcal{A}(\tau_1)(x_0 - x_{{\rm opt},\tau_1}).$$

Using (11) and (13) we find that

$$\|x(\tau_f) - x_{{\rm opt},\tau_1}\|_P^2 \leq \delta\|x_0 - x_{{\rm opt},\tau_1}\|_P^2. \tag{16}$$

Let y_0 be an element of S such that (15) is attained, i.e., Then we have that

$$\|x_0 - x_{{\rm opt},\tau_1}\|_P \leq \|x_0 - y_0\|_P + \|y_0 - x_{{\rm opt},\tau_1}\|_P \leq r_0 + \alpha. \tag{17}$$

Hence we see that

$$\begin{aligned} r_1 &:= \min_{y\in S} \|x(\tau_f) - y\|_P \leq \|x(\tau_f) - x_{{\rm opt},\tau_1}\|_P \\ &\leq \sqrt{\delta}\|x_0 - x_{{\rm opt},\tau_1}\|_P \leq \sqrt{\delta}[r_0 + \alpha]. \end{aligned} \tag{18}$$

If $r_0 > \rho$, then $r_1 < r_0$. We can repeat the above and as long as $r_n > \rho$ we have that $r_{n+1} < r_n$. Hence in finitely many steps r_n gets below any number which is larger than ρ. This proves the first assertion.

From the above proof we also see that if $x(0) \in D(\rho)$ then $x(\tau_f) \in D(\rho)$. So we have the invariance of this set.

The state x restricted to the time interval $[n\tau_f, (n+1)\tau_f]$, can be seen as the solution of the system

$$\begin{aligned} \dot{x}(t) &= A_1 x(t) + B_1, \qquad t \in [0, \tau_n) \\ \dot{x}(t) &= A_2 x(t) + B_2 \qquad t \in [\tau_n, t_f] \end{aligned}$$

with initial condition $x(n\tau_f)$. Because $x(n\tau_f)$ is uniformly bounded, the time interval is compact, and as the inputs are bounded, we conclude that $x(t)$ is bounded. The last assertion follows easily from the uniform stability of $\mathcal{A}(\tau)$, see (12). □

Using this theorem, we see that it is not difficult to steer the state to the desired state, $x_{\rm opt}$. We can simply choose τ_n be constant and equal to the τ corresponding to $x_{\rm opt}$, see (6). However, one may want to speed up the convergence, and/or to make it more robust against (unmodeled) disturbances. Because Theorem 2.3 tells us that we will get close to S whatever τ_n is, it seems natural to linearize (10) around $x = x_{\rm opt}$ and $\tau = \tau_{\rm opt}$, where $\tau_{\rm opt}$ is the τ corresponding to $x_{\rm opt}$, see (6).

When we linearize (10) around this equilibrium point, we obtain the system

$$x_{\rm var}(n+1) = A_d x_{\rm var}(n) + B_d \tau_{\rm var}(n), \tag{19}$$

where the subscript 'var' denotes the variation from the equilibrium state. Hence $x(n\tau_f) = x_{\rm opt} + x_{\rm var}(n)$. The matrices are given by

$$\begin{aligned} A_d &= \mathcal{A}(\tau_{\rm opt}) \\ B_d &= -A_2 x_{\rm opt} - B_2 + \mathcal{A}(\tau_{\rm opt}) A_1 x_{\rm opt} \\ &\quad - e^{A_2(\tau_f - \tau_{\rm opt})} e^{A_1 \tau_{\rm opt}} B_1. \end{aligned} \tag{20}$$

Now many design methods are open. In the following session, we design a PI-like controller for the time-sequence.

3. Application to Food Storage

In this section, the controller design and the stability analysis from the previous section are applied to a model of a bulk storage room for harvested food products. For more details we refer the reader to [11, 12, 13]. The storage room model is schematically drawn in Fig. 2. Air is circulated by a fan, and the air is cooled down by a heat exchanger right below the fan. The air enters the bulk at the bottom, and consequently the products at the top will be the warmest.

