Fuzzy Systems and Data Mining VII
A.J. Tallón-Ballesteros (Ed.)

doi:10.3233/FAIA210213

A Novel Possibilistic Variance of Trapezoidal Intuitionistic Fuzzy Number

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Abstract. Intuitionistic fuzzy number as the extension of fuzzy number has better capability to model uncertain data in scientifical engineering problem. In this paper, we present a novel possibilistic variance for trapezoidal intuitionistic fuzzy number (TIFN) based on possibility theory. We also discuss some notes on possibilistic variance of TIFN and get the important relationships between the possibilistic covariance and possibilistic variance of TIFNs.

Keywords. Intuitionistic fuzzy number, possibilistic mean, possibilistic variance.

1. Introduction

Intuitionistic fuzzy number(IFN) as the special intuitionistic fuzzy set [1] has been widely applied into multi-criteria decision-making [2-6], fault analysis [7,8], programming [9-11] and portfolio selection problems [12,13]. Also, some basic operations and ranking methods of intuitionistic fuzzy number were introduced in literatures [14-16]. Inspired by possibility mean value theory [17-20] of fuzzy number, in this paper we introduce a novel possibilistic variance formula for trapezoidal intuitionistic fuzzy number (TIFN) which is different from the existing ones presented in [21-23]. Then some important properties of the proposed novel possibilistic variance of TIFNs are discussed.

2. Preliminaries

Definition 1[7]. A trapezoidal intuitionistic fuzzy number (TIFN) $\widetilde{A}=(e,a,f,b,c,g,d,h)$ is a special kind of intuitionistic fuzzy set on the real number set R, whose membership function and nonmembership function are respectively defined as the following forms.

$$\mu_{\widetilde{A}}(x)=\begin{cases}(x-a)/(b-a), & a\le x<b,\\ 1, & b\le x\le c,\\ (d-x)/(d-c), & c<x\le d,\\ 0, & otherwise\end{cases}. \qquad \nu_{\widetilde{A}}(x)=\begin{cases}(f-x)/(f-e), & e\le x<f,\\ 0, & f\le x\le g,\\ (x-g)/(h-g), & g<x\le h,\\ 1, & otherwise\end{cases}.$$

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where $e \le a \le f \le b \le c \le g \le d \le h,$ and TIFN $\widetilde{A} = (\widetilde{A}_*, \widetilde{A}^*)$ is viewed as a conjunction of two fuzzy numbers $\widetilde{A}_*, \widetilde{A}^*$ with low membership $\mu_{\widetilde{A}_*}(x) = \mu_{\widetilde{A}}(x)$ and high membership $\mu_{\widetilde{A}^*}(x) = 1 - v_{\widetilde{A}}(x).$

Definition 2. Let $\widetilde{A}_i = (e_i, a_i, f_i, b_i, c_i, g_i, d_i, h_i)$, $i = 1,2,$ be two trapezoidal intuitionistic fuzzy numbers, the addition and scale multiplication of TIFNs [15] are defined as follows.

(1) $\widetilde{A}_1 + \widetilde{A}_2$

$$= (e_1 + e_2, a_1 + a_2, f_1 + f_2, b_1 + b_2, c_1 + c_2, g_1 + g_2, d_1 + d_2, h_1 + h_2).$$

(2) $x\widetilde{A}_i = (xe_i, xa_i, xf_i, xb_i, xc_i, xg_i, xd_i, xh_i)$, $\forall x \ge 0$.

$x\widetilde{A}_i = (xh_i, xd_i, xg_i, xc_i, xb_i, xf_i, xa_i, xe_i)$, $\forall x < 0$.

Definition 3. Let $\widetilde{A} = (e, a, f, b, c, g, d, h)$ be a TIFN, the λ-level cut set of low membership and high membership of TIFN $\widetilde{A}$ is defined as , $\forall \lambda \in [0,1],$

$$\widetilde{A}_*^{[\lambda]} = \{x / \mu_{\widetilde{A}_*}(x) = \mu_{\widetilde{A}}(x) \ge \lambda\} = [a_*^-(\lambda), a_*^+(\lambda)]$$
$$= [a + (b-a)\lambda, d - (d-c)\lambda)];$$
$$\widetilde{A}^{*[\lambda]} = \{x / \mu_{\widetilde{A}^*}(x) = 1 - v_A(x) \ge \lambda\} = [a^{*-}(\lambda), a^{*+}(\lambda)]$$
$$= [e + (f-e)\lambda, h - (h-g)\lambda)];$$

