Proceedings of CECNet 2022
A.J. Tallón-Ballesteros (Ed.)

doi:10.3233/FAIA220562

Coherent Accumulation Algorithm for Maneuvering Weak Target Based on Angular-Stepped-GRFT

Yuxian SUN, Shaoqiang CHANG[1], Gang WANG, Zhennan LIANG, Jinpeng GUO and Quanhua LIU
Key Laboratory of Electronic and Information Technology in Satellite Navigation Beijing Institute of Technology, China

Abstract. This paper proposes an improved angular stepped generalized Radon Fourier transform (angular-stepped-GRFT) algorithm for maneuvering weak target detection in synthetic wideband radar. The proposed method can achieve motion parameters estimation for the maneuvering weak target based on stepped frequency signal. On this basis, the motion compensation of maneuvering weak target and range profile focusing method are realized.

Keywords. Long-time coherent accumulation, GRFT, motion parameters estimation, range profile

1. Introduction

Small radar cross section (RCS) targets such as aircraft have brought severe threats and challenges to radar detection. At the same time, some small RCS targets also have strong maneuvering ability, which further increases the difficulty of radar detection. How to maintain high precision measurement of the motion parameters of small RCS maneuvering target is one of the difficult problems to be solved urgently by modern radar.

To solve the problem of estimating the motion parameters of weak targets, we can increase the observation time to realize the accumulation of target energy for a long time, so as to improve the radar echo signal to noise ratio (SNR) and improve the detection ability of weak targets[1-2]. In 1999, Perry R P et al. proposed Keystone transform[3], which corrected the phenomenon of high-speed target across range unit (ARU) through scale transformation on the slow time-fast frequency plane, and then used Fourier transform to accumulate the target energy in a coherent way. Based on Keystone transform, there are many early weak target coherent accumulation methods[4-5]. In 2011, J. Xu et al. analyzed the relationship between the target ARU phenomenon and the motion parameters of each order, combined the generalized Radon transform and Fourier transform. A long time coherent accumulation algorithm based on generalized Radon Fourier transform (GRFT) is proposed, which has achieved good detection results[6-8]. In 2021, J Guo et al. combined GRFT with stepped frequency signal and proposed a broadband coherent accumulation method based on stepped generalized Radon Fourier

[1] Corresponding Author, Shaoqiang Chang, Key Laboratory of Electronic and Information Technology in Satellite Navigation, Beijing Institute of Technology, Beijing, China; E-mail: changshaoqiang@bit.edu.cn.

transform (stepped-GRFT)[9]. Stepped frequency signal is an important high resolution radar signal[10], which is widely used in civil and military fields[11-16]. It uses a series of stepped carrier frequency narrowband pulses which are sequentially transmitted, and obtains the capability of range high resolution through synthetic broadband processing. Guo's algorithm realizes weak target detection and radial motion parameters estimation based on long-term coherent accumulation, and then studies motion parameters compensation and target high resolution range profile (HRRP) focusing method, which effectively improves the ability of wideband radar to detect and measure weak targets.

Stepped-GRFT is limited in that it can only be used for the estimation of radial motion targets. When the weak target maneuvered, stepped-GRFT could not accurately estimate the motion parameters of the target, so that the correct range profile could not be obtained. In order to detect and image maneuvering weak targets in stepped frequency radar, angular-stepped-GRFT is proposed in this paper based on stepped-GRFT, which solves the problem of inaccurate estimation of motion parameters for maneuvering weak targets in stepped-GRFT. The coherent accumulation algorithm based on angular-stepped-GRFT can achieve correct motion compensation and range profile focusing. And effectively improve the SNR of the range profile after coherent accumulation processing.

2. Modeling and analysis of maneuvering weak target

The transmitted signal of stepped frequency radar can be expressed as

$$s\left(n,t_p\right)=\mathrm{rect}\left(\frac{t_p}{T_p}\right)e^{\mathrm{j}\pi k t_p{}^2}e^{\mathrm{j}2\pi f_h t_p} \tag{1}$$

where, $n=(m-1)N+h+1$ is the sub-pulse number, m is the frame number, $m=1,2,...,M$. N is the number of the sub-pulse in the frame. t_p is the fast time, T_p is the pulse width, and define $\mathrm{rect}(\frac{t}{T})=\begin{cases}1, & 0\le \mathrm{t}\le \mathrm{T}\\ 0, & \text{others}\end{cases}$. $k=\frac{B}{T_p}$ is the frequency modulation rate. $f_h=f_0+h\Delta f$, where f_0 is the initial carrier frequency, Δf is the frequency step size, and $h\in[0,1,...,N-1]$ is the frequency coding sequence.