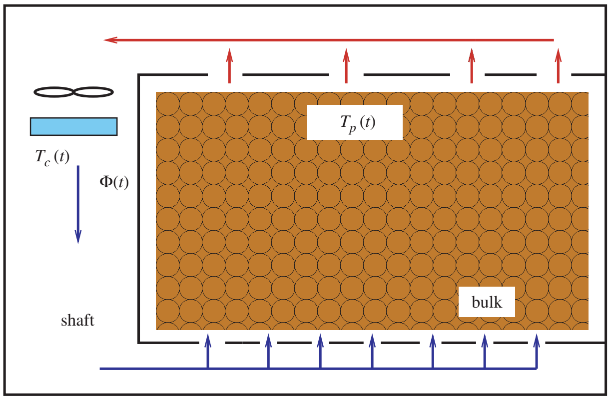

Fig. 2. Schematic representation of a bulk storage room.

Table 1. Numerical key parameter values

A_1	$-2 \cdot 10^{-5}\, 1/s$	A_2	$-2 \cdot 10^{-8}\, 1/s$
B_1	$6.6 \cdot 10^{-3}\, K/s$	B_2	$8.1 \cdot 10^{-6}\, K/s$
A_d	$1.0 - 3 \cdot 10^{-4}$	B_d	$-1.2 \cdot 10^{-4}\, K/s$
τ_f	$600\, s$	τ_{opt}	$12.2\, s$
$T_{p,\text{opt}}$	$280\, K$		

The nominal model describing the product temperature in the top of the bulk is

$$\frac{dT_p}{dt}(t) = A(\Phi(t))T_p(t) + B(\Phi(t), T_c(t)), \tag{21}$$

with $T_p(t)$ the product temperature at the top of the bulk. For the expressions of A and B we refer to [13, 11]. Note that (21) is a scalar system. We assume that the temperature of the cooling device is constant. The most important physical parameters that correspond to a storage room with a bulk of potatoes are the temperature of the cooling device $T_c = 275\, K$, the fluxes generated by the fan in on and off position $\Phi_1 = 1\, m^3/s$, and $\Phi_2 = 0.001\, m^3/s$, the height $4\, m$, the floor area $5\, m^2$, and the shaft volume $10\, m^3$. The rest of the parameter values is listed in [13, 11]. In Table 1 the numerical values of the key parameters in this article are given. We used $A_k = A(\Phi_k)$, $B_k = B(\Phi_k, T_c)$, $k = 1, 2$.

For scalar systems the result obtained in the previous section can be sharpened, see [11] for the proofs.

Theorem 3.1: *For the scalar system there holds*

1. *The set S is given by $[-A_1^{-1}B_1, -A_2^{-1}B_2]$.*
2. *For any $\gamma > 0$, the solution of (10) will lie within $[-A_1^{-1}B_1 - \gamma, -A_2^{-1}B_2 + \gamma]$ after finitely many time-steps.*
3. *The set S is invariant.*

The controller input is the product temperature at the top of the bulk, $T_p(t)$. The optimal switching time corresponds to $T_p(t) = T_{p,\text{opt}}$. Realistic disturbances in the air temperature are caused by open doors, heat leakage through the walls, etc. For mathematical simplicity, we assume that the disturbances in air temperature occur in the vicinity of the heat exchanger, and that they therefore act on the system as the temperature of the cooling element T_c, see (21). We assume that disturbances in T_c have the same qualitative influence on T_p as disturbances in τ, and so we regard it as disturbances in τ. To cancel this disturbance and to achieve good tracking, we choose the PI-based controller, see [11] for more details,

$$\zeta(n+1) = -\frac{(A_d - 1)^2}{B_d} T_{p,\text{var}}(n) + \zeta(n) \tag{22}$$

$$\tau_{\text{var}}(n) = \frac{A_d - 1}{B_d} T_{p,\text{var}}(n) + \zeta(n). \tag{23}$$

with $T_{p,\text{var}}(n) = T_p(n) - T_{p,\text{opt}}$. Because the switching time must lie between 0 and τ_f, we will apply $\tau_n = \tau_{\text{opt}} + \tau_{\text{var}}(n)$ provided this lies between these bounds. When this does not hold, we apply the following rules,

$$\text{If } \tau_{\text{var}}(n) + \tau_{\text{opt}} > \tau_f, \quad \text{then } \tau(n) = \tau_f$$

$$\text{If } \tau_{\text{var}}(n) + \tau_{\text{opt}} < 0, \quad \text{then } \tau(n) = 0.$$

