Data Linkage Dynamics with Shedding*

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Abstract. We study shedding in the setting of data linkage dynamics, a simple model of computation that bears on the use of dynamic data structures in programming. Shedding is complementary to garbage collection. With shedding, each time a link to a data object is updated by a program, it is determined whether or not the link will possibly be used once again by the program, and if not the link is automatically removed. Thus, everything is made garbage as soon as it can be viewed as garbage. By that, the effectiveness of garbage collection becomes maximal.

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1 Introduction

This paper is a sequel to [9]. In that paper, we presented an algebra, called data linkage algebra, of which the elements are intended for modelling the states of computations in which dynamic data structures are involved. We also presented a simple model of computation, called data linkage dynamics, in which states of computations are modelled as elements of data linkage algebra and state changes take place by means of certain actions. Data linkage dynamics includes the following features to reclaim garbage: full garbage collection, restricted garbage collection (as if reference counts are used), safe disposal of potential garbage, and unsafe disposal of potential garbage.

In the current paper, we add shedding to the features of data linkage dynamics. This feature is complementary to the garbage collection features of data linkage dynamics. Roughly speaking, shedding works as follows: each time a link to a data object is updated by a program, it is determined whether or not the link will possibly be used once again by the program, and if not the link is automatically removed. In this way, everything is made garbage as soon as it can be taken for garbage. The point of shedding is that by this, the effectiveness of garbage collection becomes maximal.

In the sixties of the previous century, when the first list-processing languages came up, three basic garbage collection techniques have been proposed: reference

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counting (see e.g. [15, 12]), marking (see e.g. [21, 24]), and copying (see e.g. [22, 14]). The garbage collection techniques that have been proposed in the seventies and eighties of the previous century are mainly incremental and parallel variants of the three basic techniques (see e.g. [4, 20] and [28, 19, 13], respectively), which are intended to avoid substantial interruption due to garbage collection, and conservative and tag-free variants of the three basic techniques (see e.g. [11] and [2, 16], respectively), which are intended to perform garbage collection more efficient. All the garbage collection techniques proposed in those times collect only data objects that are no longer reachable by a series of links. In the next paragraph, we will use the term "standard garbage collection techniques" to refer to this group of garbage collection techniques.

Owing to the growing use of dynamic data structures in programming, the effectiveness of garbage collection techniques becomes increasingly more important since the nineties of the previous century. It has been confirmed by recent empirical studies that standard garbage collection techniques actually leave a lot of garbage uncollected (see e.g. [25, 26, 17]). For the greater part, recently proposed garbage collection techniques that are intended to be more effective than standard garbage collection techniques turn out to make use of approximations of shedding. The approximations are obtained by means of information about future uses of links coming from static program analysis. The information is either directly provided to an adapted standard garbage collector (see e.g. [1]) or used to transform the program in question such that data objects become unreachable as soon as some safety property holds according to the information (see e.g. [27, 18]). In the latter case, the safety property used differs from one proposal to another, can in all cases be improved by taking into account that the number of data objects that can exist at the same time is bounded, and is in all cases at best weakly justified by a precise semantics of the programming language supposed to be used.

Our study of shedding arises from the work on "nullifying dead links" presented in [18]. That work concerns the removal of links that will not possibly be used once again by means of static program analysis and program transformation. In our study of shedding, different from the study in [18], the semantic effects of the fact that the number of data objects that can exist at the same time is always bounded are taken into account.

The view is taken that the behaviours exhibited by programs on execution are threads as considered in basic thread algebra.[1] A thread proceeds by performing actions in a sequential fashion. A thread may perform an action for the purpose of interacting with a service that takes the action as a command to be processed. The processing of the action results in a state change and a reply. In the setting of basic thread algebra, the use mechanism has been introduced to allow for this kind of interaction. The state changes and replies that result from performing the actions of data linkage dynamics can be achieved by means of a service.

[1] In [7], basic thread algebra is introduced under the name basic polarized process algebra. Prompted by the development of thread algebra [8], which is a design on top of it, basic polarized process algebra has been renamed to basic thread algebra.

In [9], it was explained how basic thread algebra can be combined with data linkage dynamics by means of the use mechanism in such a way that the whole can be used for studying issues concerning the use of dynamic data structures in programming. For a clear apprehension of data linkage dynamics as presented in that paper, such a combination is not needed. This is different for shedding, because it cannot be explained without reference to program behaviours. In the current paper, we adapt the data linkage dynamics services involved in the combination described in [9] to explain shedding. For the adapted data linkage dynamics services, shedding happens to be a matter close to reflection on themselves. Moreover, the adapted data linkage dynamics services are services of which the state changes and replies may depend on how the thread that performs the actions being processed will proceed. That is why we also introduce a generalization of the use mechanism to such forecasting services.

This paper is organized as follows. First, we review data linkage algebra, data linkage dynamics and basic thread algebra (Sections 2, 3, and 4). Next, we present the use mechanism for forecasting services and explain how basic thread algebra can be combined with data linkage dynamics by means of that use mechanism (Sections 5 and 6). After that, we introduce the shedding feature and adapt the data linkage dynamics services involved in the combination described before such that they support shedding (Sections 7, 8, and 9). Then, we illustrate shedding by means of some examples (Section 10). Finally, we make some concluding remarks (Section 11).

2 Data Linkage Algebra

In this section, we review the algebraic theory DLA (Data Linkage Algebra). The elements of the initial algebra of DLA can serve for the states of computations in which dynamic data structures are involved.

In DLA, it is assumed that a fixed but arbitrary finite set Spot of *spots*, a fixed but arbitrary finite set Field of *fields*, a fixed but arbitrary finite set AtObj of *atomic objects*, and a fixed but arbitrary finite set Value of *values* have been given.

DLA has one sort: the sort $\mathbf{DL}$ of *data linkages*. To build terms of sort $\mathbf{DL}$, BTA has the following constants and operators:

- for each $s \in \mathsf{Spot}$ and $a \in \mathsf{AtObj}$, the *spot link* constant $\xrightarrow{s} a : \mathbf{DL}$;
- for each $a \in \mathsf{AtObj}$ and $f \in \mathsf{Field}$, the *partial field link* constant $a \xrightarrow{f} : \mathbf{DL}$;
- for each $a, b \in \mathsf{AtObj}$ and $f \in \mathsf{Field}$, the *field link* constant $a \xrightarrow{f} b : \mathbf{DL}$;
- for each $a \in \mathsf{AtObj}$ and $n \in \mathsf{Value}$, the *value association* constant $(a)_n : \mathbf{DL}$;
- the *empty data linkage* constant $\emptyset : \mathbf{DL}$;
- the binary *data linkage combination* operator $\oplus : \mathbf{DL} \times \mathbf{DL} \to \mathbf{DL}$;
- the binary *data linkage overriding combination* operator $\oplus' : \mathbf{DL} \times \mathbf{DL} \to \mathbf{DL}$.

Terms of sort $\mathbf{DL}$ are built as usual. Throughout the paper, we assume that there are infinitely many variables of sort $\mathbf{DL}$, including X, Y, Z. We use infix notation for data linkage combination and data linkage overriding combination.

Let L and L' be closed DLA terms. Then the constants and operators of DLA can be explained as follows:

- $\xrightarrow{s} a$ is the atomic data linkage that consists of a link via spot s to atomic object a;
- $a \xrightarrow{f}$ is the atomic data linkage that consists of a partial link from atomic object a via field f;
- $a \xrightarrow{f} b$ is the atomic data linkage that consists of a link from atomic object a via field f to atomic object b;
- $(a)_n$ is the atomic data linkage that consists of an association of the value n with atomic object a;
- $\emptyset$ is the data linkage that does not contain any atomic data linkage;
- $L \oplus L'$ is the union of the data linkages L and L';
- $L \oplus' L'$ differs from $L \oplus L'$ as follows:
  - if L contains spot links via spot s and L' contains spot links via spot s, then the former links are overridden by the latter ones;
  - if L contains partial field links and/or field links from atomic object a via field f and L' contains partial field links and/or field links from atomic object a via field f, then the former partial field links and/or field links are overridden by the latter ones;
  - if L contains value associations with atomic object a and L' contains value associations with atomic object a, then the former value associations are overridden by the latter ones.

The axioms of DLA are given in Table 1. In this table, s and t stand for arbitrary spots from Spot, f and g stand for arbitrary fields from Field, a, b, c and d stand for arbitrary atomic objects from AtObj, and n and m stand for arbitrary values from Value.