If a point target starts to maneuver when $t=0$, its initial radial distance is R_0, its velocity is v, and its initial velocity direction is radial. The target performs a turning maneuver with an angular turning velocity of ω . Assuming that the velocity of the target does not change during maneuvering, the instantaneous slant range of the target can be expressed as

$$\mathbf{R}_n=\sqrt{(2-2\cos\omega t+\cos^2\omega t)R_0{}^2-\frac{2vR_0}{\omega}\sin\omega t+\frac{v^2}{\omega^2}\sin^2\omega t} \tag{2}$$

where, $t \in [0, MN * PRT)$, PRT is the pulse repetition time. This paper only discusses the case when the radial velocity of the target is positive, that is, the target moves towards radar. In this case, $0 \le t < \dfrac{\pi}{2\omega}$.

3. Motion parameters estimation and range profile focusing algorithm based on angular-stepped-GRFT

3.1. Angular-stepped-GRFT algorithm principle

Based on the stop-and-go assumption, we have the down-converted baseband target echo in two-dimensioal form as

$$s_R\left(n, t_p\right) = K_R \mathrm{rect}\left(\frac{t_p - \mathbf{R}_n / c}{T_p}\right) \mathrm{e}^{-\mathrm{j}\frac{4\pi \mathbf{R}_n f_h}{c}} \mathrm{e}^{\mathrm{j}\pi \mathrm{k}\left(t_p - \frac{2\mathbf{R}_n}{c}\right)^2} \tag{3}$$

where, K_R is the complex amplitude of target echo, c is the speed of light. Obtained by pulse compression of baseband echo target

$$s_{Rm}\left(n, t_p\right) = K_R T_p B \mathrm{sinc}\left(\pi\left(t_p - 2\mathbf{R}_n / c\right) B\right) \mathrm{e}^{-\mathrm{j}\frac{4\pi \mathbf{R}_n f_h}{c}} \tag{4}$$

where, B is the stepped frequency signal sub-pulse bandwidth.

Let $\mathbf{R}$ be the target slant range, $\mathbf{R} = ct_p / 2$, and Eq. (4) can be written as

$$s_{Rm}\left(n, t_p\right) = K_R T_p B \mathrm{sinc}\left(\frac{2\pi B\left(\mathbf{R} - \mathbf{R}_n\right)}{c}\right) \mathrm{e}^{-\mathrm{j}\frac{4\pi \mathbf{R}_n f_h}{c}} = K_R T_p B \mathrm{sinc}\left(\frac{\pi\left(\mathbf{R} - \mathbf{R}_n\right)}{\frac{c}{2B}}\right) \mathrm{e}^{-\mathrm{j}\frac{4\pi \mathbf{R}_n f_h}{c}} \tag{5}$$

where, $K_{Rm} = K_R T_p B$ is the target complex scattering coefficient. In practice, K_{Rm} is undulating and will bring phase noise. For simplicity, let's assume that K_{Rm} is a constant. $\dfrac{c}{2B}$ is the range resolution before synthetic wideband processing. In order to achieve effective coherent accumulation of multi-frame signals, the phase term in Eq. (5) needs to be compensated, and the corresponding phase compensation factor is

$$\Phi_n = e^{j\frac{4\pi \mathbf{R}_n f_h}{c}} \tag{6}$$

In summary, a coherent accumulation algorithm for detecting maneuvering weak targets with stepped frequency signals can be obtained

$$G = \sum_{n=1}^{MN} s_{Rm}\left(n, \mathbf{R}_n\right) \Phi_n \tag{7}$$

A traversal search is performed for the motion parameters R_0, v, ω of the target. The output of G is maximal if and only if the set of search parameters is equal to the actual motion parameters of the target. Considering the traversal of angular velocity, different from stepped-GRFT, which only traverses radial motion parameters, the detection algorithm for maneuvering weak targets defined in Eq. (7) for stepped frequency signals is called angular-stepped-GRFT in this paper.