Definition 4. Let $\widetilde{A} = (e, a, f, b, c, g, d, h)$ be a TIFN the lower and upper possibilistic mean value of the low membership function $\widetilde{A}_*$ of TIFN $\widetilde{A}$ are, respectively, defined as [17, 21]

$$M^-(\widetilde{A}_*) = 2\int_0^1 \lambda a_*^-(\lambda) d\lambda = 2\int_0^1 \lambda(a + \lambda(b-a)) d\lambda = \tfrac{1}{3}(a+2b);$$
$$M^+(\widetilde{A}_*) = 2\int_0^1 \lambda a_*^+(\lambda) d\lambda = 2\int_0^1 \lambda(d - \lambda(d-c)) d\lambda = \tfrac{1}{3}(d+2c).$$

The possibilistic mean of low membership of TTFN $\widetilde{A}$ is

$$M\ (\widetilde{A}_*) = [M^-(\widetilde{A}_*) + M^+(\widetilde{A}_*)]/2 \ = \int_0^1 \lambda(a_*^-(\lambda) + a_*^+(\lambda)) d\lambda$$
$$= \tfrac{1}{6}(a+d) + \tfrac{1}{3}(b+c)\ .$$

Definition 5. Let $\widetilde{A} = (e, a, f, b, c, g, d, h)$ be a TIFN, the lower and upper possibilistic mean value of the high membership function $\widetilde{A}^*$ of TIFN $\widetilde{A}$ are, respectively, defined as [22]

$$M^-(A^*) = 2\int_0^1 \lambda a^{*-}(\lambda) d\lambda = \tfrac{1}{3}(e+2f)\ ;$$
$$M^+(A^*) = 2\int_0^1 \lambda a^{*+}(\lambda) d\lambda = \tfrac{1}{3}(h+2g).$$

The possibilistic mean of high membership of TIFN $\widetilde{A}$ is

$$M(\widetilde{A}^*)=[M^-(\widetilde{A}^*)+M^+(\widetilde{A}^*)]/2 \quad =\tfrac{1}{6}(e+h)+\tfrac{1}{3}(f+g).$$

Definition 6. Let $\widetilde{A}=(e,a,f,b,c,g,d,h)$ be a TIFN, the possibilistic mean value of TIFN $\widetilde{A}$ is defined as

$$M(\widetilde{A})=\tfrac{1}{2}[M(\widetilde{A}_*)+M(\widetilde{A}^*)] =\tfrac{1}{12}(a+d+e+h)+\tfrac{1}{6}(b+c+f+g).$$

Proposition 1. Let $\widetilde{A}_1=(e_1,a_1,f_1,b_1,c_1,g_1,d_1,h_1), \widetilde{A}_2=(e_2,a_2,f_2,b_2,c_2,g_2,d_2,h_2)$ be two TIFNs, for any $x,x_1,x_2\in R$, we have

$$M(x_1\widetilde{A}_1+x_2\widetilde{A}_2)=x_1M(\widetilde{A}_1)+x_2M(\widetilde{A}_2).$$

The proof is straightforward from Definitions 4,5, 6. (Omitted)

Definition 7. Let $\widetilde{A}=(e,a,f,b,c,g,d,h)$ be a TIFN with λ cut sets $\widetilde{A}_*^{[\lambda]} = [a_*^-(\lambda),a_*^+(\lambda)]$, $\widetilde{A}^{*[\lambda]}=[a^{*-}(\lambda),a^{*+}(\lambda)]$ of low and high membership function, the possibilistic variance of low membership and high membership of TIFN $\widetilde{A}$ are respectively defined as

$$\mathrm{var}'(\widetilde{A}_*)=\int_0^1[(M(\widetilde{A}_*)-a_*^-(\lambda))^2+(M(\widetilde{A}_*)-a_*^+(\lambda))^2]\lambda d\lambda;$$

$$\mathrm{var}'(\widetilde{A}^*)=\int_0^1[(M(\widetilde{A}^*)-a^{*-}(\lambda))^2+(M(\widetilde{A}^*)-a^{*+}(\lambda))^2]\lambda d\lambda.$$

The possibilistic variance of TIFN $\widetilde{A}$ is also defined by

$$\mathrm{var}'(\widetilde{A})=\tfrac{1}{2}[\mathrm{var}'(\widetilde{A}_*)+\mathrm{var}'(\widetilde{A}^*)].$$