The set $\mathcal{B}$ of *basic terms* over DLA is inductively defined by the following rules:

- $\emptyset \in \mathcal{B}$;
- if $s \in \mathsf{Spot}$ and $a \in \mathsf{AtObj}$, then $\xrightarrow{s} a \in \mathcal{B}$;
- if $a \in \mathsf{AtObj}$ and $f \in \mathsf{Field}$, then $a \xrightarrow{f} \in \mathcal{B}$;
- if $a, b \in \mathsf{AtObj}$ and $f \in \mathsf{Field}$, then $a \xrightarrow{f} b \in \mathcal{B}$;
- if $a \in \mathsf{AtObj}$ and $n \in \mathsf{Value}$, then $(a)_n \in \mathcal{B}$;
- if $L_1, L_2 \in \mathcal{B}$, then $L_1 \oplus L_2 \in \mathcal{B}$.

Theorem 1. *For all closed* DLA *terms L, there exists a basic term $L' \in \mathcal{B}$ such that $L = L'$ is derivable from the axioms of* DLA*.*

Proof. See Theorem 1 in [9].

We are only interested in the initial model of DLA. We write $\mathcal{DL}$ for the set of all elements of the initial model of DLA. $\mathcal{DL}$ consists of the equivalence classes of basic terms over DLA with respect to the equivalence induced by the axioms of DLA. In other words, modulo equivalence, $\mathcal{B}$ is $\mathcal{DL}$. Henceforth, we will identify basic terms over DLA and their equivalence classes.

Table 1. Axioms of DLA

$X \oplus Y = Y \oplus X$	
$X \oplus (Y \oplus Z) = (X \oplus Y) \oplus Z$	
$X \oplus X = X$	
$X \oplus \emptyset = X$	
$\emptyset \oplus' X = X$	
$X \oplus' \emptyset = X$	
$X \oplus' (Y \oplus Z) = (X \oplus' Y) \oplus (X \oplus' Z)$	
$(X \oplus (\xrightarrow{s} a)) \oplus' (\xrightarrow{s} b) = X \oplus' (\xrightarrow{s} b)$	
$(X \oplus (a \xrightarrow{f})) \oplus' (a \xrightarrow{f}) = X \oplus' (a \xrightarrow{f})$	
$(X \oplus (a \xrightarrow{f} b)) \oplus' (a \xrightarrow{f}) = X \oplus' (a \xrightarrow{f})$	
$(X \oplus (a \xrightarrow{f})) \oplus' (a \xrightarrow{f} b) = X \oplus' (a \xrightarrow{f} b)$	
$(X \oplus (a \xrightarrow{f} b)) \oplus' (a \xrightarrow{f} c) = X \oplus' (a \xrightarrow{f} c)$	
$(X \oplus (a)_n) \oplus' (a)_m = X \oplus' (a)_m$	
$(X \oplus (\xrightarrow{s} a)) \oplus' (\xrightarrow{t} b) = (X \oplus' (\xrightarrow{t} b)) \oplus (\xrightarrow{s} a)$	if $s \neq t$
$(X \oplus (a \xrightarrow{f})) \oplus' (\xrightarrow{s} b) = (X \oplus' (\xrightarrow{s} b)) \oplus (a \xrightarrow{f})$	
$(X \oplus (a \xrightarrow{f} b)) \oplus' (\xrightarrow{s} c) = (X \oplus' (\xrightarrow{s} c)) \oplus (a \xrightarrow{f} b)$	
$(X \oplus (a)_n) \oplus' (\xrightarrow{s} b) = (X \oplus' (\xrightarrow{s} b)) \oplus (a)_n$	
$(X \oplus (\xrightarrow{s} a)) \oplus' (b \xrightarrow{f}) = (X \oplus' (b \xrightarrow{f})) \oplus (\xrightarrow{s} a)$	
$(X \oplus (a \xrightarrow{f})) \oplus' (b \xrightarrow{g}) = (X \oplus' (b \xrightarrow{g})) \oplus (a \xrightarrow{f})$	if $a \neq b \vee f \neq g$
$(X \oplus (a \xrightarrow{f} b)) \oplus' (c \xrightarrow{g}) = (X \oplus' (c \xrightarrow{g})) \oplus (a \xrightarrow{f} b)$	if $a \neq c \vee f \neq g$
$(X \oplus (a)_n) \oplus' (b \xrightarrow{f}) = (X \oplus' (b \xrightarrow{f})) \oplus (a)_n$	
$(X \oplus (\xrightarrow{s} a)) \oplus' (b \xrightarrow{f} c) = (X \oplus' (b \xrightarrow{f} c)) \oplus (\xrightarrow{s} a)$	
$(X \oplus (a \xrightarrow{f})) \oplus' (b \xrightarrow{g} c) = (X \oplus' (b \xrightarrow{g} c)) \oplus (a \xrightarrow{f})$	if $a \neq b \vee f \neq g$
$(X \oplus (a \xrightarrow{f} b)) \oplus' (c \xrightarrow{g} d) = (X \oplus' (c \xrightarrow{g} d)) \oplus (a \xrightarrow{f} b)$	if $a \neq c \vee f \neq g$
$(X \oplus (a)_n) \oplus' (b \xrightarrow{f} c) = (X \oplus' (b \xrightarrow{f} c)) \oplus (a)_n$	
$(X \oplus (\xrightarrow{s} a)) \oplus' (b)_n = (X \oplus' (b)_n) \oplus (\xrightarrow{s} a)$	
$(X \oplus (a \xrightarrow{f})) \oplus' (b)_n = (X \oplus' (b)_n) \oplus (a \xrightarrow{f})$	
$(X \oplus (a \xrightarrow{f} b)) \oplus' (c)_n = (X \oplus' (c)_n) \oplus (a \xrightarrow{f} b)$	
$(X \oplus (a)_n) \oplus' (b)_m = (X \oplus' (b)_m) \oplus (a)_n$	if $a \neq b$

3 Data Linkage Dynamics

DLD (Data Linkage Dynamics) is a simple model of computation that bears on the use of dynamic data structures in programming. It comprises states, basic actions, and the state changes and replies that result from performing the basic actions. The states of DLD are data linkages. In this section, we give an informal explanation of the basic actions of DLD to structure data dynamically. The basic actions of DLD to deal with values found in dynamically structured data, as well as some actions related to reclaiming garbage, are not explained. For a comprehensive presentation of DLD, the reader is referred to [9].

Like in DLA, it is assumed that a fixed but arbitrary finite set Spot of spots, a fixed but arbitrary finite set Field of fields, and a fixed but arbitrary finite set AtObj of atomic objects have been given. It is also assumed that a fixed but arbitrary *choice* function $ch : (\mathcal{P}(\mathsf{AtObj}) \setminus \emptyset) \to \mathsf{AtObj}$ such that, for all $A \in \mathcal{P}(\mathsf{AtObj}) \setminus \emptyset$, $ch(A) \in A$ has been given. The function ch is used whenever a fresh atomic object must be obtained.

Below, we will informally explain the features of DLD to structure data dynamically. When speaking informally about a state L of DLD, we say:

- if there exists a unique atomic object a for which $\xrightarrow{s} a$ is contained in L, *the content of spot* s instead of the unique atomic object a for which $\xrightarrow{s} a$ is contained in L;
- *the fields of atomic object* a instead of the set of all fields f such that either $a \xrightarrow{f}$ is contained in L or there exists an atomic object b such that $a \xrightarrow{f} b$ is contained in L;
- if there exists a unique atomic object b for which $a \xrightarrow{f} b$ is contained in L, *the content of field* f *of atomic object* a instead of the unique atomic object b for which $a \xrightarrow{f} b$ is contained in L.

In the case where the uniqueness condition is met, the spot or field concerned is called *locally deterministic*.

DLD has the following basic actions to structure data dynamically:

- for each $s \in \mathsf{Spot}$, a *get fresh atomic object action* $s!$;
- for each $s, t \in \mathsf{Spot}$, a *set spot action* $s = t$;
- for each $s \in \mathsf{Spot}$, a *clear spot action* $s = *$;
- for each $s, t \in \mathsf{Spot}$, an *equality test action* $s == t$;
- for each $s \in \mathsf{Spot}$, an *undefinedness test action* $s == *$;
- for each $s \in \mathsf{Spot}$ and $f \in \mathsf{Field}$, a *add field action* s/f;
- for each $s \in \mathsf{Spot}$ and $f \in \mathsf{Field}$, a *remove field action* $s\backslash f$;
- for each $s \in \mathsf{Spot}$ and $f \in \mathsf{Field}$, a *has field action* $s\,|\,f$;
- for each $s, t \in \mathsf{Spot}$ and $f \in \mathsf{Field}$, a *set field action* $s.f = t$;
- for each $s \in \mathsf{Spot}$ and $f \in \mathsf{Field}$, a *clear field action* $s.f = *$;
- for each $s, t \in \mathsf{Spot}$ and $f \in \mathsf{Field}$, a *get field action* $s = t.f$.

