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Performance Analysis of Aerial Base Station Cellular Network

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Abstract. In this paper, we model the cellular network consisting of aerial base stations (ABSs) which follow homogeneous Poisson Point Process (PPP) and an independent collection of random users is considered. We assume that each user communicates with the closest ABS. In particular, we deal the interference signals with Laplace transform. As a result, we can obtain a tractable expression of the coverage probability. We have found that there is a maximum coverage probability which can be reached by an optimal ABS density and with low signal-to-noise ratio (SNR), the optimal ABS density gets larger when path loss exponent (PLE) increases. Simulations under high or low SNR condition have shown that the influence of PLE on coverage probability is quite different in spite of changing signal-to-interference-plus-noise ratio (SINR) threshold or ABS density.

Keywords. coverage probability, stochastic geometry, Poisson Point Process (PPP), air-to-ground (A2G) communication

1. Introduction

When it comes to deploying ABSs in the sky, unmanned aerial vehicles (UAVs) with the advantage of flexible deployment have aroused much interest among academia and industry [1]. Once the disasters, emergencies or some major events happen, which require rapid and stable network resources, it's of significance to provide wireless connectivity rapidly [2]. Hence, it's necessary to deploy ABSs in the sky to quickly provide connectivity and recover communication.

In the research of aerial cellular network, people have taken advantage of the distribution characteristics of nodes and stochastic geometry as the tool to study the performance of the network. In [3], a framework has been presented to study the coverage probability and achievable rate of the aerial cellular network. However, they simplified the analysis without considering the effect of interference. As is known to all, interference is a key factor to influence the performance. The author in [4] has neglected the small-scale fading to conduct a performance analysis where UAVs served as ABSs. [5]

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is beneficial to have an overall look of the cellular network, especially for a better understanding of the performance analysis. However, the obtained complex analytical results involving multiple nested integrals, can't offer clear and straightforward insights for the future network design. As mentioned above, there are some questions left about the performance of the aerial cellular network.

Firstly, because the A2G link is easier to form a LOS link, we assume that A2G channel experiences a LOS propagation. We later make use of the characteristic of Rayleigh fading channels to conduct a tractable calculation. Then a tractable and accurate coverage probability expression of the aerial cellular network is obtained where ABSs are located as a PPP. Based on the results which have been derived, the impacts of ABS density, SNR and PLE are investigated. The accuracy of our theoretical analysis is proved by comparing with the results of Monte Carlo simulations.

2. System Model

The aerial cellular network model is described as Figure 1, which consists of ABSs arranged according to homogeneous PPP Φ of intensity λ with height h above the ground. The random users under consideration are modeled as an independent stationary point process. The horizontal distance denoted as r is on the assumption of that each user communicates with the closest ABS. PLE of A2G links is α. The power gain of small scale fading g is characterized by Rayleigh fading where the mean of the random variable is unit. And the power of additive white Gaussian noise is characterized by a Gaussian random variable whose mean is zero and variance is N. We assume that ABSs transmit with power normalizing to 1. Therefore, a typical node receives power of value $gr^{-\alpha}$, where the node is away from its ABS with horizontal distance r. The interference power received by the typical user is denoted as I. Excluding the received power from the tagged ABS, the sum of all other ABSs' power forms I. Slivnyak's theorem makes it reasonable to conduct the analysis that a typical user is at the origin [6].

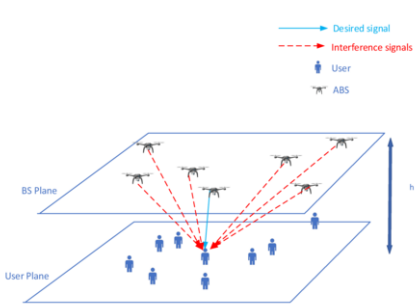

Figure 1. Air-to-ground (A2G) cellular network.

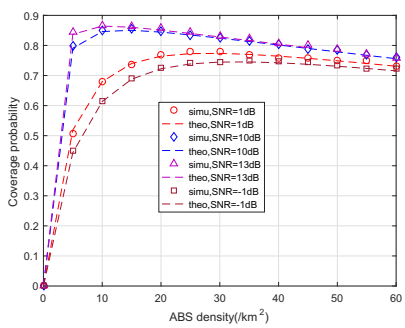

Figure 2. The coverage probability versus ABS density($/km^2$) with different SNR

3. SINR Coverage Probability

In this section, we derive the downlink coverage probability of the aerial cellular network. A SINR threshold required for the actual SINR of each user is defined. Only when the SINR threshold is lower than the actual SINR, can a user be regarded as being covered, otherwise it is not covered. Subsequently, we will consider the aggregate interference to obtain a closed form of SINR coverage probability when the serving area is assumed to be infinite.