Definition 8. Let TIFN $\widetilde{A}_1=(e_1,a_1,f_1,b_1,c_1,g_1,d_1,h_1)$ be with λ cut sets $\widetilde{A}_{1*}^{[\lambda]}=[a_{1*}^-(\lambda),a_{1*}^+(\lambda)]$, $\widetilde{A}_1^{*[\lambda]}=[a_1^{*-}(\lambda),a_1^{*+}(\lambda)]$, and $\widetilde{A}_2=(e_2,a_2,f_2,b_2,c_2,g_2,d_2,h_2)$ with λ cut sets $\widetilde{A}_{2*}^{[\lambda]}=[a_{2*}^-(\lambda),a_{2*}^+(\lambda)]$, $\widetilde{A}_2^{*[\lambda]}=[a_2^{*-}(\lambda),a_2^{*+}(\lambda)]$, the possibilistic covariance of low membership and high membership of TIFNs $\widetilde{A}_1,\widetilde{A}_2$ are respectively defined as

$$\mathrm{cov}'(\widetilde{A}_{1*},\widetilde{A}_{2*})$$
$$=\int_0^1[(M(\widetilde{A}_{1*})-a_{1*}^-(\lambda))(M(\widetilde{A}_{2*})-a_{2*}^-(\lambda))+(M(\widetilde{A}_{1*})-a_{1*}^+(\lambda))(M(\widetilde{A}_{2*})-a_{2*}^+(\lambda))]\lambda d\lambda$$
$$\mathrm{cov}'(\widetilde{A}_1^*,\widetilde{A}_2^*).$$
$$=\int_0^1[(M(\widetilde{A}_1^*)-a_1^{*-}(\lambda))(M(\widetilde{A}_2^*)-a_2^{*-}(\lambda))+(M(\widetilde{A}_1^*)-a_1^{*+}(\lambda))(M(\widetilde{A}_2^*)-a_2^{*+}(\lambda))]\lambda d\lambda$$

The possibilistic covariance of TIFNs $\widetilde{A}_1,\widetilde{A}_2$ can be computed by

$$\mathrm{cov}'(\widetilde{A}_1,\widetilde{A}_2)=\tfrac{1}{2}[\mathrm{cov}'(\widetilde{A}_{1*},\widetilde{A}_{2*})+\mathrm{cov}'(\widetilde{A}_1^*,\widetilde{A}_2^*)].$$

3. Some properties of the possibilistic variance of TIFNs

Theorem 1. Let $\widetilde{A}_1=(e_1,a_1,f_1,b_1,c_1,g_1,d_1,h_1)$ be a TIFN with λ cut sets $\widetilde{A}_{1*}^{[\lambda]}=[a_{1*}^-(\lambda),a_{1*}^+(\lambda)]$, $\widetilde{A}_1^{*[\lambda]}=[a_1^{*-}(\lambda),a_1^{*+}(\lambda)]$, $\widetilde{A}_2=(e_2,a_2,f_2,b_2,c_2,g_2,d_2,h_2)$ be a TIFN with λ cut sets $\widetilde{A}_{2*}^{[\lambda]}=[a_{2*}^-(\lambda),a_{2*}^+(\lambda)]$, $\widetilde{A}_2^{*[\lambda]}=[a_2^{*-}(\lambda),a_2^{*+}(\lambda)]$ of low and high membership, $\forall\theta\in R$, we obtain

(1) $\operatorname{cov}'(\widetilde{A}_1,\widetilde{A}_2)=\operatorname{cov}'(\widetilde{A}_2,\widetilde{A}_1)$; (2) $\operatorname{var}'(\widetilde{A}_1)=\operatorname{cov}'(\widetilde{A}_1,\widetilde{A}_1)$;

(3) $\operatorname{var}'(\widetilde{A}_1+\theta)=\operatorname{var}'(\widetilde{A}_1)$.

Proof. (1), (2) can be directly proved by Definition 8 of $\operatorname{cov}'(\widetilde{A}_1,\widetilde{A}_2)$ and Definition 7 of $\operatorname{var}'(\widetilde{A}_1)$.