3.2. Discrete angular-stepped-GRFT

Let the initial radial range of the target be $r \in \left[r_{\min}, r_{\max}\right]$, for data with range window, it is usually the case $r_{\min} = 0$, $r_{\max} = r_l$, r_l is the distance of range window. The velocity search range is $v \in \left[v_{\min}, v_{\max}\right]$, the angular velocity search range is $\omega \in \left[\omega_{\min}, \omega_{\max}\right]$. Because of the coherent accumulation time $T = MN * PRT$, the Doppler resolution of the doppler frequency filter can be written as

$$\rho_d = \frac{1}{T} \tag{8}$$

Therefore, the velocity search interval can be

$$\Delta_v = \frac{\lambda \rho_d}{2} = \frac{\lambda}{2T} = \frac{c}{2MNf_h PRT} \tag{9}$$

where, $f_h = f_0 + h\Delta f$, it can be found that the velocity search interval decreases with the increase of carrier frequency. For the convenience of calculation, the velocity search interval is selected as

$$\Delta_v = \frac{c}{2MNf_{aver} PRT} \tag{10}$$

where, $f_{aver} = f_0 + \frac{N-1}{2}\Delta f$ is the average carrier frequency. Then the velocity search number can be obtained as

$$N_v = \text{round}(\frac{v_{\max} - v_{\min}}{\Delta_v}) \tag{11}$$

where, round() denotes integer operation.

Assume that the system sampling rate f_s is given and the distance search interval is

$$\Delta_r = \frac{c}{2f_s} \tag{12}$$

Then the distance search number can be obtained as

$$N_r = \text{round}(\frac{r_l}{\Delta_r}) \tag{13}$$

The angular velocity search interval must meet

$$v\cos(\omega - \Delta\omega)T - v\cos\omega T \le \Delta_v \tag{14}$$

That is, the search interval of angular velocity which reflect the change of radial velocity should less than that of velocity search interval. Let $\omega = \omega_{\max}$ ($\omega_{\max} \le \frac{\pi}{4}$), $v = v_{\max}$. According to Eq. (14), the search interval of angular velocity is

$$\Delta\omega \le \omega_{\max} - \frac{1}{T}\arccos(\frac{\Delta v}{v_{\max}} + \cos\omega_{\max}T) \tag{15}$$

Then the angular velocity search number can be obtained as

$$N_\omega = \text{round}(\frac{\omega_{\max} - \omega_{\min}}{\Delta_\omega}) \tag{16}$$

Velocity dimension, distance dimension and angular velocity dimension search vector can be expressed as

$$\begin{aligned} v(i) &= v_{\min} + i\Delta_v, \quad i = 0,1,\cdots,N_v - 1 \\ r(p) &= p\Delta_r, \quad p = 0,1,\cdots,N_r - 1 \\ \omega(q) &= \omega_{\min} + q\Delta_\omega, \quad q = 0,1,\cdots,N_\omega - 1 \end{aligned} \tag{17}$$

Substituting Eq. (17) into Eq. (6), the discretized phase compensation factor can be expressed as

$$\Phi(n,i,p,q) = e^{j\frac{4\pi f_h\sqrt{(2-2\cos\omega(q)t+\cos^2\omega(q)t)r(p)^2 - \frac{2v(i)r(p)}{\omega(q)}\sin\omega(q)t + \frac{v(i)^2}{\omega(q)^2}\sin^2\omega(q)t}}{c}} \tag{18}$$

Substituting Eq. (17) and Eq. (18) into Eq. (7), the expression of discretized angular-stepped-GRFT can be obtained as follows

$$G(n,i,p,q)=\sum_{n=0}^{MN-1} s_{Rm}\left(n,\sqrt{\begin{array}{l}(2-2\cos\omega(q)t+\cos^2\omega(q)t)r(p)^2\\ -\dfrac{2v(i)r(p)}{\omega(q)}\sin\omega(q)t\\ +\dfrac{v(i)^2}{\omega(q)^2}\sin^2\omega(q)t\end{array}}\right)\Phi(n,i,p,q) \tag{19}$$

After the radar echo is processed by angular-stepped-GRFT, the estimation results of target motion parameters can be obtained by detecting the maximum value of range-velocity-angular velocity three-dimensional space.

