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Tracking Underground Metro Cars with Low-Cost Acceleration Sensors

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Abstract. Subways and other rapid transit systems are marked symbols of the modern metropolis. As a transdisciplinary service, accurately and safely positioning and tracking the metro trains helps the passengers to plan their travels and provides the operators with auxiliary information about the trains to enhance the metro system's resilience. However, many general-purpose positioning technologies, such as Global Navigation Satellite Systems (GNSS) and Wi-Fi signals, do not apply to the situations of underground metro trains. In this paper, we propose a two-stage framework for automatic real-time tracking of metro cars implemented only with low-cost accelerometers, saving the expense for complicated infrastructures. In the off-line stage, reference maps are developed for station-to-station track sections using the onboard acceleration data. To handle the missing data and uncalibrated consumer-grade sensors, Gaussian process regression (GPR) is adopted to denoise and interpolate the online acceleration readings, followed by the application of the Kalman filter algorithm to track the cars in real-time with the help of the reference map. We tested the proposed system in Wuhan Metro Line 2, and the results showed that our system achieved an error below 5% in positioning.

Keywords. Positioning and tracking, acceleration sensors, Gaussian process regression

Introduction

Since its birth in 1863 in London, England, the subway has become the most efficient and convenient public transport means for commuters in many metropolitan areas in the world. And the trend of its rapid growth sustains in recent years along with urbanization in the emerging economies. For example, in 2019 alone, China added over 800 kilometers to the mileage of its subway systems. There are many critical applications for the real-time positioning and tracking of metro trains, including control and positioning of repair trains (and other engineering vehicles), autonomous locomotives, collision warning systems, diagnosis and maintenance of abnormal defects, etc. In the meantime, the passengers are keen to know their whereabouts after getting on board for many possible reasons. However, even given its role to ensure the safe and efficient operations of the metro systems, the real-time positioning and tracking of metro vehicles remain a challenging problem.

Conventionally, most vehicular positioning systems rely on global navigation satellite systems (GNSS) such as the global positioning system (GPS) to provide the basis for positioning, navigation, and timing (PNT) services. Regardless of the advantage of high precision, high-speed, and availability in all-weather, the GPS-based positioning

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suffers in complex environmental conditions (e.g. tunnels and forests). However, a majority of metro lines contain a large portion of tunnels. As an alternative, the inertial measurement unit (IMU) also serves as the fundamental sensory technique for mobile objects' positioning. In particular, the high-precision IMUs provide proper localization and good autonomy at an extra cost. The advances in micro-electro-mechanical systems (MEMS) brought the trade-off between performance, size, and cost, making the acceleration sensors integrated into today's mainstream smartphones. Due to their affordability, we may install consumer-grade IMUs on the metro cars to collect the data needed in the positioning applications.

Data fusion of multiple sensor signals proves to improve the positioning performance by compensating for the drawbacks of a single sensor. Autonomous vehicles usually take advantage of a collection of sensors, like GPS receivers, frontal and lateral video cameras, LiDARs, and many more [1]. In addition to heterogeneous sensors, a priori knowledge can also be integrated to achieve higher positioning accuracy. Various forms of maps and visual landscapes give critical information to adjust the sensor's prediction. Kalman filter algorithm and particle filter algorithm are two widely used data fusion techniques. Aiming at reducing the uncertainty of error covariance and state noise in the system, the Kalman filter integrates the model-based prediction and the state measurement. [2] introduced a sensor fusion method for unmanned surface vehicle navigation based on fuzzy adaptive Unscented Kalman filter (UKF). [3] surveys the research on train positioning with the fusion of GNSS, INS, and Doppler radar signals. Also, surveillance images help to detect the position of a vehicle and can be integrated into the tracking system [4]. Recent research showed the application of some deep learning models in vehicle positioning, and [5] specifically examined the problem of inertial system drift with the help of a deep neural network.