When the actual SINR is larger than a predefined SINR threshold β, the user can be regarded as being covered while be dropped from the network for actual SINR below β. Therefore, we define the probability being covered as

$$P_c(\beta,\lambda,h,\alpha) \triangleq P[SINR > \beta]. \tag{1}$$

As mentioned above, a typical user is located at the origin. The effective SINR is $SINR = \frac{g_0 X_0^{-\alpha}}{I+N}$, where $I = \sum_{i\in\phi/\{0\}} g_i X_i^{-\alpha}$. Excluding the received power from the tagged ABS, the sum of all other ABSs' power forms I. The distance between the typical user and interference ABS is denoted as X_i, where $X_i = \sqrt{r_i^2 + h^2}$. The fading is g_i. g_i follows $g_i \sim exp(1)$.

In the following section, the subscript 0 refers to related parameters of the desired ABS and the subscript i refers to related parameters of the interference ABSs. The PLE is assumed to be a typical value of 4, which can be used to get a closed expression of coverage probability by some mathematical transformation. The coverage probability of a randomly located user can be derived as

$$P_c(\beta,\lambda,h,4) = c\sqrt{\frac{\pi}{b}}\exp(\frac{a^2}{4b})Q(\frac{a}{\sqrt{2b}}), \tag{2}$$

where $c = \pi\lambda\exp(-Ah^2 - \beta Nh^4 + \pi\lambda h^2)$, $a = A + \beta N 2h^2$, $b = \beta N$ and $A = \pi\lambda[\sqrt{\beta}(\frac{\pi}{2} - \arctan(\frac{1}{\sqrt{\beta}})) + 1]$.

The proof of Eq. (2) is given as follows. We assume that a typical user is away from its nearest ABS with distance X_0 and the projection distance is denoted as r. So the conditional coverage probability is

$$\begin{aligned} &P(SINR > \beta|r) = P(\tfrac{g_0 X_0^{-4}}{I+N} > \beta|r) = P(g_0 > \beta(I+N)X_0^4) \\ &\overset{(a)}{=} E_I[\exp(-\beta(I+N)X_0^4)] = E_I[\exp(-\beta I X_0^4)]\exp(-\beta N X_0^4), \end{aligned} \tag{3}$$

denote $m = m(r) = \beta X_0^4 = \beta(\sqrt{r^2+h^2})^4 = \beta(r^2+h^2)^2$,

$$P(SINR > \beta|r) = L_I(m)\exp(-mN), \tag{4}$$

g owns unit mean and is exponential distributed, which contributes to the result of (a). With the definition of Laplace transform, we can get the Laplace transform of I. Moreover, if we take the concrete expression into I, we can obtain

$$\begin{aligned}
&L_I(m) = E_I[\exp(-mI)] = E_I[\exp(-m \sum_{i\in\phi/\{0\}} g_i X_i^{-4})] \\
&= E_{\{g_i,\phi\}}[\prod_{i\in\phi/\{0\}} \exp(-mg_i X_i^{-4})] = E_{\{\phi\}}\{\prod_{i\in\phi/\{0\}} E_{g_i}[\exp(-mg_i X_i^{-4})]\} \\
&\overset{(b)}{=} E_{\{\phi\}}\{\prod_{i\in\phi/\{0\}} \tfrac{1}{1+mX_i^{-4}}\}.
\end{aligned} \tag{5}$$

where (b) follows from the properties of g_i including its independence from process Φ and its distribution characteristic.

As for PPP, the transformation of the expectation of the product of $f(x)$ in Φ is as follows.

$$E[\prod_{x\in\phi} f(x)] = \exp(-\lambda \int_{R^2} (1-f(x))dx). \tag{6}$$

Due to the communication strategy of a typical user communicating with the nearest ABS, the distances of interference ABSs should be farther than the distance $X_0 = \sqrt{r^2+h^2}$. Use the change of variables $v^2+h^2 = y$. So $L_I(m)$ can be further derived as

$$\begin{aligned}
&L_I(m) = \exp(-\lambda \int_{R^2} (1-\tfrac{1}{1+mX_i^{-4}})dx) = \exp(-2\pi\lambda \int_r^\infty (1-\tfrac{1}{1+m(\sqrt{v^2+h^2})^{-4}})vdv) \\
&= \exp[-\pi\lambda \int_{r^2+h^2}^\infty (1-\tfrac{1}{1+my^{-2}})dy] = \exp\{-\pi\lambda\sqrt{\beta}(r^2+h^2)[\tfrac{\pi}{2}-\arctan(\tfrac{1}{\sqrt{\beta}})]\}.
\end{aligned} \tag{7}$$