If only locally deterministic spots and fields are involved, these actions can be explained as follows:

- $s!$: if a fresh atomic object can be allocated, then the content of spot s becomes that fresh atomic object and the reply is T; otherwise, nothing changes and the reply is F;
- $s = t$: the content of spot s becomes the same as the content of spot t and the reply is T;
- $s = *$: the content of spot s becomes undefined and the reply is T;
- $s == t$: if the content of spot s equals the content of spot t, then nothing changes and the reply is T; otherwise, nothing changes and the reply is F;
- $s == *$: if the content of spot s is undefined, then nothing changes and the reply is T; otherwise, nothing changes and the reply is F;

- s/f: if the content of spot s is an atomic object and f does not yet belong to the fields of that atomic object, then f is added (with undefined content) to the fields of that atomic object and the reply is T; otherwise, nothing changes and the reply is F;
- $s\backslash f$: if the content of spot s is an atomic object and f belongs to the fields of that atomic object, then f is removed from the fields of that atomic object and the reply is T; otherwise, nothing changes and the reply is F;
- $s\,|\,f$: if the content of spot s is an atomic object and f belongs to the fields of that atomic object, then nothing changes and the reply is T; otherwise, nothing changes and the reply is F;
- $s.f = t$: if the content of spot s is an atomic object and f belongs to the fields of that atomic object, then the content of that field becomes the same as the content of spot t and the reply is T; otherwise, nothing changes and the reply is F;
- $s.f = *$: if the content of spot s is an atomic object and f belongs to the fields of that atomic object, then the content of that field becomes undefined and the reply is T; otherwise, nothing changes and the reply is F;
- $s = t.f$: if the content of spot t is an atomic object and f belongs to the fields of that atomic object, then the content of spot s becomes the same as the content of that field and the reply is T; otherwise, nothing changes and the reply is F.

In the explanation given above, wherever we say that the content of a spot or field becomes the same as the content of another spot or field, this is meant to imply that the former content becomes undefined if the latter content is undefined. If not only locally deterministic spots and fields are involved in performing an action, there is no state change and the reply is F.

Atomic objects that are not reachable via spots and fields can be reclaimed. Reclamation of unreachable atomic objects is relevant because the set AtObj of atomic objects is finite. In [9], we introduce various ways to achieve reclamation of unreachable atomic objects. In this section, we mention only one of the reclamation-related actions: the *full garbage collection action* fgc. By performing this action, all unreachable atomic objects are reclaimed. The reply that results from performing this action is always T.

We write A_{DLD} for the set of all basic actions of DLD.

In [9], we describe the state changes and replies that result from performing the basic actions of DLD by means of a term rewrite system with rule priorities [3]. For that purpose, a unary *effect* operator eff_α and a unary *yield* operator yld_α are introduced for each basic action $\alpha \in A_{\mathrm{DLD}}$. The intuition is that these operators stand for operations that give, for each state L, the state and reply, respectively, that result from performing basic action α in state L.

4 Basic Thread Algebra

In this section, we review the algebraic theory BTA (Basic Thread Algebra), a form of process algebra which is tailored to the description and analysis of the

Table 2. Axiom of BTA

$x \trianglelefteq \mathsf{tau} \trianglerighteq y = x \trianglelefteq \mathsf{tau} \trianglerighteq x$	T1

behaviours of sequential programs under execution. The behaviours concerned are called *threads*.

In BTA, it is assumed that a fixed but arbitrary finite set $\mathcal{A}$ of *basic actions*, with $\mathsf{tau} \notin \mathcal{A}$, has been given. We write $\mathcal{A}_{\mathsf{tau}}$ for $\mathcal{A} \cup \{\mathsf{tau}\}$. The members of $\mathcal{A}_{\mathsf{tau}}$ are referred to as *actions*.

Threads proceed by performing actions in a sequential fashion. Each basic action performed by a thread is taken as a command to be processed by some service provided by the execution environment of the thread. The processing of a command may involve a change of state of the service concerned. At completion of the processing of the command, the service returns a reply value T or F to the thread concerned.

BTA has one sort: the sort $\mathbf{T}$ of *threads*. To build terms of sort $\mathbf{T}$, BTA has the following constants and operators:

- the *deadlock* constant $\mathsf{D} : \mathbf{T}$;
- the *termination* constant $\mathsf{S} : \mathbf{T}$;
- for each $\alpha \in \mathcal{A}_{\mathsf{tau}}$, the binary *postconditional composition* operator $\_ \trianglelefteq \alpha \trianglerighteq \_ : \mathbf{T} \times \mathbf{T} \to \mathbf{T}$.

Terms of sort $\mathbf{T}$ are built as usual (see e.g. [29, 23]). Throughout the paper, we assume that there are infinitely many variables of sort $\mathbf{T}$, including x, y, z.

We use infix notation for postconditional composition. We introduce *action prefixing* as an abbreviation: $\alpha \circ p$, where p is a term of sort $\mathbf{T}$, abbreviates $p \trianglelefteq \alpha \trianglerighteq p$.

Let p and q be closed terms of sort $\mathbf{T}$ and $\alpha \in \mathcal{A}_{\mathsf{tau}}$. Then $p \trianglelefteq \alpha \trianglerighteq q$ will perform action α, and after that proceed as p if the processing of α leads to the reply T (called a positive reply), and proceed as q if the processing of α leads to the reply F (called a negative reply). The action tau plays a special role. It is a concrete internal action: performing tau will never lead to a state change and always lead to a positive reply, but notwithstanding all that its presence matters.

BTA has only one axiom. This axiom is given in Table 2.

Each closed BTA term of sort $\mathbf{T}$ denotes a finite thread, i.e. a thread of which the length of the sequences of actions that it can perform is bounded. Guarded recursive specifications give rise to infinite threads.

A *guarded recursive specification* over BTA is a set of recursion equations $E = \{X = p_X \mid X \in V\}$, where V is a set of variables of sort $\mathbf{T}$ and each p_X is a term of the form D, S or $p \trianglelefteq \alpha \trianglerighteq q$ with p and q BTA terms of sort $\mathbf{T}$ that contain only variables from V. We write $\mathrm{V}(E)$ for the set of all variables that occur on the left-hand side of an equation in E. We are only interested in models of BTA in which guarded recursive specifications have unique solutions, such as the projective limit model of BTA presented in [5].

Table 3. Axioms for guarded recursion

$\langle X\|E\rangle = \langle t_X\|E\rangle$	if $X = t_X \in E$	RDP
$E \Rightarrow X = \langle X\|E\rangle$	if $X \in \mathrm{V}(E)$	RSP

We extend BTA with guarded recursion by adding constants for solutions of guarded recursive specifications and axioms concerning these additional constants. For each guarded recursive specification E and each $X \in \mathrm{V}(E)$, we add a constant of sort $\mathbf{T}$ standing for the unique solution of E for X to the constants of BTA. The constant standing for the unique solution of E for X is denoted by $\langle X|E\rangle$. Moreover, we add the axioms for guarded recursion given in Table 3 to BTA, where we write $\langle t_X|E\rangle$ for t_X with, for all $Y \in \mathrm{V}(E)$, all occurrences of Y in t_X replaced by $\langle Y|E\rangle$.[2] In this table, X, t_X and E stand for an arbitrary variable of sort $\mathbf{T}$, an arbitrary BTA term of sort $\mathbf{T}$ and an arbitrary guarded recursive specification over BTA, respectively. Side conditions are added to restrict the variables, terms and guarded recursive specifications for which X, t_X and E stand.

Henceforth, we write BTA+REC for BTA extended with the constants for solutions of guarded recursive specifications and axioms RDP and RSP. Moreover, we write $\mathcal{T}$ for the set of all closed terms of BTA+REC.

In the following definition, the interpretation of a postconditional composition operator in a model of BTA+REC is denoted by the operator itself. Let $\mathfrak{M}$ be some model of BTA+REC, and let p be an element from the domain of $\mathfrak{M}$. Then the set of *residual threads* of p, written $Res(p)$, is inductively defined as follows:

- $p \in Res(p)$;
- if $q \trianglelefteq a \trianglerighteq r \in Res(p)$, then $q \in Res(p)$ and $r \in Res(p)$.

We say that p is *regular* if $Res(p)$ is finite.

We are only interested in models of BTA+REC in which the solution of a guarded recursive specification E over BTA is regular if and only if E is finite, such as the projective limit model presented in [5]. Par abus de langage, a closed term of BTA+REC without occurrences of constants $\langle X|E\rangle$ for infinite E will henceforth be called a *regular thread*.