Unlike other solutions that depend on the fusion of several different types of sensors (for example, making available extra information in case one type of sensor fails or severely degrades), in this investigation, we develop a metro car tracking and positioning framework using only low-cost MEMS acceleration sensors. The acceleration readings collected by the low-cost IMUs usually contain significant noise, are compromised by nonlinear gains, and suffer zero-shift errors. To solve these problems, we collect the data from multiple sensors for redundancy to alleviate the negative impacts in the absence of expensive labor-demanding sensor calibration. More specifically, we face two major challenges in the tracking of metro cars: first, the sensory readings are susceptible to unreliable wireless link and might be missing from time to time; second, it is impractical to obtain the real-time speed and distance observations in a realistic operating metro system, and thus we can only find sparse feedback information at the stations. The main contributions of our work include: by placing Bluetooth Low Energy (BLE) beacons at the station platforms, we develop a framework to track the relative displacement of metro cars between stations, using only the accelerometer readings; to handle the problem of lacking real-time speed and distance measurement, we design a simple reference map learned by optimization with the observed acceleration data and the fixed terminal conditions on velocity and displacement. The fusion of IMU data and map matching improves the positioning performance.

The rest of this paper is organized as follows. In Section 1, we briefly review the recent development in the related areas; and then in Section 2, we introduce the denoising and tracking algorithms along with the creation of the reference map. Section 3 discusses the setup of the experiments, data acquisition, and experimental results. Section 4 concludes the current research and provides remarks on our future investigation.

1. Related Work

A positioning and tracking system allows its users to estimate an object's location within a constrained space, with the help of a variety of sensors including those embedded in smartphones and other mobile devices. In typical outdoor environments, GPS has found extensive success in locating stationary and moving objects [6]. In a challenging environment where the GPS signal does not work well, or some special needs have to be satisfied, IMU provides complementary or alternative solutions, without the need for expensive infrastructure. Since MEMS has made remarkable progress recently, IMU becomes an important component in many positioning applications. But the IMU is susceptible to measurement noise, external disturbances, and needs integration over time to estimate the speed and distance states, so the system relying on IMU alone suffers from accumulative errors. Therefore, people exploit particular forms of learned knowledge together with the IMU measurement to jointly improve the positioning results [7]. Visual features, magnometers, and maps help correct the estimation drifts of the IMU [8, 9, 10]. [11] proposed to track the surface train by integrating the BeiDou navigation satellite system and an inertial navigation system and taking odometer and track map matching to compensate for the INS degradation and the blocked BDS signals.

Indoor positioning scenario extends the selection of sensor technologies: Wi-Fi, BLE, UWB, light, and ultrasound signals all bear the underlying information for distance estimation. While some parameters like Angle of Arrival and Time of Arrival need a specialized device to analyze, Wi-Fi and Bluetooth Received Signal Strength Indicators can be detected by regular smartphones [12, 13]. Fingerprinting maps the radio signal strengths to the coordinates of a location. Once the offline radio map gets established, matching the observed RSSIs with the entries in the fingerprints generates the online position prediction.

Both Gaussian process regression and Kalman filter are Bayesian approaches that learn the uncertainty from the temporal samples and derive the optimal decision based on statistical assumptions [14, 15]. In [16], the authors adapted GPR to denoise the CT images taking advantage of the temporal labels. While the non-parametric nature of the Gaussian process makes GPR apply to plenty of functions, the computational load grows fast when the data size increases. [17] introduced a recursive version of GPR, enabling the online regression incorporating new data. By iterative model-based forecast and data assimilation, the Kalman filter facilitates the integration of the information from different sources, and thus applies to the fusion of sensor data and other evidence reflecting the states of interest. The classic Kalman filter theory makes assumptions on the linearity of the system dynamic and Gaussian distribution of the errors. As the Kalman filter variants, extended Kalman filter (EKF) and error-state Kalman filter (ESKF) propagate the state distribution by the first-order linearization of the nonlinear system [18]. On the other hand, the Unscented Kalman Filter (UKF) is a derivative-free alternative only using a deterministic sampling approach to represent the state distributionwith a set selected sample points [19, 2]. Various forms of the Kalman filters are extensively used in multi-sensor fusion for positioning and tracking applications. To overcome the problem of high nonlinearity, in [5], the authors proposed a multi-sensor fusion algorithm for underwater vehicle localization by a radial basis function (RBF) neural network augmented ESKF.

Though people have made solid progress in tracking and locating mobile objects, most solutions in the related research rely on multiple advanced sensors to obtain reliable measurements and sophisticated site survey for the reference fingerprints.