3.3. Motion compensation and range profile focusing

Correct motion compensation is a prerequisite for range profile focusing. According to the motion parameters estimation result of angular-stepped-GRFT, motion compensation is carried out for maneuvering weak target in frequency domain

$$S_{Rm2}(n,f)=S_{Rm}(n,f)e^{-j4\pi f\left(\hat{r}-\sqrt{(2-2\cos\hat{\omega}t+\cos^2\hat{\omega}t)\hat{r}^2-\frac{2\hat{v}\hat{r}}{\hat{\omega}}\sin\hat{\omega}t+\frac{\hat{v}^2}{\hat{\omega}^2}\sin^2\hat{\omega}t}\right)/c} \tag{20}$$

where, $f\in[-f_s/2,f_s/2]$ is the fast time frequency, $S_{Rm}(n,f)$ is the fast time Fourier transform of Eq. (4) to the frequency domain.

$$\hat{t}_p=2\left(\hat{r}-\sqrt{(2-2\cos\hat{\omega}t+\cos^2\hat{\omega}t)\hat{r}^2-\frac{2\hat{v}\hat{r}}{\hat{\omega}}\sin\hat{\omega}t+\frac{\hat{v}^2}{\hat{\omega}^2}\sin^2\hat{\omega}t}\right)/c \tag{21}$$

$\hat{t}_p$ is the range shift for each pulse.

After motion compensation based on angular-stepped-GRFT, broadband is synthesized by spectrum synthetic method to obtain the range profile of each frame signal. Since the target is a weak target with low SNR, the range profile of each frame signal can be summed in the slow time dimension to obtain the range profile after coherent accumulation. That is, the focus of the maneuvering weak target's range profile is realized.

4. Performance analysis by simulations

4.1. Simulation parameters

In order to verify the effectiveness of angular-stepped-GRFT algorithm proposed in this paper in detail, this section carries out simulation experiments based on the target motion scene mentioned in section 2 and compares the performance with the similar coherent accumulation methods, stepped-RFT and stepped-GRFT.

The main parameters of radar system and target in simulation are shown in Table 1

Table 1. The parameters of radar system and target

Parameter	Value
Pulse Width	20μs
Bandwidth	16MHz
Pulse Repetition Time	1050μs
Coherent Processing Interval	2016ms
Initial Carrier Frequency	1.5GHz
Frequency Step Interval	8MHz
Step Number	64
Frames Number	30
Sampling Rate	25MHz
Initial Target Range	60km
Target Velocity	200m/s
Target Angular Velocity	0.2rad/s
SNR(After pulse compression)	-10dB

4.2. Performance analysis of angular-stepped-GRFT and similar algorithms

4.2.1. Performance analysis of stepped-RFT

Stepped-RFT is a range-velocity two-dimensional parameters search algorithm for radially moving targets in stepped frequency radar system. When using stepped-RFT, set the velocity search range to $100\ \mathrm{m/s} \sim 300\mathrm{m/s}$. The parameters estimation results obtained by stepped-RFT are shown in Figure 1. The initial radial distance of the target is estimated to be 58.404km, and the error is 1.596km. The target velocity is estimated to be $214.912\ \mathrm{m/s}$ with an error of $14.912\ \mathrm{m/s}$. Figure 2 shows the target range profile obtained after motion compensation, synthetic broadband processing and coherent accumulation. It can be seen that the motion parameters of maneuvering weak targets cannot be accurately estimated and compensated by stepped-RFT. In the range profile after coherent accumulation, the target is submerged in noise, and the range profile cannot be correctly focused.