The coverage probability of the whole work is as Eq. (8). We get the function of distribution characteristic of horizontal distance r to the nearest ABS as $f(r)$. $f(r)$ follows $f(r) = 2\pi\lambda r e^{-\lambda\pi r^2}$.

$$\begin{aligned}
&P_c(\beta,\lambda,h,4) = \textstyle\int_0^\infty P(SINR>\beta|r)f(r)dr = \int_0^\infty L_I(m)e^{-mN}f(r)dr \\
&= \int_0^\infty \exp\{-\pi\lambda\sqrt{\beta}(r^2+h^2)[\tfrac{\pi}{2}-\arctan(\tfrac{1}{\sqrt{\beta}})]\}e^{-\beta(r^2+h^2)^2N}2\pi\lambda r\exp(-\pi\lambda r^2)dr.
\end{aligned} \tag{8}$$

Using the change of variables $r^2+h^2 = u$, we obtain

$$\begin{aligned}
&P_c(\beta,\lambda,h,4) = \int_{h^2}^\infty \exp\{-\pi\lambda\sqrt{\beta}u[\tfrac{\pi}{2}-\arctan(\tfrac{1}{\sqrt{\beta}})]\}e^{-\beta u^2N}\pi\lambda\exp(-\pi\lambda(u-h^2))du \\
&= \int_{h^2}^\infty \exp\{-\pi\lambda[\sqrt{\beta}(\tfrac{\pi}{2}-\arctan(\tfrac{1}{\sqrt{\beta}}))+1]u\}\exp(-\beta Nu^2)\pi\lambda\exp(\pi\lambda h^2)du \\
&\overset{(d)}{=} \int_{h^2}^\infty \exp(-Au)\exp(-\beta Nu^2)\pi\lambda\exp(\pi\lambda h^2)du \\
&\overset{(e)}{=} \int_0^\infty \exp(-A(w+h^2))\exp(-\beta N(w+h^2)^2)\pi\lambda\exp(\pi\lambda h^2)dw \\
&= \int_0^\infty \exp[-(A+\beta N2h^2)w]\exp(-\beta Nw^2)\pi\lambda\exp(-Ah^2-\beta Nh^4+\pi\lambda h^2)dw.
\end{aligned} \tag{9}$$

In (d), $A = \pi\lambda[\sqrt{\beta}(\frac{\pi}{2} - \arctan(\frac{1}{\sqrt{\beta}})) + 1]$, where (e) uses the change of variables $u - h^2 = w$.

$$\int_0^\infty e^{-ax} e^{-bx^2} dx = \sqrt{\frac{\pi}{b}} \exp(\frac{a^2}{4b}) Q(\frac{a}{\sqrt{2b}}), \tag{10}$$

We call Q(x) as standard Gaussian tail probability, where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-y^2/2) dy$. Set $a = A + \beta N 2h^2$, and $b = \beta N$, gives

$$P_c(\beta, \lambda, h, 4) = c\sqrt{\frac{\pi}{b}} \exp(\frac{a^2}{4b}) Q(\frac{a}{\sqrt{2b}}). \tag{11}$$

where c is given by (2).

4. Numerical Results and Discussion

In this section, the simulation results regarding coverage probability with different system parameters are provided. Theoretical results are verified by comparing with the results from Monte-Carlo simulations. In the end, some system design guidelines to maximize the coverage probability are showed. The simulation process is as follows. We randomly generate the locations of ABSs according to a PPP in an area with a radius of 1000 m. The channel fading gain g between the receiver and transmitter nodes obeys exponential distribution with unit mean. The power of additive white Gaussian noise N is set by different value in different SNR environments. Path loss exponent α is from 2 to 5. We assume that the desired signal power is from the nearest ABS. After the desired power and interference power are obtained, we calculate the actual SINR. If the actual SINR surpasses some SINR threshold defined by our problem, the link is deemed to be successfully covered. Therefore, the successful connection times add one time. The simulations run for 100000 times.

We set SINR threshold=-10dB, h=100m, PLE=4 and theoretical results denoted by dotted lines fit well with Monte Carlo results denoted by different patterns in Figure 2. As we can see, there exists an optimal ABS density to maximize coverage probability. Because in a range of low ABS density, the probability of connecting a nearer ABS by the typical user will raise when increasing ABS density. As a result, communicating distance gets smaller and the received power gets lager which makes coverage probability larger. Once the ABS density exceeds a certain value, the coverage probability is inversely related to the increase of ABS density because users receive increasing interference. Owing that a higher SNR has a smaller noise power, it's obvious that a smaller noise power causes a higher coverage probability.