5 A Use Mechanism for Forecasting Services

A thread may perform an action for the purpose of interacting with a service that takes the action as a command to be processed. The processing of the action may involve a change of state of the service and at completion of the processing of the action the service returns a reply value to the thread. In this section, we introduce a mechanism that is concerned with this kind of interaction. It is a

[2] Throughout the paper, we use the symbol $\Rightarrow$ for implication.

generalization of the use mechanism introduced in [9] to forecasting services. A forecasting service is a service of which the state changes and replies may depend on how the thread that performs the actions being processed will proceed.

It is assumed that a fixed but arbitrary finite set $\mathcal{F}$ of *foci* and a fixed but arbitrary finite set $\mathcal{M}$ of *methods* have been given. Each focus plays the role of a name of some service provided by an execution environment that can be requested to process a command. Each method plays the role of a command proper. For the set $\mathcal{A}$ of actions, we take the set $\{f.m \mid f \in \mathcal{F}, m \in \mathcal{M}\}$. Performing an action $f.m$ is taken as making a request to the service named f to process command m.

Recall that $\mathcal{T}$ stands for the set of all closed terms of BTA+REC.

A *forecasting service* H consists of

- a set S of *states*;
- an *effect* function $\mathit{eff} : \mathcal{M} \times S \times \mathcal{T} \to S$;
- a *yield* function $\mathit{yld} : \mathcal{M} \times S \times \mathcal{T} \to \{\mathsf{T}, \mathsf{F}, \mathsf{B}\}$;
- an *initial state* $s_0 \in S$;

satisfying the following conditions:

$$\begin{aligned}
&\exists s \in S \bullet \forall m \in \mathcal{M}, p \in \mathcal{T} \bullet \\
&\quad (\mathit{yld}(m, s, p) = \mathsf{B} \wedge \forall s' \in S \bullet (\mathit{yld}(m, s', p) = \mathsf{B} \Rightarrow \mathit{eff}(m, s', p) = s)) \,, \\
&\forall s \in S, m, m' \in \mathcal{M}, f \in \mathcal{F}, p, q \in \mathcal{T} \bullet \\
&\quad (\mathit{yld}(m, s, \mathsf{S}) = \mathsf{B} \wedge \mathit{yld}(m, s, \mathsf{D}) = \mathsf{B} \wedge \\
&\quad\ \mathit{yld}(m, s, \mathsf{tau} \circ p) = \mathsf{B} \wedge (m \neq m' \Rightarrow \mathit{yld}(m, s, p \trianglelefteq f.m' \trianglerighteq q) = \mathsf{B})) \,.
\end{aligned}$$

The set S contains the states in which the services may be, and the functions *eff* and *yld* give, for each method m, state s and thread p, the state and reply, respectively, that result from processing m in state s if p is the thread that makes the request to process m. In certain states, requests to process certain methods may be rejected. B, which stands for blocked, is used to indicate this.

Given a forecasting service $H = (S, \mathit{eff}, \mathit{yld}, s_0)$, a method $m \in \mathcal{M}$ and a thread $p \in \mathcal{T}$:

- the *derived service* of H after processing m in the context of p, written $\frac{\partial}{\partial m} H[p]$, is the forecasting service $(S, \mathit{eff}, \mathit{yld}, \mathit{eff}(m, s_0, p))$;
- the *reply* of H after processing m in the context of p, written $H[p](m)$, is $\mathit{yld}(m, s_0, p)$.

A forecasting service $H = (S, \mathit{eff}, \mathit{yld}, s_0)$ can be understood as follows:

- if thread p makes a request to the service to process m and $H[p](m) \neq \mathsf{B}$, then the request is accepted, the reply is $H[p](m)$, and the service proceeds as $\frac{\partial}{\partial m} H[p]$;
- if thread p makes a request to the service to process m and $H[p](m) = \mathsf{B}$, then the request is rejected.

Table 4. Axioms for use operators

$\mathsf{S} \mathbin{/_f} H = \mathsf{S}$		TSU1
$\mathsf{D} \mathbin{/_f} H = \mathsf{D}$		TSU2
$(\mathsf{tau} \circ p) \mathbin{/_f} H = \mathsf{tau} \circ (p \mathbin{/_f} H)$		TSU3
$(p \trianglelefteq g.m \trianglerighteq q) \mathbin{/_f} H = (p \mathbin{/_f} H) \trianglelefteq g.m \trianglerighteq (q \mathbin{/_f} H)$	if $f \neq g$	TSU4
$(p \trianglelefteq f.m \trianglerighteq q) \mathbin{/_f} H = \mathsf{tau} \circ (p \mathbin{/_f} \frac{\partial}{\partial m} H[p \trianglelefteq f.m \trianglerighteq q])$	if $H[p \trianglelefteq f.m \trianglerighteq q](m) = \mathsf{T}$	TSU5
$(p \trianglelefteq f.m \trianglerighteq q) \mathbin{/_f} H = \mathsf{tau} \circ (q \mathbin{/_f} \frac{\partial}{\partial m} H[p \trianglelefteq f.m \trianglerighteq q])$	if $H[p \trianglelefteq f.m \trianglerighteq q](m) = \mathsf{F}$	TSU6
$(p \trianglelefteq f.m \trianglerighteq q) \mathbin{/_f} H = \mathsf{D}$	if $H[p \trianglelefteq f.m \trianglerighteq q](m) = \mathsf{B}$	TSU7

By the first condition on forecasting services, after a request has been rejected by the service, it gets into a state in which any request will be rejected. By the second condition on forecasting services, any request that does not correspond to the action being performed by thread p is rejected.

In the case of a forecasting service $H = (S, \mathit{eff}, \mathit{yld}, s_0)$, the derived service and reply that result from processing a method may depend on how the thread that makes the request to process that method will proceed. Hence the name forecasting service. Henceforth, we will omit the qualification forecasting if no confusion can arise with other kinds of services.

We introduce yet another sort: the sort $\mathbf{S}$ of *services*. However, we will not introduce constants and operators to build terms of this sort. We demand that the interpretation of the sort $\mathbf{S}$ in a model is a set $\mathcal{FS}$ of forecasting services such that for all $H \in \mathcal{FS}$, $\frac{\partial}{\partial m} H[p] \in \mathcal{FS}$ for each $m \in \mathcal{M}$ and $p \in \mathcal{T}$.

We introduce the following additional operators:

- for each $f \in \mathcal{F}$, the binary *use* operator $\_ \mathbin{/_f} \_ : \mathbf{T} \times \mathbf{S} \to \mathbf{T}$.

We use infix notation for the use operators.

Intuitively, $p \mathbin{/_f} H$ is the thread that results from processing all actions performed by thread p that are of the form $f.m$ by service H. When an action of the form $f.m$ performed by thread p is processed by service H, that action is turned into the internal action tau and postconditional composition is removed in favour of action prefixing on the basis of the reply value produced. In previous work, we sometimes opted for the alternative to conceal the processed actions completely. However, we experienced repeatedly in cases where this alternative appeared to be appropriate at first that it turned out to impede progress later.

The axioms for the use operators are given in Table 4. In this table, f and g stand for arbitrary foci from $\mathcal{F}$, m stands for an arbitrary method from $\mathcal{M}$, and p and q stand for arbitrary closed terms of sort $\mathbf{T}$. H ranges over the interpretation of sort $\mathbf{S}$. Axioms TSU3 and TSU4 express that the action tau and actions of the form $g.m$, where $f \neq g$, are not processed. Axioms TSU5 and TSU6 express that a thread is affected by a service as described above when an action of the form $f.m$ is processed by the service. Axiom TSU7 expresses that deadlock takes place when an action to be processed is not accepted.

Henceforth, we write $\mathrm{BTA}_{\mathrm{use}}$ for BTA, taking the set $\{f.m \mid f \in \mathcal{F}, m \in \mathcal{M}\}$ for $\mathcal{A}$, extended with the use operators and the axioms from Table 4.

Table 5. Definition of effect and yield functions for DLD

$\mathit{eff}(m, L, p \trianglelefteq f.m \trianglerighteq q) = \mathit{eff}_m(L)$	if $m \in A_{\mathrm{DLD}}$
$\mathit{eff}(m, L, p \trianglelefteq f.m \trianglerighteq q) = \uparrow$	if $m \notin A_{\mathrm{DLD}}$
$\mathit{yld}(m, L, p \trianglelefteq f.m \trianglerighteq q) = \mathit{yld}_m(L)$	if $m \in A_{\mathrm{DLD}}$
$\mathit{yld}(m, L, p \trianglelefteq f.m \trianglerighteq q) = \mathsf{B}$	if $m \notin A_{\mathrm{DLD}}$
$\mathit{yld}(m, L, p) = \mathsf{B} \Rightarrow \mathit{eff}(m, L, p) = \uparrow$	

The use mechanism introduced in [8] deals in essence with forecasting services of which:

- the set of states is the set of all sequences with elements from $\mathcal{M}$;
- the derived service and reply that result from processing a method do not depend on how the thread that makes the request to process that method will proceed.