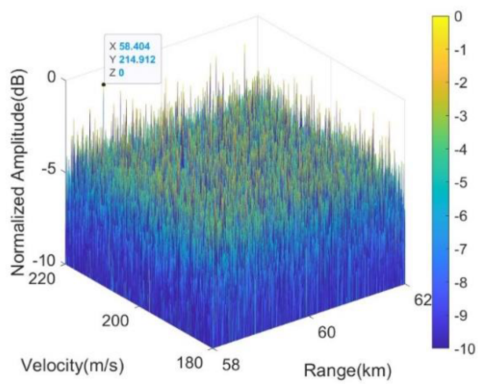

Figure 1. Stepped-RFT parameters estimation result

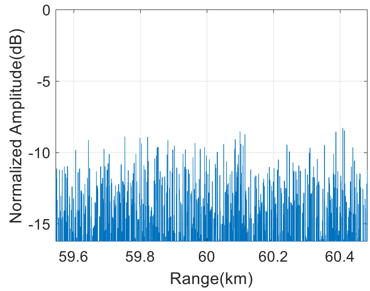

Figure 2. Range profile after motion compensation (Hamming Windows are used in the frequency domain)

4.2.2. Performance analysis of stepped-GRFT

Stepped-GRFT is a range-velocity-high order acceleration search algorithm for radially moving targets under stepped frequency radar system. In this paper, range-velocity-acceleration three-dimensional stepped-GRFT is selected for simulation which has the same search dimension as angular-stepped-GRFT. When three-dimensional stepped-GRFT is used, the velocity search range is set as $100\ \mathrm{m/s} \sim 300\mathrm{m/s}$, and the acceleration search range is set as $0 \sim 16\mathrm{m/s^2}$, and ensure that the acceleration search range covers the variation range of radial acceleration during target maneuver. Figure 3 and Figure 4 show the estimation results of the initial radial distance, initial velocity and acceleration of the target using three-dimensional stepped-GRFT. The initial radial distance of the target is estimated to be 62.28km, and the error is 2.28km. The target velocity is estimated to be $250.934\ \mathrm{m/s}$, and the error is $50.934\ \mathrm{m/s}$. The target acceleration is estimated to be $1.2366\ \mathrm{m/s^2}$. Figure 5 shows the target range profile obtained after motion compensation, synthetic broadband processing and coherent accumulation. It can be seen that three-dimensional stepped-GRFT cannot achieve accurate estimation and compensation of the motion parameters of maneuvering weak target. In the range profile after coherent accumulation, the target is submerged in noise, and the range profile cannot be correctly focused. Three-dimensional stepped-GRFT algorithm approximates the motion parameters of the target by searching the range-velocity-acceleration parameters. However, when the target maneuvers, the radial acceleration changes, which leads to the incomplete motion compensation based on three-dimensional stepped-GRFT. And then the range profile can not be focused. Elected to take a higher order acceleration during stepped-GRFT parameters search, while it is possible to reduce the error of motion parameters estimation, the estimation results are more close to actual motion state. But the use of higher order radial acceleration to describe the target motion state means that the four-dimensional search and more multi-dimensional search. Compared with the three-dimensional search of angular-stepped-GRFT, multi-dimensional search will bring huge calculation burden which have no realizability.

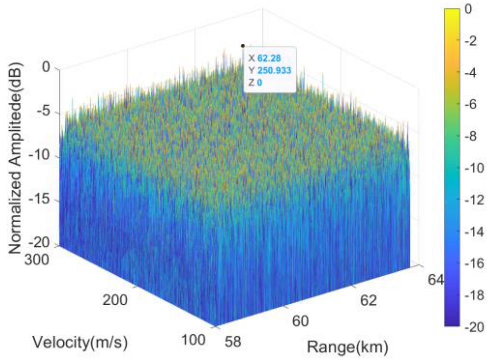

Figure 3. Stepped-GRFT parameters estimation result (range-velocity plane)

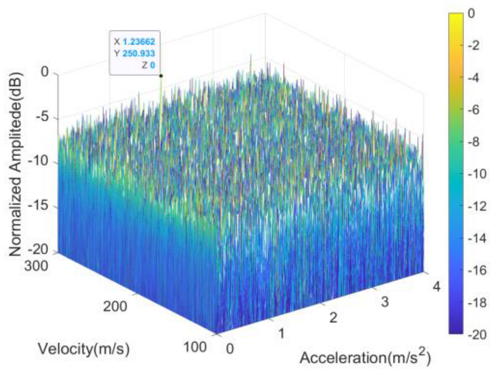

Figure 4. Stepped-GRFT parameters estimation result (acceleration-velocity plane)