2. Proposed Method

2.1. System Architecture

Table 1 summarizes the workflow of the proposed underground metro car tracking system. The system's hardware consists of the accelerometers, Bluetooth Low Energy (BLE) beacons, and the onboard computer equipped with a BLE probe. Several IMU/BLE devices are attached to the interior walls of a metro car. Each IMU sensor samples the three-dimensional accelerations at a predetermined frequency, and the built-in BLE unit then broadcasts the readings. On the platform of each station, separate BLE beacons are fixed to identify the station.

Instead of estimating the train's absolute position through complex measurement infrastructure, in this study, we only attempt to determine the train's location relative to its previous full stop at a station using the low-cost accelerometers. To simplify, we treat a subway train as a longitudinal rigid body and further approximate a car as a particle. Then the moving train is constrained by a one-dimensional track. Therefore, we only need to solve a one-dimensional dynamic problem, i.e., finding the trains' displacement and velocity. Given the subway track details, this one-dimensional solution can be converted to a three-dimensional earth frame, and then the train can be marked with the absolute coordinates. In the data acquisition setup, the sensors periodically send the three-axis acceleration data through BLE connections to a smartphone or a computer for processing. In a realistic situation, the passengers might be blocking the wireless signal's propagation paths in a crowded metro car. Because the BLE broadcast is prone to interference, there is no guarantee that the probe receives every message transmitted by the BLE beacons. Therefore, besides the measurement noise and variable zero shifts, we also need to handle the data-missing problem. Assuming the underlining properties about acceleration's smoothness and continuity, as shown in Figure 1, we apply Gaussian process regression (GPR) to interpolate in case of missing data. Additionally, we adopt the Kalman filter algorithm to handle the inertial drifts and estimate the train's states by look up the speed and displacement references in a learned map.

Table 1. Metro car online positioning algorithm

Inputs	Offline reference maps for all subway sections
Step 1	Section identification: read platform beacon's ID, find the corresponding section
Step 2	Model-based prediction: use the current GPR result as input to the dynamic model, update the state estimates
Step 3	Map-matching: use the segment of historical and current data to search for the closest reference point, return the matched result
Step 4	Fusion: apply Kalman filter to update the state prediction and parameters
Outputs	Predicted state values

As stated before, we deploy just a simplified type of IMUs (without gyroscopes) in the positioning and tracking of metro cars to save cost. Since the sensors only measure the three-axis accelerations at low precision, different sensors may disagree on the estimates of velocity and displacement as the accumulative errors grow over time. To obtain a better system state estimation, we need to introduce an independent feedback mechanism to provide additional evidence on the train's location. The metro operator has to comply with a strict protocol to serve the public, and therefore a train properly running between two stations stays in three modes: accelerating, maintaining, decelerating. Each stage presents a similar pattern that will be repeatedly followed in the same subway section. We take advantage of these patterns to find the train's mode for the improvement of state prediction. However, since it is unrealistic to survey the running trains for state annotations, we derive the reference patterns by optimizing the functionals satisfying the terminal conditions as well as being in line with the observed data.

2.2. Gaussian process regression and signal preprocessing

Gaussian process regression is a machine learning method using nonparametric models based on strict statistical theory instead of specific domain knowledge. Using Gaussian process regression for interpolation, not only can we predict the optimal acceleration value at each time instant, but we can determine the uncertainty of the data as well. As required by car tracking, we need to estimate the longitudinal acceleration value $a(k)$. First, a set of data measured by the IMUs is selected. Then, this data set is taken as the training set of Gaussian process regression, to denoise the original signal and interpolate for the missing ones.

The critical component in Gaussian process regression is the covariance among variables. The most common forms of covariance functions include Gaussian kernel, linear kernel, and periodic kernel. In this paper, we use the Gaussian kernel functions in the following form:

$$\kappa(x_i, x_j) = \sigma^2 \exp\left(-\frac{||x_i - x_j||_2^2}{2l^2}\right),$$

where parameters σ and l controls the shape of the curve fitting the observed data points, in particular, the scaling factor l influences the curve's smoothness.

Gaussian distribution can be used to estimate the mean value and covariance, and the mean value is taken as the predicted longitudinal acceleration value at each moment, and the covariance is taken as the uncertainty.

$$f(x) \sim \mathcal{N}(\mu(x), \kappa(x, x))$$

where $\mu(\cdot)$ represents the mean function, returning the mean value of each dimension; $\kappa(\cdot, \cdot)$ is the covariance function, representing the correlation of data points.