We set height=100m and λ=10 ABSs$/km^2$ in Figure 3 and λ=0.5 ABSs$/km^2$ in Figure 4. It shows that there is a positive correlation between PLE and the coverage probability when SNR=50dB in Figure 3. With the high SNR, the effect of noise power is negligible. The larger PLE, the more path loss both the desired signal and interference signal will suffer. However, the decay peed of the interference power and desire power

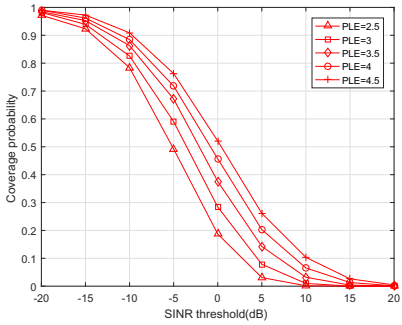

Figure 3. The coverage probability versus SINR threshold(dB) with SNR=50dB.

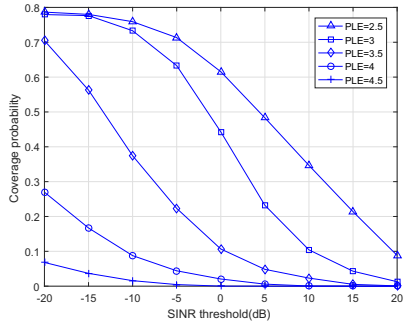

Figure 4. The coverage probability versus SINR threshold(dB) with SNR=1dB.

is quite different. Due to the larger propagation distance of interference ones, the attenuation degree of interference power is more severe than that of desired power. Therefore, the actual SINR is enhanced when increasing PLE which also improves the coverage. While the interference could be neglected with the low SNR condition as Figure 4 presents. The actual SINR is mainly depended on desired power and noise power. It's obvious that when the PLE gets larger, the desired power will become smaller. The noise power can be considered as a constant with the specific environment. Hence the coverage probability is reduced when increasing PLE.

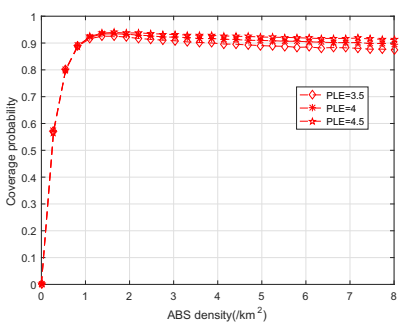

Figure 5. The coverage probability versus ABS density($/km^2$) with SNR=50dB.

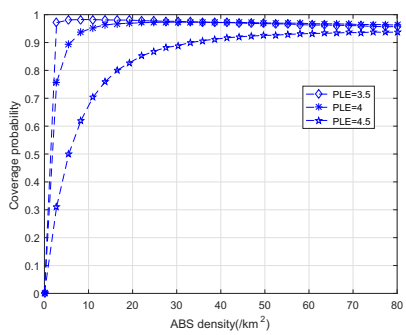

Figure 6. The coverage probability versus ABS density($/km^2$) with SNR=1dB.

As is demonstrated in Figure 5 and Figure 6, we set SINR threshold=-10dB and height=100m. With high SNR value of 50dB, the larger PLE corresponds to larger coverage probability. However, with low SNR value of 1dB, larger PLE leads to smaller coverage probability. The reason why there are two different results is as the reason of Figure 3 and Figure 4.

We can observe that the optimal ABS densities are very close with different PLEs in Figure 5. As the result of high SNR, the tiny noise power makes it possible to deploy a small number of ABSs to achieve maximum coverage. As for actual SINR, it's mainly

dominated by the ratio of signal power to aggregate interference power and so as the maximum coverage probability. Because the sparse ABS distribution leads to maximum coverage, the change of PLE to large communication distance and interference distances has little impact on the ratio. Therefore, the optimal ABS densities are almost the same under different PLEs. While it shows another phenomenon in Figure 6. As the PLE grows, the optimal ABS density gets larger. Because the SINR is mainly decided by the ratio of signal to noise with low SNR. The desired signal is bound to get smaller and so as SINR with the increase of PLE. Hence, it's necessary to deploy more ABSs to reach the maximum coverage.

5. Conclusion

In this paper, we presented a downlink coverage analysis with different parameters. We modeled the location of ABSs as a PPP. An exact expression of SINR coverage probability for the aerial cellular network under Rayleigh fading channel is obtained. The impacts of ABS density, SNR and PLE were investigated by analyzing the results. We have found that there is a maximum coverage probability which can be reached by an optimal ABS density and with low SNR, the optimal ABS density gets larger when PLE increases. Simulations under high or low SNR condition have shown that the influence of PLE on coverage probability is quite different in spite of changing SINR threshold or ABS density. Some performance trends of cellular parameters have been revealed with the analysis, which contributes to the future system design.

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