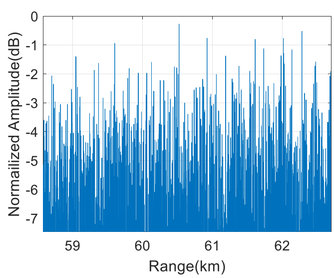

Figure 5. Range profile after motion compensation (hamming windows are used in the frequency domain)

4.2.3. Performance analysis of angular-stepped-GRFT

When angular-stepped-GRFT is used for parameters estimation, the velocity search range is set as $100\ \mathrm{m/s} \sim 300\mathrm{m/s}$, and the angular velocity search range is $0 \sim 0.3\mathrm{rad/s}$. As a comparison of stepped-RFT and stepped-GRFT, Figure 6 and Figure 7 show the estimation results of the initial distance, initial velocity and angular velocity of the target based on angular-stepped-GRFT. The initial distance of the target is estimated to be 60.0km, and the error is 0. The initial velocity was estimated to be $199.994\ \mathrm{m/s}$ with an error of $0.006\ \mathrm{m/s}$. The angular velocity is estimated to be $0.19994\ \mathrm{rad/s}$ with error of

6×10^{-5} rad/s . Figure 8 and Figure 9 shows the coherent accumulation results of target range profile obtained by using the proposed angular-stepped-GRFT algorithm. The peak value of the target can be clearly observed from the result of range profile accumulation, and the initial distance is measured to be 60.0km, which is consistent with the simulation conditions. Obviously, when the stepped frequency signal is used to measure the motion parameters of maneuvering weak target, the target motion parameters estimation based on angular-stepped-GRFT and the range profile focusing algorithm proposed in this paper have better performance.

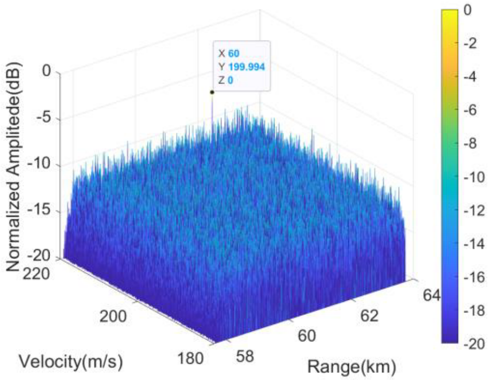

Figure 6. Angular-stepped-GRFT parameters estimation result (range-velocity plane)

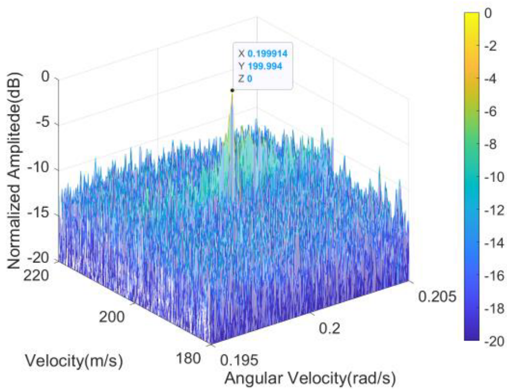

Figure 7. Angular-stepped-GRFT parameters estimation result (acceleration-velocity plane)

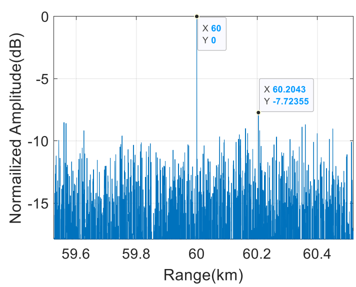

Figure 8. Range profile after motion compensation (hamming window is used in the frequency domain)

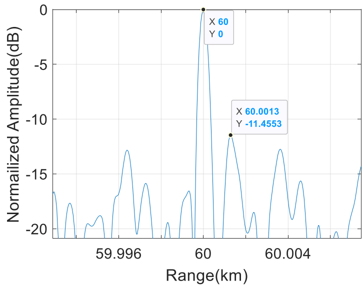

Figure 9. Enlarged range profile after motion compensation (hamming windows is used in the frequency domain)