Now we study the stability of the closed-loop system. This is done for the linearized and for the original system. For more details we refer to chapter 4 of [11]. Using (10), (22) and (23), the closed-loop system can be written as

$$\begin{aligned} T_{p,\text{var}}(n+1) &= A_d T_{p,\text{var}}(n) \\ &\quad + \left[B_d + \varepsilon(T_{p,\text{var}}(n), \tau_{\text{var}}(n))\right] \tau_{\text{var}}(n) \end{aligned} \tag{24}$$

$$\begin{aligned} &\tau_{\text{var}}(n+1) \\ &\quad = \left[A_d + \frac{A_d - 1}{B_d} \times \varepsilon(T_{p,\text{var}}(n), \tau_{\text{var}}(n))\right] \tau_{\text{var}}(n). \end{aligned} \tag{25}$$

where ε contains linear and higher order terms of $T_{p,\text{var}}(n)$ and $\tau_{\text{var}}(n)$. Hence, for the linearized system this term is zero. Because $|A_d| < 1$, it is easy to see that the linearized closed-loop system is stable. We will now investigate whether the same holds for the system (24) and (25). For this the following lemma is useful. The proof is easy, as we have a scalar differential equation.

Lemma 3.2: *Consider equation (25). Let*

$$\begin{aligned} \Omega = \{(T_{p,\text{var}}, \tau_{\text{var}}) \mid\ & T_{p,\text{var}} + T_{p,\text{opt}} \\ & \in [-A_1^{-1}B_1, -A_2^{-1}B_2] \textit{ and } \tau_{\text{var}} + \tau_{\text{opt}} \in [0, \tau_f]\}. \end{aligned}$$

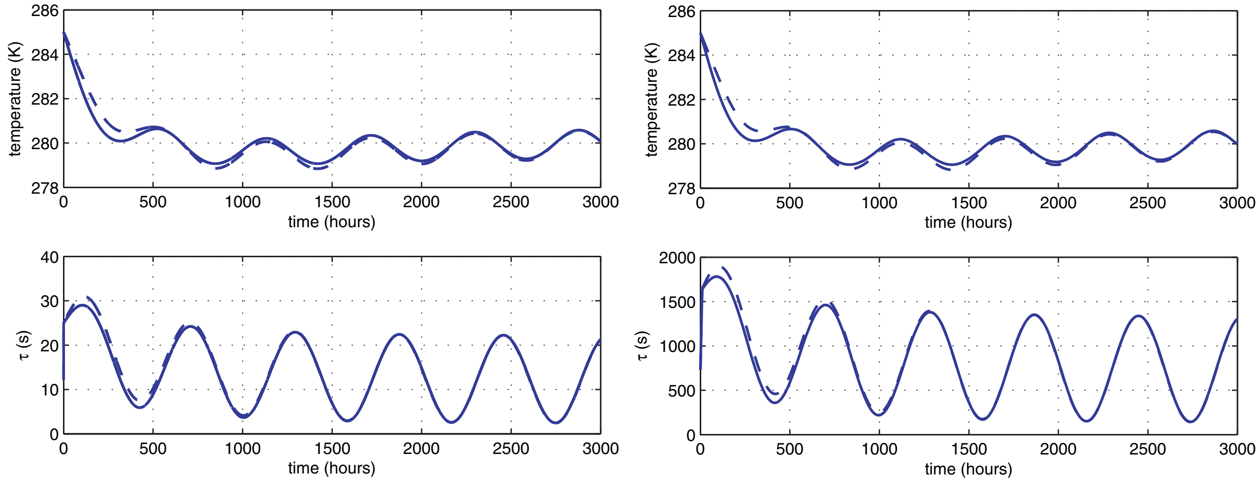

Fig. 3. T_p (upper) and τ (lower) of the linearized controlled system (dashed line), and the nominal controlled system (dotted line). Left: $\tau_f = 10$ minutes, and right: $\tau_f = 10$ hours.

If

$$\sup_{(T_{\text{var}},\tau_{\text{var}})\in\Omega} \left| A_d + \frac{A_d - 1}{B_d}\varepsilon(T_{p,\text{var}}, \tau_{\text{var}}) \right| < 1, \quad (26)$$

then (25) is asymptotically stable.

Note that the stability of (25) implies also the stability of (24), and so our closed-loop system is stable when (26) holds.