(3) Since θ is a constant TIFN, we easily know $\widetilde{\theta}_*^{[\lambda]}=[\theta,\theta]$; and $\widetilde{\theta}^{*[\lambda]}=[\theta,\theta]$.
Thus, $(\widetilde{A}_1+\theta)_*^{[\lambda]}=\widetilde{A}_{1*}^{[\lambda]}+\theta=[a_{1*}^-(\lambda)+\theta,\ \ a_{1*}^+(\lambda)+\theta]$;

$$(\widetilde{A}_1+\theta)^{*[\lambda]}=\widetilde{A}_1^{*[\lambda]}+\theta=[a_1^{*-}(\lambda)+\theta,\ \ a_1^{*+}(\lambda)+\theta].$$

$$\begin{aligned}\operatorname{var}'(\widetilde{A}_1+\theta)_*&=\int_0^1[(M(\widetilde{A}_1+\theta)_*-(a_{1*}^-(\lambda)+\theta))^2+(M(\widetilde{A}_1+\theta)_*-(a_{1*}{}^+(\lambda)+\theta))^2]\lambda d\lambda\\&=\int_0^1[(M(\widetilde{A}_{1*})+\theta-a_{1*}^-(\lambda)-\theta)^2+(M(\widetilde{A}_{1*})+\theta-a_{1*}^+(\lambda)-\theta)^2]\lambda d\lambda\\&=\int_0^1[(M(\widetilde{A}_{1*})-a_{1*}^-(\lambda))^2+(M(\widetilde{A}_{1*})-a_{1*}^+(\lambda))^2]\lambda d\lambda=\operatorname{var}'(\widetilde{A}_{1*});\end{aligned}$$

$$\begin{aligned}\operatorname{var}'(\widetilde{A}_1+\theta)^*&=\int_0^1[(M(\widetilde{A}_1+\theta)^*-(a_1^{*-}(\lambda)+\theta))^2+(M(\widetilde{A}_1+\theta)^*-(a_1{}^{*+}(\lambda)+\theta))^2]\lambda d\lambda\\&=\int_0^1[(M(\widetilde{A}_1^*)+\theta-a_1^{*-}(\lambda)-\theta)^2+(M(\widetilde{A}_1^*)+\theta-a_1^{*+}(\lambda)-\theta)^2]\lambda d\lambda\\&=\int_0^1[(M(\widetilde{A}_1^*)-a_1^{*-}(\lambda))^2+(M(\widetilde{A}_1^*)-a_1^{*+}(\lambda))^2]\lambda d\lambda=\operatorname{var}'(\widetilde{A}_1^*);\end{aligned}$$

Hence, $\operatorname{var}'(\widetilde{A}_1+\theta)=\operatorname{var}'(\widetilde{A}_1)$.

Theorem 2. Let $\widetilde{A}_1=(e_1,a_1,f_1,b_1,c_1,g_1,d_1,h_1)$ be a TIFN with λ cut sets $\widetilde{A}_{1*}^{[\lambda]}=[a_{1*}^-(\lambda),a_{1*}^+(\lambda)]$, $\widetilde{A}_1^{*[\lambda]}=[a_1^{*-}(\lambda),a_1^{*+}(\lambda)]$, $\widetilde{A}_2=(e_2,a_2,f_2,b_2,c_2,g_2,d_2,h_2)$ be a TIFN with λ cut sets $\widetilde{A}_{2*}^{[\lambda]}=[a_{2*}^-(\lambda),a_{2*}^+(\lambda)]$, $\widetilde{A}_2^{*[\lambda]}=[a_2^{*-}(\lambda),a_2^{*+}(\lambda)]$ of low and high membership, for any $x_1,x_2\in R$, then
$\operatorname{var}'(x_1\widetilde{A}_1+x_2\widetilde{A}_2)=x_1{}^2\operatorname{var}'(\widetilde{A}_1)+x_2{}^2\operatorname{var}'(\widetilde{A}_2)+2|x_1x_2|\operatorname{cov}'(\phi(x_1)\widetilde{A}_1,\phi(x_2)\widetilde{A}_2)$,
where $\phi(x)$ is a signal function of $x\in R$.