2.3. System dynamic model

Let $s(k)$, $v(k)$, and $a(k)$ represent the displacement, velocity, and acceleration of the metro car respectively at time instant k; let ω_s and ω_v represent the system noise contained in the displacement and velocity at time k; and denote the sampling period T. The state can be modeled in the vector form:

$$\begin{bmatrix} s(k) \\ v(k) \end{bmatrix} = \begin{bmatrix} 1 & T & \frac{T^2}{2} \\ 0 & 1 & T \end{bmatrix} \begin{bmatrix} s(k-1) \\ v(k-1) \\ a(k-1) \end{bmatrix} + \begin{bmatrix} \omega_s(k-1) \\ \omega_v(k-1) \end{bmatrix}$$

We rewrite the state equation as,

$$X(k) = \Phi(k, k-1)X(k-1) + Ba(k-1) + W(k-1)$$

where the state vector $X(k) = [s(k), v(k)]^T$, $\Phi(k, k-1)$ is the state transition matrix from time point $k-1$ to instant k. Given the current state and the measured acceleration value, we may derive the next state prediction using the system model.

2.4. Reference map generation and online matching

Since there is no direct measurement available for the car's velocity and displacement in between two stations, we have to find an additional estimation of $s(k)$ and $v(k)$ independent of the ones predicted by the model. Moreover, for the same reason, we cannot take fingerprinting-like site survey on a running train. Alternatively, we generate a reference state-acceleration map for the subway section between two consecutive stations using an optimization scheme.

Let $A = \{a(k)|0 \leq k \leq K\}$ be the measured acceleration data in a section, we know that the state starts at $[0,0]^T$ and the velocity goes back to 0 after a train comes to a full stop. In our system setup, the BLE beacons installed on the platforms give us a clear indicator of a station's identity. Denote the unknown reference states as $Z = \{z(k) = [s(k), v(k)]^T | 0 \leq k \leq K\}$. Hence we have the terminal conditions $z(0) = [0,0]^T$ and $z(k) = [dist, 0]^T$, where $dist$ is the known distance between two stations. Within all functionals Z, we require the reference to hold a few important properties: smoothness, conforming to the observation and meeting the terminal conditions.

Consequently, we find the values Z^* for by optimizaing the constrained quadratic form:

$$\min \sum_k ||z(k) - x(k)|| + \alpha||z(k) - z'(k)||$$

$$s.t. \qquad z(0) = [0,0]^T, z(K) = [dist, 0]^T$$

where $x(k)$ is the state propagated by the model using A, $z'(k)$ is the Gaussian weighted average of the reference states in a specified time interval, and parameter α balances the importance of data term and smoothness term. This quadratic problem can be solved with the simple matrix operations.

We project the data in A into a d-dimensional delayed coordinate phase space P to make the data points $p(k) = [a(k), a(k-1), ..., a(k-d+1)]^T$ spread in that space. The movement of the train is divided into three portions along the time axis: speed-up, gentle change, and slow-down. The center of each portion is calculated then. In the online stage, we first find the coarse match of the measured data by searching the shortest distance to the portion centers; subsequently, we refine the match by the shortest distance to a phase space point within the selected portion. The state in Z^* corresponding to that point is returned as the state estimation. Now we have established a reference map for the section as a mapping $P \rightarrow Z^*$.

In practice, the two-step reference map lookup requires little computation and storage because a subway section contains the observation data sampled at low frequency in the period lasting only a couple of minutes.

2.5. Kalman filter and data fusion

Kalman filter is a recursive linear estimator. Relying on the periodic observations of the state, it continuously estimates the state value changes over time. The recursive steps involved in the Kalman filter algorithm of distance and speed estimation include: predicting the new state using the dynamic model and the previous state; then using the reference map match result to correct the prediction for the optimal estimation of the next state; finally updating the parameters in the dynamic state equation.

Then Kalman filtering can be expressed as the following process:

(1) Temporal updating

$$X(k,k-1) = \Phi(k,k-1)X(k-1)$$
$$P(k,k-1) = \Phi(k,k-1)P(k-1)\Phi^T(k,k-1) + \Gamma Q(k-1)\Gamma^T$$

(2) Measurement updating

$$X(k) = X(k,k-1) + K(k)(Z(k) - HX(k,k-1))$$
$$K(k) = P(k,k-1)H^T(HP(k,k-1)H^T + X(k,k-1))$$
$$P(k) = (I - K(k)H)P(k,k-1)$$

where, $X(k,k-1)$ represents the predicted value of the state based on information available at time $k-1$, $K(k)$ is the Kalman gain, and $P(k,k-1)$ represents the error covariance with respect to $X(k,k-1)$; $P(k-1)$ is the estimation error variance matrix at time k; H is the sensor measurement matrix, I stands for the identity matrix, and $Z(k)$ represents the reference map matching values at instant k.