For these services, the use mechanism introduced in this section coincides with the use mechanism introduced in [8]. The architecture-dependent services considered in [6] can be looked upon as simple forecasting services.

6 Thread Algebra and Data Linkage Dynamics Combined

The state changes and replies that result from performing the actions of data linkage dynamics can be achieved by means of services. In this short section, we explain how basic thread algebra can be combined with data linkage dynamics by means of the use mechanism introduced in Section 5 such that the whole can be used for studying issues concerning the use of dynamic data structures in programming. The services involved do not have a forecasting nature. The adapted services needed to deal with shedding, which are described in Section 9, have a forecasting nature.

Recall that $\mathcal{DL}$ stands for the set of all elements of the initial model of DLA, and recall that, for each $\alpha \in A_{\mathrm{DLD}}$, eff_α and yld_α stand for unary operations on $\mathcal{DL}$ that give, for $L \in \mathcal{DL}$, the state and reply, respectively, that result from performing basic action α in state L. It is assumed that a blocking state $\uparrow \notin \mathcal{DL}$ has been given.

Take $\mathcal{M}$ such that $A_{\mathrm{DLD}} \subseteq \mathcal{M}$. Moreover, let $L \in \mathcal{DL} \cup \{\uparrow\}$. Then the *data linkage dynamics service* with initial state L, written $\mathcal{DLD}(L)$, is the service $(\mathcal{DL} \cup \{\uparrow\}, \mathit{eff}, \mathit{yld}, L)$, where the functions eff and yld are the effect and yield functions satisfying the (unconditional and conditional) equations in Table 5. Notice that, because of the conditions imposed on forecasting services in Section 5, these equations characterize the effect and yield functions uniquely.

By means of threads and the data linkage dynamics services introduced above, we can give a precise picture of computations in which dynamic data structures are involved. Examples of such computations can be found in [9].

The combination of basic thread algebra and data linkage dynamics by means of the use mechanism can be used for studying issues concerning the use of dynamic data structures in programming at the level of program behaviours. A hierarchy of simple program notations rooted in PGA is presented in [7]. Included are program notations which are close to existing assembly languages up to and including program notations that support structured programming by offering a rendering of conditional and loop constructs. Regular threads are taken as the behaviours of programs in those program notations. Together with one of the program notations, the combination of basic thread algebra and data linkage dynamics can be used for studying issues concerning the use of dynamic data structures in programming at the level of programs. We mention one such issue. In general terms, the issue is whether we can do without garbage collection by program transformation at the price of a linear increase of the number of available atomic objects. In [9], we phrase this issue precisely for one of the program notation rooted in PGA.

The notation for the basic actions of DLD, makes the focus-method notation $f.m$ less suitable in the case where m is a basic action of DLD. Therefore, we will henceforth mostly write $f(m)$ instead of $f.m$ if $m \in A_{\mathrm{DLD}}$.

7 The Shedding Feature

In this section, we introduce the shedding feature in the setting of data linkage dynamics in an informal way. In Section 9, we will adapt the data linkage dynamics services introduced in Section 6 to explain shedding in a more precise way.

Roughly speaking, shedding works as follows: each time the content of a spot or field is changed, it is determined whether or not the spot or field will possibly be used once again, and if not its content is made undefined. If a spot or field is made undefined in this way, we say that it is shed. The use of a previously shed spot or field is called a shedding error.

The shedding feature is rather non-obvious. Consider the thread

$$\mathsf{dld}(s\,!) \circ ((\mathsf{dld}(u = s) \circ \mathsf{S}) \trianglelefteq \mathsf{dld}(t\,!) \trianglerighteq \mathsf{S})$$

and assume that the cardinality of AtObj is 1. If s is not shed on performing $s\,!$, then a negative reply is produced on performing $t\,!$ and the thread terminates without having made use of s. However, from this it cannot be concluded that s could be shed on performing $s\,!$ after all. If s would be shed on performing $s\,!$, a positive reply would be produced on performing $t\,!$ and after that a shedding error would occur. This shows that shedding becomes paradoxical if we do not deal properly with the fact that shedding of a spot or field influences whether or not that spot or field will possibly be used once again.

In the light of this, it is of the utmost importance to have the right criterion for shedding in mind:

> a spot or field can safely be shed if it is not possible for the program behaviour under consideration to evolve in the case where that spot

or field is shed, irrespective as to whether other spots and fields are subsequently shed, in such a way that the first shedding error concerns that spot or field.

When speaking about applications of this criterion, shedding errors that concern the spot or field to which the criterion is applied are called *primary* shedding errors and other shedding errors are called *secondary* shedding errors.

In Section 6, it was explained how basic thread algebra can be combined with data linkage dynamics by means of the use mechanism from Section 5 in such a way that the whole can be used for studying issues concerning the use of dynamic data structures in programming. For a clear apprehension of data linkage dynamics as presented in Section 3, such a combination is not needed. This is different for shedding: it cannot be explained without reference to program behaviours. In Section 9, we adapt the data linkage dynamics services involved in the combination described in Section 6 to explain shedding.

For the adapted data linkage dynamics services, shedding happens to be a matter close to reflection on itself. Material to the adaptation is the above-mentioned criterion for shedding a spot or field. Instrumental in checking this criterion are the data linkage dynamics services for a minor variation of DLD. It concerns services which support the mimicking of shedding.

8 Mimicking of Shedding

The shedding supporting data linkage dynamics services, which will be introduced in Section 9, check the criterion for shedding adopted in Section 7 by determining what would happen if mimicking of shedding supporting data linkage dynamics services were used. In this section, we describe the mimicking of shedding supporting data linkage dynamics services in question.

These services are data linkage dynamics services for a variation of DLD. The variation concerned, referred to as $\mathrm{DLD}^{\mathrm{msh}}$, differs from DLD as follows:

- it has two additional atomic objects $*_{\mathrm{p}}$ and $*_{\mathrm{s}}$;
- for each $s \in \mathsf{Spot}$, it has two additional basic actions $s = *_{\mathrm{p}}$ and $s = *_{\mathrm{s}}$;
- for each $s \in \mathsf{Spot}$ and $f \in \mathsf{Field}$, it has two additional basic actions $s.f = *_{\mathrm{p}}$ and $s.f = *_{\mathrm{s}}$;
- on performing $s!$, the contents of spot s never becomes $*_{\mathrm{p}}$ or $*_{\mathrm{s}}$;
- on performing fgc, $*_{\mathrm{p}}$ and $*_{\mathrm{s}}$ are never reclaimed.

If only locally deterministic spots and fields are involved, the additional basic actions can be explained as follows:

- $s = *_{\mathrm{p}}$: the content of spot s becomes $*_{\mathrm{p}}$ and the reply is T;
- $s = *_{\mathrm{s}}$: the content of spot s becomes $*_{\mathrm{s}}$ and the reply is T;
- $s.f = *_{\mathrm{p}}$: if the content of spot s is an atomic object and f belongs to the fields of that atomic object, then the content of that field becomes $*_{\mathrm{p}}$ and the reply is T; otherwise, nothing changes and the reply is F;

Table 6. Definition of effect and yield functions for DLD with mimicking of shedding

$$
\begin{array}{ll}
\mathit{eff}^{\mathrm{msh}}(m, L, p \trianglelefteq f.m \trianglerighteq q) = \mathit{eff}^{\mathrm{msh}}_m(L) & \mathsf{if}\ m \in A_{\mathrm{DLD}^{\mathrm{msh}}} \\
\mathit{eff}^{\mathrm{msh}}(m, L, p \trianglelefteq f.m \trianglerighteq q) = \uparrow & \mathsf{if}\ m \notin A_{\mathrm{DLD}^{\mathrm{msh}}} \\
\mathit{yld}^{\mathrm{msh}}(m, L, p \trianglelefteq f.m \trianglerighteq q) = \mathit{yld}^{\mathrm{msh}}_m(L) & \mathsf{if}\ m \in A_{\mathrm{DLD}^{\mathrm{msh}}} \\
\mathit{yld}^{\mathrm{msh}}(m, L, p \trianglelefteq f.m \trianglerighteq q) = \mathsf{B} & \mathsf{if}\ m \notin A_{\mathrm{DLD}^{\mathrm{msh}}} \\
\mathit{yld}^{\mathrm{msh}}(m, L, p) = \mathsf{B} \Rightarrow \mathit{eff}^{\mathrm{msh}}(m, L, p) = \uparrow &
\end{array}
$$

– $s.f = *_{\mathrm{s}}$: if the content of spot s is an atomic object and f belongs to the fields of that atomic object, then the content of that field becomes $*_{\mathrm{s}}$ and the reply is T; otherwise, nothing changes and the reply is F.