Proof. (1) If $x_1>0$ and $x_2<0$, by Zadeh's fuzzy extension principle one can see that

$$(x_1\widetilde{A}_1 + x_2\widetilde{A}_2)_*^{[\lambda]} = [x_1 a_{1*}^-(\lambda) + x_2 a_{2*}^+(\lambda), x_1 a_{1*}^+(\lambda) + x_2 a_{2*}^-(\lambda)];$$
$$(x_1\widetilde{A}_1 + x_2\widetilde{A}_2)^{*[\lambda]} = [x_1 a_1^{*-}(\lambda) + x_2 a_2^{*+}(\lambda), x_1 a_1^{*+}(\lambda) + x_2 a_2^{*-}(\lambda)] .$$

With Definitions 7,8 and Proposition 1 we easily get

$$\begin{aligned}
&\mathrm{var}'(x_1\widetilde{A}_1 + x_2\widetilde{A}_2)_* \\
&= \int_0^1 [M(x_1\widetilde{A}_1 + x_2\widetilde{A}_2)_* - (x_1 a_{1*}{}^-(\lambda) + x_2 a_{2*}{}^+(\lambda))]^2 \lambda d\lambda \\
&+ [M(x_1\widetilde{A}_1 + x_2\widetilde{A}_2)_* - (x_1 a_{1*}{}^+(\lambda) + x_2 a_{2*}{}^-(\lambda))]^2 \lambda d\lambda \\
&= \int_0^1 \{[x_1(M(\widetilde{A}_{1*}) - a_{1*}{}^-(\lambda)) + x_2(M(\widetilde{A}_{2*}) - a_{2*}{}^+(\lambda))]^2 \\
&\quad + [x_1(M(\widetilde{A}_{1*}) - a_{1*}{}^+(\lambda)) + x_2(M(\widetilde{A}_{2*}) - a_{2*}{}^-(\lambda))]^2\}\lambda d\lambda \\
&= x_1{}^2\,\mathrm{var}'(\widetilde{A}_{1*}) + 2x_1x_2\int_0^1 \lambda[(M(\widetilde{A}_{1*}) - a_{1*}{}^-(\lambda))(M(\widetilde{A}_{2*}) - a_{2*}{}^+(\lambda)) + (M(\widetilde{A}_{1*}) - a_{1*}{}^+(\lambda)) \\
&\quad (M(\widetilde{A}_{2*}) - a_{2*}{}^-(\lambda))]d\lambda \ + x_2^2\,\mathrm{var}'(\widetilde{A}_{2*}) \\
&= x_1{}^2\,\mathrm{var}'(\widetilde{A}_{1*}) - 2x_1x_2\,\mathrm{cov}'\,(\widetilde{A}_{1*}, -\widetilde{A}_{2*}) + x_2{}^2\,\mathrm{var}'(\widetilde{A}_{2*}) \\
&= x_1{}^2\,\mathrm{var}'(\widetilde{A}_{1*}) + 2|x_1x_2|\,\mathrm{cov}'\,(\phi(x_1)\widetilde{A}_{1*}, \phi(x_2)\widetilde{A}_{2*}) + x_2{}^2\,\mathrm{var}'(\widetilde{A}_{2*});
\end{aligned}$$

Similarly, we can prove that

$$\begin{aligned}
&\mathrm{var}'(x_1\widetilde{A}_1 + x_2\widetilde{A}_2)^* \\
&= x_1{}^2\,\mathrm{var}'(\widetilde{A}_1^*) + 2|x_1x_2|\,\mathrm{cov}'\,(\phi(x_1)\widetilde{A}_1^*, \phi(x_2)\widetilde{A}_2^*) + x_2{}^2\,\mathrm{var}'(\widetilde{A}_2^*);
\end{aligned}$$

Hence, we obtain

$$\begin{aligned}
\mathrm{var}'(x_1\widetilde{A}_1 + x_2\widetilde{A}_2) &= \tfrac{1}{2}[\mathrm{var}'(x_1\widetilde{A}_1 + x_2\widetilde{A}_2)_* + \mathrm{var}'(x_1\widetilde{A}_1 + x_2\widetilde{A}_2)^*] \\
&= x_1{}^2\,\mathrm{var}'(\widetilde{A}_1) + 2|x_1x_2|\,\mathrm{cov}'(\phi(x_1)\widetilde{A}_1, \phi(x_2)\widetilde{A}_2) + x_2{}^2\,\mathrm{var}'(\widetilde{A}_2).
\end{aligned}$$

The other cases (2) $x_1 < 0$, $x_2 > 0$; (3) $x_1 > 0$, $x_2 > 0$ and (4) $x_1 \le 0$, $x_2 \le 0$ can be similarly proved.

