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N-Compactness of Intuitionistic L-Fuzzy Topological Space

Jiaxin GAO, Xiaoxia WANG[1] and Qiaoqiao LI
College of Mathematics and Computer Science, Yan'an University, Yan'an 716000, China

Abstract. In this paper, we extend the N-compactness in L-Fuzzy Topological space to intuitionistic L-Fuzzy Topological space. Firstly, intuitionistic fuzzy N-compactness is defined by α-quasi-coincident family in an intuitionistic L-Fuzzy Topological space, and its equivalent characterization is given by using intuitionistic fuzzy α-net and intuitionistic fuzzy filter. Secondly, it is proved that intuitionistic fuzzy N-compactness has closed heritability; The union of a finite number of intuitionistic fuzzy N-compact sets is still intuitionistic fuzzy N-compact set and intuitionistic fuzzy N-compactness is topological invariance. This research enriches the theoretical system of intuitionistic L-fuzzy Topological space, and provides a certain reference value for the related theoretical research of intuitionistic L-fuzzy Topological space.

Keywords. Intuitionistic L-fuzzy Topological space; N-compactness; Closed heritability; Topological invariance

1. Introduction

In 1983, Atanassov [1] extended the fuzzy set firstly proposed by Zadeh [2] in 1965 and introducing non membership degrees in fuzzy set theory proposed the concept of intuitionistic fuzzy sets. Subsequently, Coker [3] was influenced by the concept of fuzzy topology in the sense of Chang [4], and used intuitionistic fuzzy sets to propose the concept of intuitionistic fuzzy topological spaces and studied their connectivity and separability. The membership function value of traditional fuzzy sets is only a single value, and in practical applications, it cannot simultaneously describe affirmation, negation, and uncertainty. On the basis of fuzzy sets, intuitionistic fuzzy sets add non membership functions and hesitancy, describing fuzzy concepts that are neither definite nor negative, and better characterization of fuzziness and uncertainty. Later, Atanassov and Stoeva[5] further extended the concept of intuitionistic fuzzy sets to residual lattices and proposed intuitionistic *L*-fuzzy sets (i.e. it is based on intuitionistic fuzzy sets on residual lattices). It has more flexibility and applicability than intuitionistic fuzzy sets when dealing with fuzziness problems than intuitionistic fuzzy sets.

Fuzzy sets, intuitionistic fuzzy sets, and their extended forms play a crucial role in decision-making, data analysis, and healthcare in recent years. For example, Xu and Fang [6]mapped medical images into intuitionistic fuzzy sets and applied them to subsequent image segmentation processes, converting images into fuzzy numbers and performing

[1]Corresponding Author, Xiaoxia WANG, Department of Mathematics and Computer Science, Yan'an University, China; E-mail: yd-wxx@163.com.

segmentation; Yager [7] proposed Generalized orthopair fuzzy sets, which is a more general form of intuitionistic fuzzy sets, allowing decision-making analysis to have a larger decision-making space and stronger decision-making information processing capabilities[8]; Peng[9]proposed Pythagorean fuzzy soft set, which combines Pythagorean fuzzy sets with soft set theory, expanding the application scope of intuitionistic fuzzy soft sets, making evaluation results more objective and authentic, and its application in stock investment. Topological spaces composed of different fuzzy sets, their properties have important applications in different disciplinary fields. For example, Gong[10] used the properties in topological space to determine the existence of defects in C language, which is beneficial for developing C language software more effectively and safely. Wang [11] analyzed the topological forms of Chinese classical gardens, identified various topological phenomena in modern urban landscape design, and analyzed the application of topological principles in landscape design. Qian [12] applied topological properties to trajectory planning of intelligent driving vehicles in complex traffic scenarios.

On this basis, in the third section of this paper provides the concept and related knowledge of intuitionistic *L*-fuzzy topological spaces. In the fourth section of this paper extends the *N-c*ompactness in *L*-Fuzzy topological spaces to intuitionistic *L*-fuzzy topological spaces. Firstly, in intuitionistic *L*-fuzzy topological spaces, intuitionistic fuzzy *N*-compactness is defined be α -quasi-coincident family, and its equivalent characterization is given using intuitionistic fuzzy α -net and intuitionistic fuzzy filter; Secondly, it was proved that its intuitionistic fuzzy *N*-compactness has closed heritability, topological invariance, and a finite number of intuitionistic fuzzy *N*-compactness sets, which are still intuitionistic fuzzy *N*-compactness sets, and so on. This study enriches the theoretical system of intuitionistic *L*-fuzzy topological spaces and provides certain reference value for related theoretical research on intuitionistic *L*-fuzzy topological spaces. Symbols and concepts not explained in this article can be found in reference [13].