If not only locally deterministic spots and fields are involved in performing an action, there is no state change and the reply is F.

The special atomic objects $*_{\mathrm{p}}$ and $*_{\mathrm{s}}$ are used as follows:

– when checking of the criterion for some spot or field starts, the shedding of that spot or field is mimicked by setting its content to $*_{\mathrm{p}}$;
– during checking, the shedding of another spot or field is mimicked by setting its content to $*_{\mathrm{s}}$.

If a spot or field is used whose content is $*_{\mathrm{p}}$, a mimicked primary shedding error is encountered and, if a spot or field is used whose content is $*_{\mathrm{s}}$, a mimicked secondary shedding error is encountered.

Different sets of spots, sets of fields or sets of atomic objects give rise to different instances of DLA. The states of DLD are the elements of the initial model of some instance of DLA. Because of the two additional atomic objects, the states of $\mathrm{DLD}^{\mathrm{msh}}$ are the elements of the initial model of another instance of DLA. Henceforth, we write $\mathcal{DL}$ for the set of all elements of the initial model of former instance of DLA and $\mathcal{DL}^{\mathrm{msh}}$ for the set of all elements of the initial model of latter instance of DLA. In [9], we describe the state changes and replies that result from performing basic actions of DLD by means of a term rewrite system with rule priorities. It is obvious how that term rewrite system must be adapted to obtain a term rewrite system describing the state changes and replies that result from performing basic actions of $\mathrm{DLD}^{\mathrm{msh}}$. For each basic action α of $\mathrm{DLD}^{\mathrm{msh}}$, we write $\mathit{eff}^{\mathrm{msh}}_\alpha$ and $\mathit{yld}^{\mathrm{msh}}_\alpha$ for the effect and yield operators that go with α in the latter term rewrite system. Moreover, we write $A_{\mathrm{DLD}^{\mathrm{msh}}}$ for the set of all basic actions of $\mathrm{DLD}^{\mathrm{msh}}$.

Let $L \in \mathcal{DL}^{\mathrm{msh}} \cup \{\uparrow\}$. Then the *mimicking of shedding supporting data linkage dynamics service* with initial state L, written $\mathcal{DLD}^{\mathrm{msh}}(L)$, is the service $(\mathcal{DL}^{\mathrm{msh}} \cup \{\uparrow\}, \mathit{eff}^{\mathrm{msh}}, \mathit{yld}^{\mathrm{msh}}, L)$, where the functions $\mathit{eff}^{\mathrm{msh}}$ and $\mathit{yld}^{\mathrm{msh}}$ are the effect and yield functions satisfying the equations in Table 6.

9 Shedding Supporting Data Linkage Dynamics Services

In this section, we turn to the data linkage dynamic services that support shedding themselves.

We assume that $\mathsf{dld} \in \mathcal{F}$. It is supposed that requests to a shedding supporting data linkage dynamics service to process basic actions of DLD are always made using the focus dld. We write $A^{\mathrm{sh}}_{\mathrm{DLD}}$ for the set of all basic actions of DLD that are of the form $s\,!$, $s = t$, $s.f = t$ or $s = t.f$.

In the definition of shedding supporting data linkage dynamics services given below, we use an auxiliary function $sh : A_{\mathrm{DLD}} \to A_{\mathrm{DLD}}$ and a set $shok \subseteq \mathcal{T} \times \mathcal{DL}$.

The function sh gives, for each basic action of DLD for changing the content of a spot or field, the basic action of DLD for making the content of that spot or field undefined. For each other basic action of DLD, sh gives the basic action itself. The function sh is defined as follows:

$$\begin{aligned}
&sh(s\,!) = (s = *)\,,\\
&sh(s = t) = (s = *)\,,\\
&sh(s.f = t) = (s.f = *)\,,\\
&sh(s = t.f) = (s = *)\,,\\
&sh(\alpha) = \alpha \ \mathsf{if}\ \alpha \notin A^{\mathrm{sh}}_{\mathrm{DLD}}\,.
\end{aligned}$$

In the definition of the set $shok$, we use an auxiliary function $msh : \{0,1,2\} \times A_{\mathrm{DLD}} \to A_{\mathrm{DLD}^{\mathrm{msh}}}$ and, for each $L \in \mathcal{DL}^{\mathrm{msh}}$, sets $nosherr(L), secmsherr(L) \subseteq A_{\mathrm{DLD}}$.

The function msh gives, for each natural number in the set $\{0,1,2\}$ and each basic action of DLD for changing the content of a spot or field: the basic action itself if the number is 0, the basic action of $\mathrm{DLD}^{\mathrm{msh}}$ for making the content of that spot or field $*_{\mathrm{p}}$ if the number is 1, and the basic action of $\mathrm{DLD}^{\mathrm{msh}}$ for making the content of that spot or field $*_{\mathrm{s}}$ if the number is 2. For each other basic action of DLD, msh gives always the basic action itself. The function msh is defined as follows:

$$\begin{aligned}
&msh(0,\alpha) = \alpha\,, && msh(i,\alpha) = \alpha \ \ \mathsf{if}\ \alpha \notin A^{\mathrm{sh}}_{\mathrm{DLD}}\,,\\
&msh(1,s\,!) = (s = *_{\mathrm{p}})\,, && msh(2,s\,!) = (s = *_{\mathrm{s}})\,,\\
&msh(1,s = t) = (s = *_{\mathrm{p}})\,, && msh(2,s = t) = (s = *_{\mathrm{s}})\,,\\
&msh(1,s.f = t) = (s.f = *_{\mathrm{p}})\,, && msh(2,s.f = t) = (s.f = *_{\mathrm{s}})\,,\\
&msh(1,s = t.f) = (s = *_{\mathrm{p}})\,, && msh(2,s = t.f) = (s = *_{\mathrm{s}})\,.
\end{aligned}$$

For each $L \in \mathcal{DL}^{\mathrm{msh}}$, the set $nomsherr(L)$ contains all basic actions $\alpha \in A_{\mathrm{DLD}}$ whose use in state L does not amount to a mimicked shedding error and the set $secmsherr(L)$ contains all basic actions $\alpha \in A_{\mathrm{DLD}}$ whose use in state L amounts to a mimicked secondary shedding error. For each $L \in \mathcal{DL}^{\mathrm{msh}}$, the set $nomsherr(L)$ is inductively defined as follows:

- $s\,!$, $s = * \in nomsherr(L)$;
- if $L \oplus (\xrightarrow{s} a) = L$, $a \neq *_{\mathrm{p}}$ and $a \neq *_{\mathrm{s}}$,
  then $t = s$, $s == *$, s/f, $s\backslash f$, $s \,|\, f \in nomsherr(L)$;

– if $L \oplus (\xrightarrow{s} a) \oplus (\xrightarrow{t} b) = L$, $a \neq *_{\mathrm{p}}$, $a \neq *_{\mathrm{s}}$, $b \neq *_{\mathrm{p}}$ and $b \neq *_{\mathrm{s}}$,
  then $s == t$, $s.f = t \in \mathit{nomsherr}(L)$;
– if $L \oplus (\xrightarrow{s} a) \oplus (a \xrightarrow{f} b) = L$, $a \neq *_{\mathrm{p}}$, $a \neq *_{\mathrm{s}}$, $b \neq *_{\mathrm{p}}$ and $b \neq *_{\mathrm{s}}$,
  then $t = s.f \in \mathit{nomsherr}(L)$;

and the set $\mathit{secmsherr}(L)$ is inductively defined as follows:

– if $L \oplus (\xrightarrow{s} *_{\mathrm{s}}) = L$,
  then $t = s$, $s == t$, $t == s$, $s == *$, s/f, $s\backslash f$, $s \,|\, f$, $s.f = t$, $t.f = s$,
  $t = s.f \in \mathit{secmsherr}(L)$;
– if $L \oplus (\xrightarrow{s} a) \oplus (a \xrightarrow{f} *_{\mathrm{s}}) = L$,
  then $t = s.f \in \mathit{secmsherr}(L)$.