Theorem 3. Let $\widetilde{A} = (e, a, f, b, c, g, d, h)$ be a TIFN with λ cut sets $\widetilde{A}_*^{[\lambda]} = [a_*^-(\lambda), a_*^+(\lambda)]$, $\widetilde{A}^{*[\lambda]} = [a^{*-}(\lambda), a^{*+}(\lambda)]$ of low and high membership, then we obtain $\mathrm{var}(\widetilde{A}) \le \mathrm{var}'(\widetilde{A})$.

Proof.

$$\begin{aligned}
\mathrm{var}'(\widetilde{A}_*) &= \int_0^1 \{[M(\widetilde{A}_*) - a_*^-(\lambda)]^2 + [M(\widetilde{A}_*) - a_*^+(\lambda)]^2\}\lambda d\lambda \\
&= \int_0^1 [(a_*^+(\lambda))^2 + (a_*^-(\lambda))^2]\lambda d\lambda - \int_0^1 2(M(\widetilde{A}_*))[a_*^-(\lambda) + a_*^+(\lambda)]\lambda d\lambda + 2\int_0^1 M(\widetilde{A}_*)^2 \lambda d\lambda \\
&= \int_0^1 [(a_*^+(\lambda))^2 + (a_*^-(\lambda))^2]\lambda d\lambda - (M(\widetilde{A}_*))^2 .
\end{aligned}$$

Also, by the possibilistic variance definition stated in [17, 21] we know that

$$\operatorname{var}(\widetilde{A}_*) = \frac{1}{2}\int_0^1 [a_*^+(\lambda) - a_*^-(\lambda)]^2 \lambda d\lambda$$
$$= \frac{1}{2}\int_0^1 [(a_*^+(\lambda))^2 + (a_*^-(\lambda))^2]\lambda d\lambda - \int_0^1 a_*^+(\lambda) a_*^-(\lambda) \lambda d\lambda .$$

From the Jensen inequality and $\int_0^1 2\lambda d\lambda = 1,$ we have

$$[\int_0^1 2\lambda(a_*^-(\lambda) + a_*^+(\lambda))d\lambda]^2 \le \int_0^1 2\lambda(a_*^-(\lambda) + a_*^+(\lambda))^2 d\lambda .$$

That is, $[\int_0^1 \lambda(a_*^-(\lambda) + a_*^+(\lambda))d\lambda]^2 \le \frac{1}{2}\int_0^1 \lambda(a_*^-(\lambda) + a_*^+(\lambda))^2 d\lambda$.

So, $\operatorname{var}(\widetilde{A}_*) - \operatorname{var}'(\widetilde{A}_*)$

$$= -\frac{1}{2}\int_0^1 [(a_*^+(\lambda))^2 + (a_*^-(\lambda))^2]\lambda d\lambda - \int_0^1 a_*^+(\lambda) a_*^-(\lambda) \lambda d\lambda + (M(\widetilde{A}_*))^2$$
$$= -\frac{1}{2}\int_0^1 [(a_*^+(\lambda))^2 + (a_*^-(\lambda))^2]\lambda d\lambda - \int_0^1 a_*^+(\lambda) a_*^-(\lambda) \lambda d\lambda + (M(\widetilde{A}_*))^2$$
$$= -\frac{1}{2}\int_0^1 [a_*^+(\lambda) + a_*^-(\lambda)]^2 \lambda d\lambda + [\int_0^1 \lambda(a_*^-(\lambda) + a_*^+(\lambda))d\lambda]^2 \le 0 .$$

Similarly, we can prove $\operatorname{var}(\widetilde{A}^*) - \operatorname{var}'(\widetilde{A}^*) \le 0$.

So, $\operatorname{var}(\widetilde{A}) = [\operatorname{var}(\widetilde{A}_*) + \operatorname{var}(\widetilde{A}^*)]/2 \le [\operatorname{var}'(\widetilde{A}_*) + \operatorname{var}'(\widetilde{A}^*)]/2 = \operatorname{var}'(\widetilde{A})$.

4. Conclusion

In this article, we introduce a new definition of possibilistic variance of trapezoidal intuitionistic fuzzy number and discuss some important properties. In the future work, we will study the possibilistic mean and variance of the related uncertain numbers such as fuzzy neutrosophic numbers and dual-connection numbers, which are very important in the uncertain multi-criteria decision and fuzzy portfolio selection problem. Also, we will investigate the intuitionistic fuzzy portfolio decision-making and selection problem by maximizing possibilistic mean and minimizing possibilistic variance of portfolio.

Acknowledgement
This work is supported in part by the Guangdong Basic and Applied Basic Research Foundation under Grant No. 2018A030313996.

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