The positioning and tracking routine works as follows: given the initial state $X(0)$ of the metro car and the arbitrarily set initial error $P(0)$, the optimal state estimation $X(k)$ is achieved with the update of Kalman gain $K(k)$ based on the sensor measurement. As mentioned earlier, the consumer-grade low-cost sensors may have different uncalibrated zero shifts, therefore, we deploy multiple sensors at the same time, and take the average value of the online GPR processed reults as the acceleration input.

3. Experiment Results

We tested the tracking system in Wuhan Metro Line 2. Wuhan Metro Line 2 is the first metro line crossing the Yangtze River via a tunnel in China. Wuhan Metro Line 2, starting from Tianhe Airport Station and ending at the Fozuling station, has a length of 60.8km, with 38 stations in total. The number of daily average passengers is over 100,000.

Four low-cost MEMS accelerometers with built-in BLE transmitters were attached to the walls inside a car. A laptop computer connected with a signal receiver (probe) was used to collect the acceleration data. Both accelerometers and BLE units were commercial off-the-shelf products. In the experiments, three accelerometers were installed in the front, rear, and middle of the car on the same side, and the fourth one was installed in the middle of the car on the other side. The tests were carried out in 24 sections (between 25 stations). Throughout the experiments, we set the sampling frequency of the accelerometer to 1 Hz. To verify the effectiveness of this method, we conducted field tests in Wuhan Metro Line 2 and placed the beacon on the platforms of all stations of line 2 to identify the stations. The field tests were performed between December 2020 and March 2021 at different times of the day, and on different days in a week. In the actual experiments, our probe might fail to receive all data from a single beacon, but the redundancy in multiple sensors ensured at least one measurement be successfully received per second.

Figure 1 displays the GPR smoothing result for a sensor's measurements in a section. The stages were partitioned by the end and the start of the steepest slope representing the beginning of decelerations. In Figure 2, we can find that the averaging outcome of the multiple sensors' readings has almost corrected the zero drifts. The coarse match of the reference map depends on the comparison of distances from the current IMU data to the centers of learned modes. By projecting to the high-dimensional space, each point in the phase space represents a segment of acceleration history.

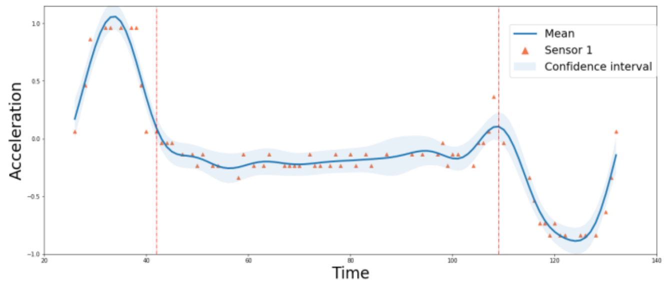

Figure 1. GPR result for a single sensor (acceleration in m/s^2, time in second)

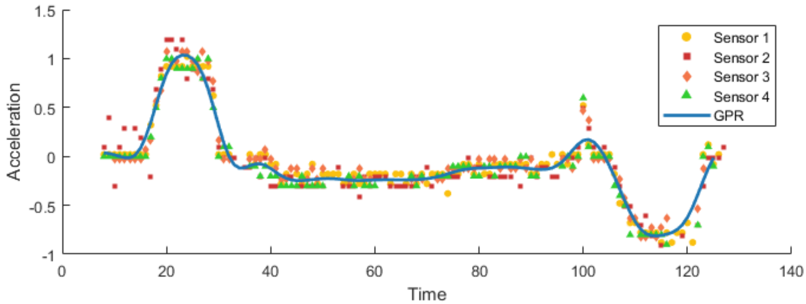

Figure 2. Sensor measurments and the GPR result (acceleration in m/s^2, time in second)