2. Preliminary Knowledge

In this section, we will review some basic concepts of complete regular residual lattices and intuitionistic *L*-fuzzy sets.

Definition 2.1 [14] The algebraic structure $\mathcal{L}=(L,\wedge,\vee,*,\to,0,1)$ is called a residual lattice if $\mathcal{L}$ satisfies the following conditions:

(1) $(L,\wedge,\vee)$ is a bounded lattice with a partial order relationship, and 0,1 are its minimum and maximum elements, respectively;

(2) $(L,*,1)$ is a commutative monoid, and $*$ preserves order for each argument;

(3) $x*y\le z$ if and only if $x\le y\to z$, for any $x,y,z\in L$.

If $(L,\wedge,\vee)$ is complete, then the remaining lattice $\mathcal{L}=(L,\wedge,\vee,*,\to,0,1)$ is called complete.

Definition2.2[15] Defines the negation operator on a residue lattice $\neg:L\to L:\neg a=a\to 0$, for any $a\in L$ .if $\neg\neg a=a$,for any $a\in L$,then $\mathcal{L}=(L,\wedge,\vee,*,\to,0,1)$ is called a regular residue lattice. In fact, the negation operator on a regular residue lattice is an order-reversing-involution.

Definition 2.3[16] $\mathcal{L}=(L,\wedge,\vee,*,\to,0,1)$ is complete regular residue lattice, set $\bar{\mathcal{L}}=\{(x_1, x_2)\in\mathcal{L}\times\mathcal{L};x_1\le\neg x_2\}$, define the binary relationship $\le_{\bar{\mathcal{L}}}$ on the set $\bar{\mathcal{L}}$: $\forall x=(x_1,x_2),\ y=$

$(y_1,y_2)\in\bar{\mathcal{L}}$, $x\le_{\bar{\mathcal{L}}} y \Leftrightarrow x_1\le y_1$ and $x_2\ge y_2$ .Then it can be verified that $\le_{\bar{\mathcal{L}}}$ is a partial order relationship on $\bar{\mathcal{L}}$, and $(\bar{\mathcal{L}},\le_{\bar{\mathcal{L}}})$ is a complete lattice with minimum and maximum elements of $0_{\bar{\mathcal{L}}}=(0,1)$ 、 $1_{\bar{\mathcal{L}}}=(1,0)$ .The intersection operation on the lattice $\bar{\mathcal{L}}$ about the partial order relationship $\le_{\bar{\mathcal{L}}}$ is as follows: $x\wedge_{\bar{\mathcal{L}}} y=(x_1\wedge y_2,x_2\vee y_2)$ .Since the lattice $\bar{\mathcal{L}}$ is a complete lattice, for $\{x^i=(x_1^i,x_2^i)\in\bar{\mathcal{L}};i\in\Gamma\}$, any intersection and union operations on the lattice $\mathcal{L}$ are as follows: $(\underset{i\in\Gamma}{\wedge})_{\bar{\mathcal{L}}}x^i=(\underset{i\in\Gamma}{\wedge}x_1^i,\underset{i\in\Gamma}{\vee}x_2^i)$.

Definition 2.4 [16] If X be a nonempty set, then $A=\{<x,\mu_A(x),\gamma_A(x)>;x\in X\}$ is an intuitionistic L-fuzzy set on X, where $\mu_A:X\to\mathcal{L}$, $\gamma_A:X\to\mathcal{L}$ satisfies the condition $\mu_A(x)\le\neg\gamma_A(x)$,for each $x\in X$. Where $\mu_A(x)$ represents the L-membership of x to A, and $\gamma_A(x)$ represents the L-non membership of x to A. All intuitionistic L-fuzzy sets on X are denoted as ζ^X .Note $A(x)=(\mu_A(x),\gamma_A(x))$, $A\in\zeta^X$ if and only if $A(x)\in\bar{\mathcal{L}}$, for each $x\in X$. Note that the maximum element of the intuitionistic L-fuzzy set is $(\tilde{1},\tilde{0})=\{<x,1,0>,x\in X\}$, and the minimum element is $(\tilde{0},\tilde{1})=\{<x,0,1>,x\in X\}$.