The set *shok* contains all pairs $(p, L) \in \mathcal{T} \times \mathcal{DL}$ such that, if the first action that is performed by p is an action of the form $\mathsf{dld}.m$, where m is a basic action of DLD for changing the content of a spot or field, the criterion for shedding of that spot or field is met. The general idea underlying the definition of *shok* given below is that the criterion for shedding can be checked by mimicking shedding. In checking, all possibilities must be considered:

– if an action of the form $f.m$ with $f \neq \mathsf{dld}$ is encountered, then two possibilities arise: (i) the reply is T and (ii) the reply is F;
– if an action of the form $\mathsf{dld}.m$ with m a basic action of DLD of the form $s\,!$, $s = t$, $s.f = t$ or $s = t.f$ is encountered, then two possibilities arise: (i) the spot or field eligible for shedding is not shed and (ii) the spot or field eligible for shedding is shed.

In general, this means that many paths must be followed. For regular threads, the number of paths to be followed will remain finite and eventually either termination, deadlock, a mimicked primary shedding error, a mimicked secondary shedding error or a cycle without mimicked shedding errors will be encountered along each of the paths to be followed. The criterion for shedding is met if along each of the paths to be followed it is not a mimicked primary shedding error that is encountered first. For non-regular threads, it is undecidable whether the criterion for shedding is met.

The set *shok* is defined by $\mathit{shok} = \mathit{shok}'(1, \emptyset)$, where the sets $\mathit{shok}'(i, C) \subseteq \mathcal{T} \times \mathcal{DL}^{\mathrm{msh}}$ for $i \in \{0, 1, 2\}$ and $C \subseteq \mathcal{T} \times \mathcal{DL}^{\mathrm{msh}}$ are defined by simultaneous induction as follows:

– $(\mathsf{S}, L), (\mathsf{D}, L) \in \mathit{shok}'(i, C)$;
– if $f \neq \mathsf{dld}$,
  $(p, L) \in \mathit{shok}'(0, C \cup \{(p \trianglelefteq f.m \trianglerighteq q, L)\})$,
  $(p, L) \in \mathit{shok}'(2, C \cup \{(p \trianglelefteq f.m \trianglerighteq q, L)\})$,
  $(q, L) \in \mathit{shok}'(0, C \cup \{(p \trianglelefteq f.m \trianglerighteq q, L)\})$,
  $(q, L) \in \mathit{shok}'(2, C \cup \{(p \trianglelefteq f.m \trianglerighteq q, L)\})$,
  then $(p \trianglelefteq f.m \trianglerighteq q, L) \in \mathit{shok}'(i, C)$;

Table 7. Definition of effect and yield functions for DLD with shedding

$$
\begin{array}{ll}
\mathit{eff}^{\mathrm{sh}}(m, L, p \trianglelefteq f.m \trianglerighteq q) = \mathit{eff}_{sh(m)}(L) & \mathsf{if}\ m \in A_{\mathrm{DLD}} \wedge (p \trianglelefteq f.m \trianglerighteq q, L) \in \mathit{shok} \\
\mathit{eff}^{\mathrm{sh}}(m, L, p \trianglelefteq f.m \trianglerighteq q) = \mathit{eff}_{m}(L) & \mathsf{if}\ m \in A_{\mathrm{DLD}} \wedge (p \trianglelefteq f.m \trianglerighteq q, L) \notin \mathit{shok} \\
\mathit{eff}^{\mathrm{sh}}(m, L, p \trianglelefteq f.m \trianglerighteq q) = \uparrow & \mathsf{if}\ m \notin A_{\mathrm{DLD}} \\
\\
\mathit{yld}^{\mathrm{sh}}(m, L, p \trianglelefteq f.m \trianglerighteq q) = \mathit{yld}_{sh(m)}(L) & \mathsf{if}\ m \in A_{\mathrm{DLD}} \wedge (p \trianglelefteq f.m \trianglerighteq q, L) \in \mathit{shok} \\
\mathit{yld}^{\mathrm{sh}}(m, L, p \trianglelefteq f.m \trianglerighteq q) = \mathit{yld}_{m}(L) & \mathsf{if}\ m \in A_{\mathrm{DLD}} \wedge (p \trianglelefteq f.m \trianglerighteq q, L) \notin \mathit{shok} \\
\mathit{yld}^{\mathrm{sh}}(m, L, p \trianglelefteq f.m \trianglerighteq q) = \mathsf{B} & \mathsf{if}\ m \notin A_{\mathrm{DLD}} \\
\\
\mathit{yld}^{\mathrm{sh}}(m, L, p) = \mathsf{B} \Rightarrow \mathit{eff}^{\mathrm{sh}}(m, L, p) = \uparrow &
\end{array}
$$

- if $m \in \mathit{nomsherr}(L)$,
  $(p \trianglelefteq \mathsf{dld}.\mathit{msh}(i, m) \trianglerighteq q) \mathbin{/_{\mathsf{dld}}} \mathcal{DLD}^{\mathrm{msh}}(L) = \mathsf{tau} \circ (r \mathbin{/_{\mathsf{dld}}} \mathcal{DLD}^{\mathrm{msh}}(L'))$,
  $(r, L') \in \mathit{shok}'(0, C \cup \{(p \trianglelefteq \mathsf{dld}.m \trianglerighteq q, L)\})$,
  $(r, L') \in \mathit{shok}'(2, C \cup \{(p \trianglelefteq \mathsf{dld}.m \trianglerighteq q, L)\})$,
  then $(p \trianglelefteq \mathsf{dld}.m \trianglerighteq q, L) \in \mathit{shok}'(i, C)$;
- if $m \in \mathit{secmsherr}(L)$,
  then $(p \trianglelefteq \mathsf{dld}.m \trianglerighteq q, L) \in \mathit{shok}'(i, C)$;
- if $\langle X|E\rangle \in \mathcal{T}$, $X = t_X \in E$, $(\langle t_X|E\rangle, L) \in \mathit{shok}'(i, C)$,
  then $(\langle X|E\rangle, L) \in \mathit{shok}'(i, C)$;
- if $(p, L) \in C$, then $(p, L) \in \mathit{shok}'(i, C)$.

In $\mathit{shok}'(i, C)$, i corresponds to the way in which a basic action of DLD for changing the content of a spot or field is dealt with in checking:

- without mimicking of its shedding if $i = 0$;
- with mimicking of its shedding by means of $*_{\mathrm{p}}$ if $i = 1$;
- with mimicking of its shedding by means of $*_{\mathrm{s}}$ if $i = 2$.

The members of C correspond to the combinations of thread and state encountered before in checking. If such a combination is encountered again, this indicates a cycle without shedding errors because a path is not followed further after termination, deadlock, a mimicked shedding error or a cycle without mimicked shedding errors has been encountered.

By the occurrence of the equation $(p \trianglelefteq \mathsf{dld}.\mathit{msh}(i, m) \trianglerighteq q) \mathbin{/_{\mathsf{dld}}} \mathcal{DLD}^{\mathrm{msh}}(L) = \mathsf{tau} \circ (r \mathbin{/_{\mathsf{dld}}} \mathcal{DLD}^{\mathrm{msh}}(L'))$ in the third rule of the inductive definition of the sets $\mathit{shok}'(i, C)$, a service that is engaged in checking whether a pair $(p, L) \in \mathcal{T} \times \mathcal{DL}$ belongs to shok is close to reflecting on itself.

Now, we are ready to define, for $L \in \mathcal{DL} \cup \{\uparrow\}$, a data linkage dynamic service $\mathcal{DLD}^{\mathrm{sh}}(L)$ that supports shedding.

Let $L \in \mathcal{DL} \cup \{\uparrow\}$. Then the *shedding supporting data linkage dynamics service* with initial state L, written $\mathcal{DLD}^{\mathrm{sh}}(L)$, is the service $(\mathcal{DL} \cup \{\uparrow\}, \mathit{eff}^{\mathrm{sh}}, \mathit{yld}^{\mathrm{sh}}, L)$, where the functions $\mathit{eff}^{\mathrm{sh}}$ and $\mathit{yld}^{\mathrm{sh}}$ are the effect and yield functions satisfying the equations in Table 7.

10 Examples

In this section, we give two examples that illustrate how the definition of shedding supporting data linkage dynamics services can be used to determine whether in a fixed case a spot or field, whose contents should be changed, is shed. The first example concerns a case where a spot is shed and the second example concerns a case where a spot is not shed.