Ignoring the higher order terms in ε, and using the parameters of Table 1, we found

$$\varepsilon(T_{p,\text{var}}, \tau_{\text{var}}) = -2.4\cdot 10^{-5}\, T_{p,\text{var}} + 1.4\cdot 10^{-9}\tau_{\text{var}}. \quad (27)$$

We have that $A_d = 1 - 3\cdot 10^{-4}$, and $B_d = 6.5\cdot 10^{-3}$, so the stability criterion of Lemma 3.2 $|A_d + \frac{A_d-1}{B_d}\varepsilon| < 1$ becomes $|1 - 3\cdot 10^{-4} + 4.6\cdot 10^{-2}\varepsilon| < 1$, which is fulfilled if $|\varepsilon| < 64.6$. We have that $0 < \tau_{\text{var}} < \tau_f = 600$, and that T_p will converge to the range $(-A_1^{-1}B_1 - \gamma, -A_2^{-1}B_2 + \gamma)$ for any γ, see Theorem 3.1. As for our case $(-A_1^{-1}B_1, -A_2^{-1}B_2) = (275.1, 398.2)$, we have that $|T_{p,\text{var}}| < 123.1$ for any choice of $T_{p,\text{opt}}$. Altogether, $T_{p,\text{var}}$ and τ_{var} cannot grow large enough to violate condition (26), and hence the system is asymptotically stable according to Lemma 3.2.

The interval length τ_f is chosen in a rather conservative way, and therefore we do the same analysis for a very large interval length, namely $\tau_f = 10$ hours. This results in

$$\varepsilon(T_{p,\text{var}}, \tau_{\text{var}}) = -2.3\cdot 10^{-5}\, T_{p,\text{var}} + 1.4\cdot 10^{-9}\tau_{\text{var}}. \quad (28)$$

For this new τ_f we have that $A_d = 1 - 1.8\cdot 10^{-2}$, and $B_d = 6.4\cdot 10^{-3}$, so the stability criterion (26) becomes $|1 - 1.8\cdot 10^{-2} + 2.8\varepsilon| < 1$, which is fulfilled if $|\varepsilon| < 6.4\cdot 10^{-3}$. Clearly, the margin becomes much smaller. However, the bounds on τ_{var} (i.e., $-\tau_{\text{opt}} \le \tau_{\text{var}} \le \tau_f - \tau_{\text{opt}}$) prohibit the second term of ε to grow large, and we have $\varepsilon \approx -2.3\cdot 10^{-5}\, T_{p,\text{var}}$. The system is stable when $|T_{p,\text{var}}| < 278\ K$, which in practice will always be the case. Hence, we conclude that the system is asymptotically stable.

Now we analyze the loss of performance due to the linearization. This is done by connecting controller (22) and (23) to the linearized system (19) and to the nominal system (21). The differences in $T_p(t)$ and $\tau(t)$ should give an indication whether any essential dynamics are discarded. Further, a heavy input disturbance d will be added, such that the system dynamics become clearly visible. The initial product temperature was set uniform at $285\,K$, while the optimal product temperature is $280\,K$. The input disturbance is $d = a\sin(\omega t)$, with $a = 10\,s$, and $\omega = 3\cdot 10^{-6}\,Hz$.

The dynamics of τ and T_p are shown in Fig. 3a. For both controlled systems the dynamics of T_p and τ are more or less the same, indicating that the linearization error between (21) and (19) does not discard any essential dynamics. Even when initially the product temperature differs considerably from the linearization point of $280\,K$, the differences are small. Furthermore, the controller seems to perform quite well under these large input disturbances. For various frequencies of d similar results were obtained. Fig. 3b shows the results for the large interval of $\tau_f = 10$ hours. The amplitude of the disturbance is scaled with τ_f. The results are very similar, which indicates that the size of τ_f has no considerable influence on the performance robustness of the controller. For different amplitudes and frequencies, the linearization was also found to be very accurate.

4. Conclusions

In this article, we have shown that switching between two stable systems with a common Lyapunov function gives independently of the switching times a stable system. That is, the solutions all converge to a compact invariant set. The steady states are elements of this invariant set, and so a controller design based on the linearization around a steady state is likely to work.

As an example, a controller was designed and connected to a temperature model of a bulk storage room. For controller design, the original (or nominal) model was linearized. It was shown that the stability cannot be jeopardized by the linearization error. Numerical simulations show that under large input disturbances the nominal and the approximated system have similar dynamics in T_p and τ. This also holds for different disturbance amplitudes and frequencies, indicating that the linearization does not discard any essential dynamics. Hence, a controller with excellent properties can be designed for the experimentally validated bulk storage room model.

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