3. Intuitionistic L-fuzzy Topological Space Related Knowledge

Below, we provide the definition of intuitionistic L-fuzzy topological spaces and some basic definitions and conclusions related to intuitionistic L-fuzzy topological spaces.

Definition 3.1 Let X be a nonempty set, and ζ^X be all intuitionistic L-fuzzy sets on X, δ : $\zeta^X\to\bar{\mathcal{L}}$ satisfies the following conditions:(1) $\delta((\tilde{1},\tilde{0}))=\delta((\tilde{0},\tilde{1}))=1_{\bar{\mathcal{L}}}$;(2) $\forall A,B\in\zeta^X$, $\delta(A\wedge B)\ge\delta(A)\wedge\delta(B)$; (3) $\forall A_t\in\zeta^X,t\in T,\delta(\underset{t\in T}{\vee}A_t)\ge\underset{t\in T}{\wedge}\delta(A_t)$.

Then δ is called an intuitionistic L-fuzzy topology on X, and the pair (X,δ) is an intuitionistic L-fuzzy topological space ($IL-fts$ for short).

Definition 3.2 Let X be a nonempty set, $A=\{<y,\mu_A(y),\gamma_A(y)>,y\in X\}\in\zeta^X$, $\alpha,\beta\in L$, $\alpha\le\neg\beta$, satisfying $A(y)=\begin{cases}(\alpha,\beta) & if\ \ y=x,\\ (0,1) & if\ \ y\ne x.\end{cases}$, then A is called an intuitionistic L fuzzy point. Meanwhile, A is denoted as $x_{(\alpha,\beta)}$ .Record $M(L)$ as the set of molecules in L; When $\alpha,\beta\in M(L)$, $x_{(\alpha,\beta)}$ is called an intuitionistic LF molecule. $\{x\in X,\mu_A(x)>0,\ \gamma_A(x)<1\}$ is called the support of A, denoted as $SuppA$. x is the support point of $x_{(\alpha,\beta)}$ and α is its height. The set of all intuitionistic L fuzzy points $x_{(\alpha,\beta)}$ is denoted as $Pt(\zeta^X)$.

Definition 3.3 Let $A\in\zeta^X$, $x_{(\alpha,\beta)}\in Pt(\zeta^X)$ .If $x_{(\alpha,\beta)}\notin A'$ (i.e. $\alpha>\gamma_A(x)$ or $\beta<\mu_A(x)$),we say that $x_{(\alpha,\beta)}$ quasi-coincides with A, denoted as $Aqx_{(\alpha,\beta)}$.

Definition 3.4 Let (X,δ) be $IL-fts$, $A\in\zeta^X$, $x_{(\alpha,\beta)}\in Pt(\zeta^X)$,and if there is $B\in\delta$ such that $x_{(\alpha,\beta)}qB\subset A$,then A is called the intuitionistic quasi-coincident neighborhood of $x_{(\alpha,\beta)}$ .The family of all the intuitionistic quasi-coincident neighborhood of $x_{(\alpha,\beta)}$ is called the system of intuitionistic quasi-coincident neighborhood of $x_{(\alpha,\beta)}$,denoted by $\mathcal{N}(x_{(\alpha,\beta)})$.

Definition 3.5 Let (X,δ) be $IL-fts$, $A\in\zeta^X$, $x_{(\alpha,\beta)}\in Pt(\zeta^X)$. If there is PqA for each $P\in\mathcal{N}(x_{(\alpha,\beta)})$, then $x_{(\alpha,\beta)}$ is the intuitionistic L fuzzy adherence point of A. If $x_{(\alpha,\beta)}$ is the intuitionistic L fuzzy adherence point of A, and $x_{(\alpha,\beta)}\in A$, $P\in\mathcal{N}(x_{(\alpha,\beta)})$, if P quasi-coincident with the intuitionistic L fuzzy points in A that are different from $x_{(\alpha,\beta)}$, then $x_{(\alpha,\beta)}$ is called intuitionistic L fuzzy accumulation point.