5. Conclusion

In this paper, aiming at the problem of accurate estimation of motion parameters of maneuvering weak target by stepped frequency radar and range profile focusing, angular-stepped-GRFT is proposed, which realizes the estimation of motion parameters of weak target in circular maneuver scene with long time coherent accumulation. On this basis, the motion parameters compensation and target range profile focusing method are studied, and compared with stepped-GRFT and other methods. The comparative results are shown in Table 2. It is proved that angular-stepped-GRFT can effectively improve the range profile's ability of stepped frequency radar for maneuvering weak target.

Table 2. The comparative results

Algorithm	Search dimension	Range Profile Result
Stepped-RFT	Range-Velocity	No Target
Stepped-GRFT	Range-Velocity-Acceleration	No Target
Angular-Stepped-GRFT	Range-Velocity-Angular Velocity	Clear Target

6. Acknowledgements

This study was supported in part by the Beijing Institute of Technology Research Fund Program for Young Scholars, in part by the National Science Fund for Distinguished Young Scholars under Grant 62201048.

References

[1] Huang P, Liao G, Yang Z. Approach for space-based radar manoeuvring target detection and high-order motion parameter estimation. IET Radar, Sonar & Navigation. 2015; 9(6): 732-741.

[2] Tian J. Study on Long-time Coherent Integration for Radar Maneuvering Targets. Beijing: Beijing University of Technology Press; 2014. 32p.

[3] Perry R P, Dipietro R C, Fante R L. SAR imaging of moving targets. IEEE Transactions on Aerospace and Electronic Systems. 1999; 35(1): 188-200.

[4] Zhang S, Zeng T, Long T. Dim target detection based on keystone transform. IEEE International Radar Conference. 2005; 889-894.

[5] Zhang S, Zhang W, Wang Y. Multiple targets' detection in terms of Keystone transform at the low SNR level. 2008 International Conference on Information and Automation. 2008; 1-4.

[6] Xu J, Yu J, Peng Y N. Radon-Fourier Transform for Radar Target Detection(I): Generalized Doppler Filter Bank. IEEE Transaction on Aearospace and Electronic Systems. 2011; 47(2): 1186-1202.
[7] Xu J, Yu J, Peng Y N. Radon-Fourier transform for radar target detection (II): Blind speed sidelobe suppression. IEEE Transactions on Aerospace and Electronic Systems. 2011; 47(4): 2473-2489.
[8] Yu J, Xu J, Peng Y N. Radon-Fourier transform for radar target detection (III): Optimality and fast implementations. IEEE Transactions on Aerospace and Electronic Systems. 2012; 48(2): 991-1004.
[9] Liu Q, Guo J, Liang Z. Motion Parameter Estimation and HRRP Construction for High-Speed Weak Targets Based on Modified GRFT for Synthetic-Wideband Radar With PRF Jittering. IEEE Sensors Journal. 2021; 21(20): 23234-23244.
[10] Wehner D R. High-Resolution Radar. Boston: Artech House Press; 1995. 68p.
[11] Ingwersen P A, Lemnios W Z. Radars for ballistic missile defense research. Lincoln Laboratory Journal. 2000; 12(2): 245-266.
[12] Myers H, Moore R, Butler W. Common aperture multiband radar (CAMBR). Proceedings of the 1997 IEEE National Radar Conference. IEEE. 1997; 143-148.
[13] Ulander L M H, Frolind P O, Gustavsson A. Performance of the CARABAS-II VHF-band synthetic aperture radar. IEEE 2001 International Geoscience and Remote Sensing Symposium. 2001; 1: 129-131.
[14] Hellsten H, Ulander L M H, Gustavsson A. Development of vhf carabas ii sar. Radar Sensor Technology. International Society for Optics and Photonics. 1996; 2747: 48-60.
[15] Myers H, Moore R, Butler W. Common aperture multiband radar (CAMBR). Radar Processing, Technology, and Applications. 1996; 2845: 2-13.
[16] Singh P. FQTSFM: A fuzzy-quantum time series forecasting model. Information Sciences. 2021; 566: 57-79.