Example 1. Let

$$\begin{aligned} p &= \mathsf{dld}(s\,!) \circ (\mathsf{S} \trianglelefteq \mathsf{dld}(t\,!) \trianglerighteq \mathsf{D}) \;, \\ p' &= \mathsf{S} \trianglelefteq \mathsf{dld}(t\,!) \trianglerighteq \mathsf{D} \;, \\ p'' &= \mathsf{S} \trianglelefteq \mathsf{dld}(t = *_{\mathrm{s}}) \trianglerighteq \mathsf{D} \;. \end{aligned}$$

Thread p' is a residual thread of p and thread p'' is p' with $t\,!$ replaced by $t = *_{\mathrm{s}}$ to mimic shedding. Assume that the cardinality of AtObj is 1, and let a be the unique atomic object such that $\mathsf{AtObj} = \{a\}$. Then in $p \mathbin{/_{\mathsf{dld}}} \mathcal{DLD}^{\mathrm{sh}}(\emptyset)$, spot s is shed on performing $s\,!$. This is straightforwardly shown using the definition of shedding supporting data linkage dynamics services. It follows immediately from the definition of *nomsherr* that:

$$\begin{aligned} &(s\,!) \in \mathit{nomsherr}(\emptyset) \;, \\ &(t\,!) \in \mathit{nomsherr}(\xrightarrow{s} *_{\mathrm{p}}) \;, \end{aligned}$$

and it follows easily from the axioms for the use operators and the definition of mimicking of shedding supporting data linkage dynamics services that:

$$\begin{aligned} &p \mathbin{/_{\mathsf{dld}}} \mathcal{DLD}^{\mathrm{sh}}(\emptyset) = \mathsf{tau} \circ (p' \mathbin{/_{\mathsf{dld}}} \mathcal{DLD}^{\mathrm{sh}}(\xrightarrow{s} *_{\mathrm{p}})) \;, \\ &p' \mathbin{/_{\mathsf{dld}}} \mathcal{DLD}^{\mathrm{sh}}(\xrightarrow{s} *_{\mathrm{p}}) = \mathsf{tau} \circ (\mathsf{S} \mathbin{/_{\mathsf{dld}}} \mathcal{DLD}^{\mathrm{sh}}((\xrightarrow{s} *_{\mathrm{p}}) \oplus (\xrightarrow{t} a))) \;, \\ &p'' \mathbin{/_{\mathsf{dld}}} \mathcal{DLD}^{\mathrm{sh}}(\xrightarrow{s} *_{\mathrm{p}}) = \mathsf{tau} \circ (\mathsf{S} \mathbin{/_{\mathsf{dld}}} \mathcal{DLD}^{\mathrm{sh}}((\xrightarrow{s} *_{\mathrm{p}}) \oplus (\xrightarrow{t} *_{\mathrm{s}}))) \;. \end{aligned}$$

Hence, by the definitions of shok' and shok:

$$\begin{aligned} &(p', \xrightarrow{s} *_{\mathrm{p}}) \in \mathit{shok}'(0, \{(p, \emptyset)\}) \;, \\ &(p', \xrightarrow{s} *_{\mathrm{p}}) \in \mathit{shok}'(2, \{(p, \emptyset)\}) \;, \\ &(p, \emptyset) \in \mathit{shok}'(1, \emptyset) \;, \\ &(p, \emptyset) \in \mathit{shok} \;. \end{aligned}$$

From this it follows by the definition of $\mathit{eff}^{\mathrm{sh}}$ that $\mathit{eff}^{\mathrm{sh}}(s\,!, \emptyset, p) = \mathit{eff}_{\mathrm{sh}(s\,!)}(\emptyset)$.

On account of shedding, we have that

$$p \mathbin{/_{\mathsf{dld}}} \mathcal{DLD}^{\mathrm{sh}}(\emptyset) = \mathsf{tau} \circ \mathsf{tau} \circ \mathsf{S} \;,$$

whereas

$$p \mathbin{/_{\mathsf{dld}}} \mathcal{DLD}(\emptyset) = \mathsf{tau} \circ \mathsf{tau} \circ \mathsf{D} \;.$$

The point is that a positive reply is produced on performing $t\,!$ only if spot s is shed on performing $s\,!$.

Example 2. Let

$$\begin{aligned} p &= \mathsf{dld}(s\,!) \circ ((\mathsf{dld}(u = s) \circ \mathsf{S}) \trianglelefteq \mathsf{dld}(t\,!) \trianglerighteq \mathsf{S}) \;, \\ p' &= (\mathsf{dld}(u = s) \circ \mathsf{S}) \trianglelefteq \mathsf{dld}(t\,!) \trianglerighteq \mathsf{S} \;, \\ p'' &= \mathsf{dld}(u = s) \circ \mathsf{S} \;. \end{aligned}$$

Thread p is the same thread as the one discussed in Section 7 and threads p' and p'' are residual threads of p. Assume again that the cardinality of AtObj is 1, and let a be the unique atomic object such that $\mathsf{AtObj} = \{a\}$. Then in $p \,/_{\mathsf{dld}}\, \mathcal{DLD}^{\mathrm{sh}}(\emptyset)$, spot s is not shed on performing $s\,!$. This is easily shown using the definition of shedding supporting data linkage dynamics services. It follows immediately from the definition of *nomsherr*, the definition of *secmsherr*, and basic set theory that:

$$\begin{aligned} &(u = s) \notin \mathit{nomsherr}((\xrightarrow{s} *_{\mathrm{p}}) \oplus (\xrightarrow{t} a)) \;, \\ &(u = s) \notin \mathit{secmsherr}((\xrightarrow{s} *_{\mathrm{p}}) \oplus (\xrightarrow{t} a)) \;, \\ &(p'', (\xrightarrow{s} *_{\mathrm{p}}) \oplus (\xrightarrow{t} a)) \notin \{(p, \emptyset), (p', \xrightarrow{s} *_{\mathrm{p}})\} \;, \end{aligned}$$

and it follows easily from the axioms for the use operators and the definition of mimicking of shedding supporting data linkage dynamics services that:

$$\begin{aligned} &p \,/_{\mathsf{dld}}\, \mathcal{DLD}^{\mathrm{sh}}(\emptyset) = \mathsf{tau} \circ (p' \,/_{\mathsf{dld}}\, \mathcal{DLD}^{\mathrm{sh}}(\xrightarrow{s} *_{\mathrm{p}})) \;, \\ &p' \,/_{\mathsf{dld}}\, \mathcal{DLD}^{\mathrm{sh}}(\xrightarrow{s} *_{\mathrm{p}}) = \mathsf{tau} \circ (\mathsf{S} \,/_{\mathsf{dld}}\, \mathcal{DLD}^{\mathrm{sh}}((\xrightarrow{s} *_{\mathrm{p}}) \oplus (\xrightarrow{t} a))) \;. \end{aligned}$$

Hence, by the definitions of shok' and shok:

$$\begin{aligned} &(p'', (\xrightarrow{s} *_{\mathrm{p}}) \oplus (\xrightarrow{t} a)) \notin \mathit{shok}'(0, \{(p, \emptyset), (p', \xrightarrow{s} *_{\mathrm{p}})\}) \;, \\ &(p', \xrightarrow{s} *_{\mathrm{p}}) \notin \mathit{shok}'(0, \{(p, \emptyset)\}) \;, \\ &(p, \emptyset) \notin \mathit{shok}'(1, \emptyset) \;, \\ &(p, \emptyset) \notin \mathit{shok} \;. \end{aligned}$$

From this it follows by the definition of $\mathit{eff}^{\mathrm{sh}}$ that $\mathit{eff}^{\mathrm{sh}}(s\,!, \emptyset, p) = \mathit{eff}_{s\,!}(\emptyset)$.

11 Conclusions

We have introduced shedding in the setting of data linkage dynamics and have adapted the data linkage dynamics services described in [9] so that they support shedding. The adaptation shows that the shedding feature is rather non-obvious. In particular, it is striking how much the matter is complicated by taking into consideration the semantic effects of the fact that the number of data objects that can exist at the same time is always bounded.

We consider the work presented in this paper a semantic validation of shedding. It is an entirely different question whether a real implementation of shedding is of any use in practice. We have not answered this question. Empirical studies, see e.g. [17], indicate that in general a large part of the data objects that are reachable at a program point are actually not used beyond that point. However, the static approximation of shedding proposed in [18] might be more useful in practice.

In the definition of shedding supporting data linkage dynamics services, belonging to *shok* corresponds to meeting the criterion for shedding. The set *shok* is defined using the idea of mimicked shedding. As a result, the description of the criterion for shedding looks to be rather concrete. It is an open question whether a more abstract description of the criterion for shedding can be given. If so, our concrete description should be correct with respect to that abstract description.

In the case of shedding, the use of forecasting turns out to be semantically feasible. No restrictions are needed to preclude forecasting from introducing something paradoxical. This is certainly not always the case. For example, in the case of the halting problem, the use of forecasting is not semantically feasible, see e.g. [10], and in the case of security hazard risk assessment, the use of forecasting requires certain restrictions, see e.g. [6].

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