Definition 3.6 Let (X,δ) be $IL-fts$ and X be a nonempty set, $\mathcal{F}$ is a family of intuitionistic L-fuzzy sets, if it satisfies the following conditions: (1) If $A_1,A_2\in\mathcal{F}$,then $A_1\wedge A_2\in\mathcal{F}$; (2) If $A\in\mathcal{F}$, for each intuitionistic L-fuzzy set B such that $A\subseteq B$, we have $B\in\mathcal{F}$; We call $\mathcal{F}$ an intuitionistic L fuzzy filter on X. Let $\alpha\in M(L)$, if Min $\{ht(A), A\in F\}=\alpha$,for each $F\in\mathcal{F}$, where $ht(A)=Max\ \{\mu_A(x),x\in X\}$, then $\mathcal{F}$ is called an intuitionistic L fuzzy α -filter on X.

Definition 3.7 Let (X,δ) be $IL-fts$, $\mathcal{F}$ an intuitionistic L fuzzy filter on X. If for each $F\in\mathcal{F}$,we have $F\not\subset P$,then $x_{(\alpha,\beta)}$ is called an intuitionistic L fuzzy adherence point of $\mathcal{F}$.

Definition 3.8 Let (X,δ) be $IL-fts$, D be an directed set, and X be a nonempty set, $P\in\mathcal{N}(x_{(\alpha,\beta)})$ and the mapping $S:D\to Pt(\zeta^X)$ be an intuitionistic L fuzzy net on X and is denoted as $S=(S(n),n\in D)$ .If $S(n)=\{x^n_{(\alpha_n,\beta_n)},x\in X\}\in A$, for any $n\in D$,then S is called intuitionistic L fuzzy net of A.

Definition 3.9 Let (X,δ) be $IL-fts$, $x_{(\alpha,\beta)}\in Pt(\zeta^X)$, $S=(S(n),n\in D)$ .If for each $P\in\mathcal{N}(x_{(\alpha,\beta)})$, the eventually SqP (i.e. eventually $S(n)\notin P$), then the intuitionistic L fuzzy net S converges to $x_{(\alpha,\beta)}$ .If for each $P\in\mathcal{N}(x_{(\alpha,\beta)})$,the frequently SqP (i.e. frequently $S(n)\notin P$),intuitionistic L fuzzy net S with $x_{(\alpha,\beta)}$ as the accumulation point .

Definition 3.10 Let (X,δ) and (Y,τ) be $IL-fts$, $f:X\to Y$ is a mapping. If for each $B\in\tau$,has $f^{-1}(B)\in\delta$, then f is called a continuous mapping.

4. Intuitionistic Fuzzy N-compactness and Equivalent Characterization

In this section, we discussed and explored N-compactness in intuitionistic L-fuzzy topological spaces, and obtained some related results.

Definition 4.1 Let (X,δ) be $IL-fts$, $A\in\zeta^X$, $\Phi\in\delta'$, $\alpha\in M(L)$. If there is $P\in\Phi$, such that $P\in\mathcal{N}(x_{(\alpha,\beta)})$, for each intuitionistic LF molecule $x_{(\alpha,\beta)}$ with a height of α in A, then Φ is called the α -quasi-coincident neighborhood family of A. If there is $r\in\beta^*(\alpha)$, such that Φ is the r -quasi-coincident neighborhood family of A, then Φ is called the α^- -quasi-coincident neighborhood family of A.

Definition 4.2 Let (X,δ) be $IL-fts$, $A\in\zeta^X$,If there is a finite subfamily ψ for any α -quasi-coincident neighborhood family Φ of A, such that ψ constitutes α^- -quasi-coincident neighborhood family of A, then A is called intuitionistic fuzzy N-compact set .If the maximum intuitionistic L-fuzzy set $(\tilde{1},\tilde{0})$ is a N-compact set, then (X,δ) is called an intuitionistic fuzzy N- compact space.

Theorem 4.3 Let (X,δ) be $IL-fts$, $A\in\zeta^X$, $\forall\alpha\in M(L)$. If A is an intuitionistic fuzzy N-compact set, if and only if it satisfies the following conditions: (1) For every α -quasi-coincident neighborhood Φ of A there is finite α -quasi-coincident neighborhood subfamilies; (2) The α -quasi-coincident neighborhood family $\Phi=\{P\}$ of A composed by a closed set is also α^- -quasi-coincident neighborhood family of A.