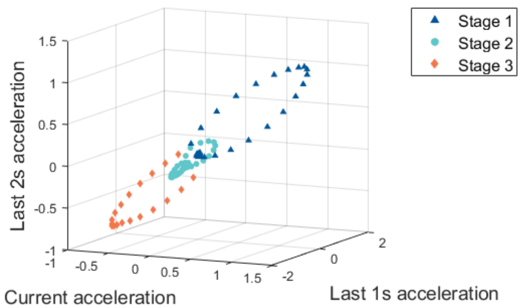

Figure 3. Phase space representations of points in different stages (only showing 3 dimensions)

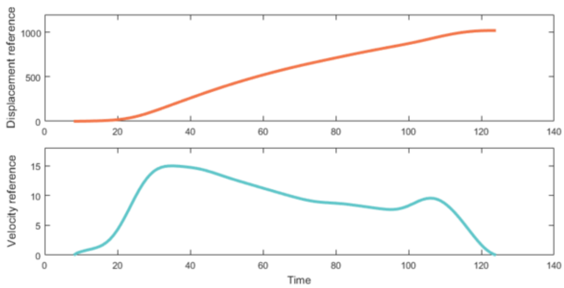

Figure 4. Reference displacement (m) and velocity (m/s)

Figure 3 shows that the points in different modes spread over a big area except for those in the speed-maintaining stage. In Figure 4 are the reference maps for speed and distance. There was a fluctuation in periods for the trains running between two stations, as found in our tests.

As illustrated in Figure 5, the model-based prediction depending only on IMU drifts severely over time. The fusion of IMU and reference map matching helps contain the estimate errors.

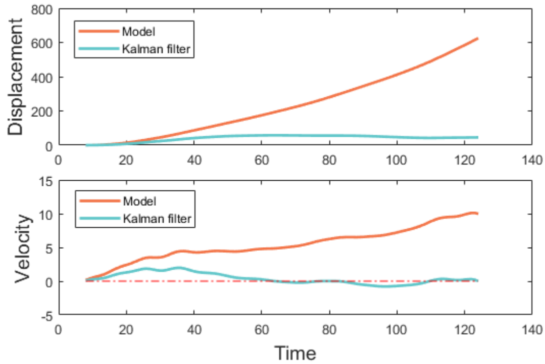

Figure 5. Prediction errors of displacement (m) and velocity (m/s)

Table 2. The predicted states at the end of each section

Section		1	2	3	4	5	6	7	8
v (m/s)	**Predicted**	0.03	0.26	0.09	0.15	0.06	0.19	0.28	0.08
s (m)	**Actual**	1021	1999	1603	2038	1059	1487	1368	817
	Predicted	975	1957	1582	2066	1050	1464	1315	815

Section		9	10	11	12	13	14	15	16
v (m/s)	**Predicted**	0.05	0.65	0.43	0.25	0.12	0.05	0.23	0.19
s (m)	**Actual**	1009	1317	1442	794	1613	951	1238	1418
	Predicted	982	1251	1395	800	1577	936	1202	1386

Section		17	18	19	20	21	22	23	24
v (m/s)	**Predicted**	0.01	0.23	0.16	0.01	0.33	0.39	0.44	0.16
s (m)	**Actual**	966	1168	930	966	3292	897	1543	946
	Predicted	953	1112	905	953	3205	889	1499	922

In the experiments, we randomly picked the data collected from one trip as the training set to learn the reference map and tested the proposed online tracking algorithm with the rest of the data. The cross-validation results listed in Table 2 indicate that the data fusion framework presented a distance estimation within a 5% error range, and the predicted velocity was close to 0 when the train stopped (with RMSE of about 0.4m/s).

4. Discussion and Conclusions

In this work, we proposed a framework for tracking underground metro cars, which can be extended into similar scenarios like a coal mine. Tracking and positioning metro trains are the foundation for many location-based services, including assisting the decision-making for the metro operators. Real-time positioning requires a balance among several factors, most importantly, cost and performance. In the proposal, we only deployed the consumer-grade low-cost acceleration sensors and BLE beacons in cars and on platforms. To confront the accumulative errors introduced by IMU and the dynamic model, we designed a reference map that took advantage of the train's moving patterns in a between-station track section. By applying a simple Kalman filter to integrate the model-IMU prediction and the map-matching outcome, we achieved a positioning accuracy at the error below 5% on arriving at the next stop. In the future, we will focus on improving the dynamic model and refining the online reference map for better tracking results.

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