Proof: $\Leftarrow$ The hypothesis is that holds, Φ is α -quasi-coincident neighborhood family of A. From (1), it can be inferred that Φ all have finite subfamilies ψ to form α -quasi-coincident neighborhood family of A, let $\{P\}$ is also α -quasi-coincident neighborhood family of A. From (2), it can be inferred that $\{P\}$ is also α^- -quasi-coincident neighborhood family of A. i.e. there is a molecule $r\in\beta^*(\alpha)$ in $M(L)$, so that $r\nleq P(x)=\wedge\{Q(x),Q\in\psi\}$,for any intuitionistic LF molecule $x_{(r,\beta)}$ in A. Exist $Q\in\psi$,so that $r\nleq Q(x)$ i.e. $Q\in\mathcal{N}(x_{(r,\beta)})$, so ψ is α^- -quasi-coincident neighborhood family of A. Therefore, A is an intuitionistic fuzzy N- compact set.

$\Rightarrow$ Let A is an intuitionistic fuzzy N-compact set, Φ is α -quasi-coincident neighborhood family, then Φ both have finite subfamilies ψ to form α^- -quasi-coincident neighborhood family of A. Obviously, ψ is also α -quasi-coincident neighborhood subfamily of Φ .Therefore, condition (1) holds. If $\Phi=\{P\}$ is α^- -quasi-coincident neighborhood family of A, then there are finite subfamilies ψ of Φ to form α^--quasi-coincident neighborhood family of A. And at this point, there can only be $\psi=\Phi$, so $\Phi=\{P\}$ is also α^- -quasi-coincident neighborhood family of A, thus condition (2) holds.

Definition 4.4 Let (X,δ) be $IL-fts$, D be the directed set, $S=(S(n),n\in D)$ be the intuitionistic L fuzzy net in X. Use $V(S(n))$ to represent the height of $S(n)$. Let $V(S)=\{V(S(n)),n\in D\}$ be the value net of S. Let $\alpha\in M(L)$, if for any $r\in\beta^*(\alpha)$, $V(S(n))$ is eventually greater than or equal to r (i.e. exists $n_0\in D$,when $n\geq n_0$, $V(S(n))\geq r$),then net S is called an intuitionistic L fuzzy α -net.

Theorem 4.5 Let (X,δ) be $IL-fts$,if A is an intuitionistic fuzzy N-compact set iff for each $\alpha\in M(L)$, intuitionistic L fuzzy α -net in A has accumulation point in A with height α .

Proof: $\Rightarrow$ If A is an intuitionistic fuzzy N-compact set, $S=\{S(n),n\in D\}$ be an intuitionistic L fuzzy α -net in A, and S has no accumulation point in A with height α , then $P(x)\in\mathcal{N}(x_{(\alpha,\beta)})$,for any intuitionistic LF molecule $x_{(\alpha,\beta)}$ in A, exists $n(x)\in D$,when $n\geq n(x)$, $S(n)\leq P(x)$ .Let $\Phi=\{P(x),x_{(\alpha,\beta)}\leq A\}$ be α -quasi-coincident neighbor-hood family of A. Since A is an intuitionistic fuzzy N-compact set, So Φ has finite subfamilies $\psi=\{P(x_i),\ i=1,2,\cdots,k\}$,so that ψ forms α^- -quasi-coincident neighbor-hood family of A,(i.e. there exists $r\in\beta^*(\alpha)$ such that for any intuitionistic LF molecule $y_{(\gamma,\mu)}$ in A has $i\leq k$ such that $y_{(\gamma,\mu)}\nleq P(x_i)$).Let $P=\wedge_{i=1}^{n}P(x_i)$,for any intuitionistic LF molecule $y_{(\gamma,\mu)}$ in A, which has $y_{(\gamma,\mu)}\nleq P$ (i.e. $\forall y_{(\gamma,\mu)}\in A, r\nleq P(y)$).Also, because D is an directed set, then there is $n_0\in D$,such that $n_0\geq n(x_i)(i=1,2,\cdots,k)$ .Then when $n\geq n_0$, $S(n)\leq P(x_i)(i=1,2,\cdots,k)$,so $S(n)\leq P$ when $n\geq n_0$ .Also, because $S(n)\leq A$,when $n\geq n_0$, $r\nleq V(S(n))$ .This contradicts $S=\{S(n),n\in D\}$ is an intuitionistic L fuzzy a-net in A .Therefore, S has at least one accumulation point with a height of α in A.

$\Leftarrow$ Let each intuitionistic *L*-fuzzy α -net in *A* have accumulation point with height α , and Φ be α -quasi-coincident neighborhood family in *A*. Let any finite subfamily of Φ be not α^- -quasi-coincident neighborhood family in *A*(i.e. for each $r \in \beta^*(\alpha)$, $\Phi_0 \in 2^{(\Phi)}$,so that $\wedge\Phi_0 < A(r)$ is not hold). For each pair of r and Φ_0 , there is a point $S(r,\Phi_0) \in \wedge\Phi$ in *A* with a height of r . Let $D = \beta^*(\alpha) \times 2^{(\Phi)}$, it is specified that $(r_1,\Phi_1) \geq (r_2,\Phi_2)$ if and only if $r_1 \geq r_2$, $\Phi_1 \supset \Phi_2$. Since $\beta^*(\alpha)$ and $2^{(\Phi)}$ are both directional sets, then *D* is also a directional set. Let $S = \{S(r,\Phi_0),(r,\Phi_0) \in D\}$ be an intuitionistic *L*-fuzzy net in *A*. Because *S* has accumulation point in *A* with height α .In fact, for a point $x_{(\alpha,\beta)} \in A$, there are $P \in \Phi$, $P(x) \in \mathcal{N}(x_{(\alpha,\beta)})$ (i.e. $x_{(\alpha,\beta)} \notin P$), which indicates that *S* is eventually in *P*. Therefore, $x_{(\alpha,\beta)}$ is not the accumulation point of *S*. This contradicts the hypothesis. Therefore, any finite subfamily of Φ is α^- -quasi-coincident neighborhood family of *A*, that is, *A* is an intuitionistic fuzzy *N*-compact set.

Theorem 4.6 Let (X,δ) be $IL-fts$,if *A* is an intuitionistic fuzzy *N*-compact set iff every intuitionistic *L* fuzzy α -filter containing *A* has an adherence point in *A* with height α .

Proof: $\Leftarrow$ Assuming that the condition is satisfied and $S = \{S(n), n \in D\}$ be an intuitionistic *L* fuzzy α -net in *A*. Let $F_m = V\{S(n), n \geq m\}(m \in D)$, because D is an directed set, we can obtain that the set family $\{F_m, m \in D\}$ can generate an intuitionistic *L* fuzzy filter $\mathcal{F}$,and $A \in \mathcal{F}$ .Because *S* is an intuitionistic *L* fuzzy α -net, for each $r \in \beta^*(\alpha)$, $V(S)$ is eventually greater than r .Therefore, for each $r \in \beta^*(\alpha)$, $\vee_{x \in X} F_m(x) = \vee\{\vee(S(n)),\ n \geq m\} \geq r$, that is $\vee_{x \in X} F_m(x) \geq \alpha$ .Since $\mathcal{F}$ is generated by $\{F_m, m \in D\}$, then each $F \in \mathcal{F}$ contains some F_m , so $\vee_{x \in X} F_m(x) \geq \alpha$. Therefore, $\mathcal{F}$ is an intuitionistic *L* fuzzy α -filter. Because $\mathcal{F}$ has adherence points with height α , $x_{(\alpha,\beta)} \in A \in \mathcal{F}$ .From Definition3.7, it can be concluded that for $P(x) \in \mathcal{N}(x_{(\alpha,\beta)})$, $F \in \mathcal{F}$,there is $F_m \subset F \not\subset P$, which indicates that the intuitionistic *L*-fuzzy net *S* is frequently not in *P*. Therefore, $x_{(\alpha,\beta)}$ is the adherence point of the intuitionistic *L* fuzzy net *S*. From Theorem 4.5, it can be concluded that the intuitionistic *L*-fuzzy set *A* is intuitionistic fuzzy *N*-compact.

$\Rightarrow$ Assuming that *A* is intuitionistic fuzzy *N*-compact set, $\mathcal{F}$ is an intuitionistic *L* fuzzy α -filter containing *A* $(\alpha \in M(L))$,then for each $F \in \mathcal{F}$, $F \wedge A \in \mathcal{F}$ .Since $r \in \beta^*(\alpha) \subset \beta(\alpha)$ satisfies the condition that for each $\varphi \in L$, $\vee\varphi \geq \alpha$, there is $b \in \varphi$,such that $b \geq r$ .For each $F \in \mathcal{F}$,there is $\vee_{x \in X}(F \wedge A)(x) \geq \alpha$ .Therefore, for each $r \in \beta^*(\alpha)$,there is $s(F,r)$ contained in $F \wedge A$, $s(F,r)$ with height r .Assuming $S = \{s(F,r),\ (F,r) \in\ \mathcal{F} \times \beta^*(\alpha)\}$,then *S* is an intuitionistic *L* fuzzy α -net in *A*. Because *A* is intuitionistic fuzzy *N*-compact set, we can obtain that *S* adherence point in *A* with height α .Below, we will prove that $x_{(\alpha,\beta)}$ is also adherence point of $\mathcal{F}$. In fact, for each $P(x) \in \mathcal{N}(x_{(\alpha,\beta)})$,*S* is frequently not in *P*, for each $F \in \mathcal{F}$, there is $F_1 \subset F$, for certain $r \in \beta^*(\alpha)$, $s(F_1,r) \notin P$ .Therefore, based on $s(F_1,r) \in F_1$, $F_1 \subset F$, can be obtained. Therefore, $x_{(\alpha,\beta)}$ is also an adherence point of $\mathcal{F}$.

Theorem 4.7 Let (X,δ) be $IL-fts$, $A \in \zeta^X$ .If the support of *A* is finite, i.e. $\sigma_0(A) = \{x \in X, < x, \mu_A(x) \neq 0, \gamma_A(x) \neq 1 >\}$ is a finite set, then *A* is an intuitionistic fuzzy *N*- compact set.

Proof: Let $\tau_\alpha(A)=\{x_1,x_2,\cdots,x_n\}$, Φ be any α -quasi-coincident neighborhood family of A. $\forall i\le n$,take $P_i\in\Phi$,so that $\alpha\not\le P_i(x_i)$ .Because $\alpha=\sup\beta^*(\alpha)$,there is $r_i\not\le P_i(x_i)$,for $r_i\in\beta^*(\alpha)$ .Also, since α is a molecule in L, it can be concluded that $\beta^*(\alpha)$ is an upper directed set, so there is $r_i\in\beta^*(\alpha)$ such that $r\ge r_i(i\le n)$ .Meanwhile, there is $r_i\not\le P_i(x_i)$,for $\forall i\le n$.The finite subfamily $\Psi=\{P_1,P_2,\cdots,P_n\}$ of Φ is α^- -quasi-coincident neighborhood family of A. Therefore, A is an intuitionistic fuzzy N-compact set.

Theorem 4.8 The union of a finite number of intuitionistic fuzzy N-compact sets remains an intuitionistic fuzzy N-compact set.

Proof : If A is the union of a finite number of intuitionistic fuzzy N-compact sets A_1, $A_2,\cdots,A_n$ in $IL-fts$,i.e. $A=\bigcup_{i=1}^n A_i$ .If Φ is any α -quasi-coincident neighborhood family of A, then Φ is also α -quasi-coincident neighborhood family of $A_i(i=1,2,\cdots,n)$ .And because $A_i(i=1,2,\cdots,n)$ is an intuitionistic fuzzy N-compact set, then Φ has a finite subfamily ψ_i to form a family of α^- -quasi-coincident neighborhood family related to A_i .Therefore, $\psi=\vee_{i=1}^n\psi_i$ is a finite subfamily of Φ and forms α^- -quasi-coincident neighborhood family of A. Therefore, A is an intuitionistic fuzzy N- compact set.

Theorem 4.9 Let (X,δ) be $IL-fts$, A is an intuitionistic fuzzy N-compact subset, B is an intuitionistic L-fuzzy closed set, then $C=A\wedge B$ is also intuitionistic fuzzy N-compact set.

Proof : Let S be an intuitionistic fuzzy α -net in C, then S is also an intuitionistic fuzzy α -net in A .Because A is an intuitionistic fuzzy N-compact subset, then S has accumulation point $x_{(\alpha,\beta)}$ in A with height α . S is also an intuitionistic fuzzy net in B, where $x_{(\alpha,\beta)}$ is the accumulation point of S. It can be concluded that $x_{(\alpha,\beta)}\le B$,thus $x_{(\alpha,\beta)}\le A\wedge B$,i.e. $x_{(\alpha,\beta)}$ is the accumulation point of S in C. Therefore, C is an intuitionistic fuzzy N-compact set.

Inference 4.10 If (X,δ) be $IL-fts$ and A is an intuitionistic fuzzy N- compact set, then each intuitionistic fuzzy closed subset contained in A is also an intuitionistic fuzzy N-compact set.

Lemma 4.11 Let (X,δ) and (Y,τ) be $IL-fts$, $P\in\delta$, $Q\in\tau$, $f:(X,\delta)\to(Y,\tau)$ be a continuous mapping, with $P=f^{-1}(Q)$ .For any the intuitionistic L fuzzy point $x_{(\alpha,\beta)}$ in X satisfies $x_{(\alpha,\beta)}\notin P$ then $f(x_{(\alpha,\beta)})$ is an intuitionistic L fuzzy point in Y, satisfies $f(x_{(\alpha,\beta)})\notin Q$.

Theorem 4.12 Let (X,δ) and (Y,τ) be $IL-fts$, A be an intuitionistic fuzzy N- compact set in X, $f:(X,\delta)\to(Y,\tau)$ is a continuous mapping, then $f(A)$ is an intuitionistic fuzzy N-compact set in Y.

Proof : Let $S=\{S(n),n\in D\}$ be an intuitionistic L fuzzy α -net contained in $B=f(A)$ .For each $n\in D$,there is $S(n)=y^n_{(\gamma_n,\mu_n)}$,and y^n is a support point of $S(n)$,then $f^{-1}(y^n)$ is a crisp set in X. For each $k\in N$,there is a crisp point $x^n\in f^{-1}(y^n)$,such that $\mu_A(x^n)>\mu_B(y^n)-1/k$ .From $S(n)\in B$,we can obtain $V(S(n))\in\mu_B(y^n)$,therefore $\mu_A(x^n)>V(S(n))-1/k$ .For each $n\in D$, $k\in N$,we can choose an intuitionistic L fuzzy point $T((n,k))\in A$,such that $V(S(n))-1/k\le V(S(n,k))\le V(S(n))$, $f(T((n,k)))\in S(n)$ .Assuming $D_1=\{(n,k);n\in D,k\in N\}$,and if $n_1\ge n_2$, $k_1\ge k_2$,whenever $(n_1,k_1)\ge(n_2,k_2)$,then D_1 is an directed set, so

$T=\{T((n,k)),(n,k)\in D_1\}$ is an intuitionistic L fuzzy α -net contained in A. Because A is an intuitionistic fuzzy *N*-compact set, so T has accumulation point $x_{(\alpha,\beta)}\in A$. Let $y_{(\gamma,\mu)}=f(x_{(\alpha,\beta)})$,then $y_{(\gamma,\mu)}$ is the intuitionistic *L*-fuzzy point in *Y*, and $y_{(\gamma,\mu)}\in f(A)$ .Now we will verify that $y_{(\gamma,\mu)}$ is the accumulation point of *S*. Since $x_{(\alpha,\beta)}$ is the accumulation point of T, for each $P\in\mathcal{N}(x_{(\alpha,\beta)})$, $(n,k)\in D$,for certain $(n_0,k_0)\in D$,then $(n,k)\geq(n_0,k_0)$ frequently $T(n,k)\notin P$ .From the lemma, it can be inferred that $f(T(n,k))$ is an intuitionistic *L*-fuzzy point in Y ,when $Q=f(P)\subset Y$,there is frequently $f(T(n,k))\notin Q$ .Because $f(T((n,k)))\in S(n)$,So $y_{(\gamma,\mu)}$ is accumulation point of *S*. Therefore, $f(A)$ is an intuitionistic fuzzy *N*- compact set in *Y*.

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