Bilateral Teleoperation by Sliding Mode Control Design Approach

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***Abstract*—Sliding mode control has been used extensively in robotics to cope with parameters uncertainty, model perturbations and system disturbance. Bilateral robotic teleoperators are often required to provide a haptic interaction in telerobotic applications in which human kinesthetic sense is stimulated locally by remote environment. The paper deals with bilateral control for a force-reflection master-slave telerobotic architecture. It involves a short overview of basic bilateral modes. Chattering-free SMC design procedure for force-reflecting master-slave teleoperator is presented. The proposed bilateral control scheme was experimentally validated for a 1DOF master-slave teleoperator.**

***Index Terms*—Telerobotics, Bilateral Teleoperation, Sliding Mode Control.**

I. INTRODUCTION

Telerobotics [1] can be applied in manipulation where the operator has some task to do in the remote environment, or where a human operator can not be physically present or access the object concerned in the robotic task. Beside traditional applications which comprise handling in hazardous or inaccessible environments as space, underwater, nuclear plants many other uses of advanced telerobotic systems have been recently suggested or explored such as safety applications or robotic surgery. In teleoperation a human operator conducts a task by monitoring and control actions in a remote environment via a teleoperation system, i.e. teleoperator or telerobot, which can be viewed as a system for extending the sensorimotor system of the human organism with purpose to facilitate the human operator's ability to sense, maneuver in, and manipulate the environment. It is a machine that enables teleoperation and usually consists of two robotic manipulators, that are interconnected in master-slave connection. The local robotic system is a master and is physically operated by a human operator. On the other side in remote place the robotic system plays a role of a slave and follows the motion dictated by the master robot. Consequently, teleoperation couples the human operator and the master robot on one side, and on the other side appears strong interaction between the slave robot and remote environment. The master robot should represent the distant environment locally, i.e. at the human operator side, and vice versa, the slave robot should represent the operator action in distant place. The human operator may use such teleoperator for motion of objects in remote place not concerning reaction force significant for the task performance. In this case, only visual feed back may be enough and the teleoperation is unilateral. However, the slave robot can also have a contact with remote environment in which the reaction force may have substantial importance e.g. in remote assembling or disassembling and robotic surgery. In this case, the task performance can be significantly improved if the teleoperator can provide contact force information to the human operator from the remote environment. Although this information can be obtained from visual displays, it is more useful provided directly, i.e. by reflecting the measured contact force from remote place to motors on the master robot. When this is done, the contact force is said to be reflected to the human operator, and the teleoperator is said to be controlled *bilaterally*. A block scheme of a bilateral teleoperation system with information flow in both feedforward and feedback direction is depicted by Fig.1. Such telerobotic system is *force reflecting teleoperator* that provides force feedback from the remote environment to the human operator. Display of force feedback to the operator can be straightforward in principle; in force-reflecting master-slave teleoperators the measured force signals drive motors on the master arm that push back on the hand of the operator to stimulate human kinesthetic sense with the same forces and torques with which the slave pushes on the environment.

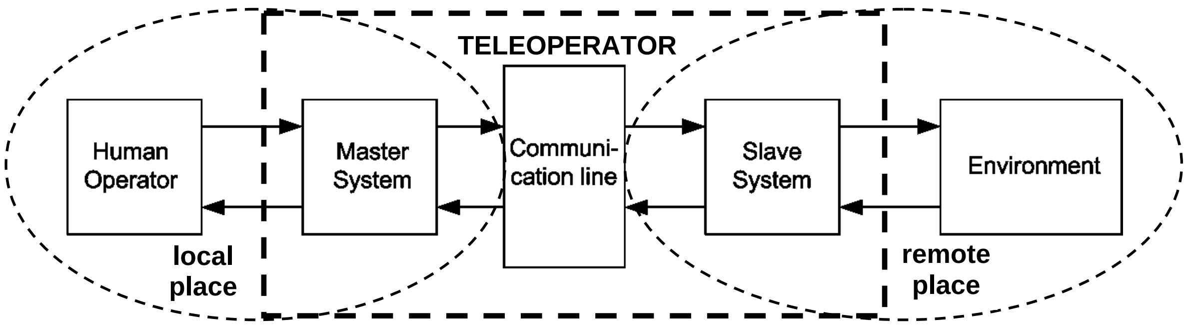

Figure 1. Bilateral teleoperation system

Figure 2. Network model of a bilateral teleoperation system

This might work perfectly in an ideal world where such slave-back-to-master force tracking is perfect, and the master and slave arms impose no mass, compliance, viscosity, or static friction characteristics of their own. However, in reality we must count on all these effects which consequently make harder or impossible to achieve the ideal teleoperator characteristics.

In this paper, basic bilateral model and control architectures for master-slave force-reflecting teleoperation are overviewed along with the main evaluation indexes in Section II. It is shown that impedance control is a fundamental approach in the design of master and slave control, respectively. Furthermore, it will be evident that only 4-channel bilateral architecture can provide perfect transparency. However, its implementation is questionable in practice due to the unavailability of some signals that are required within the control scheme and due to the stability problems that are inherently connected to the 4-channel teleoperator. Section III deals with sliding mode control and impedance robot control. In Section IV the derivation of sliding mode bilateral control is shown. Section V is focused on experimental results and Section VI concludes the paper.

II. BASICS OF BILATERAL CONTROL

A. Master-slave network model

The bilateral teleoperation system consists of a set of two robots, referred to as the master robot and the slave robot, together with appropriate sensors and a computer for control implementation. The master robot is that which is directly driven by the operator from his workplace, whereas the slave robot is that which is located in the remote environment, ready to follow any trajectories dictated by the operator. Thus, the operator moves a master device and its velocity is transmitted to the slave device, which is forced to follow the master movement. The ideal motion tracking assumes that the slave will follow the commanded motion at every time instant. In the case of contact with environment a reaction force appears. While accurate tracking is essential for the skillful control of tasks, it is not enough to achieve really good performance on its own since position is not the only relationship that exists between both robots. In fact, at the moment that the slave robot starts its interaction with the environment, reaction forces appear that is to be fed back on the master side. The human operator can feel reaction force from distant place. The goal of the teleoperator design is therefore also to convey precise information of the forces that appear between the slave robot and the environment.

In order to provide a proper design of such teleoperator, the study of the system traditionally adopts a network model of the bilateral teleoperation system (Fig.2). A two-port network model can be used for description of a bilateral teleoperator, i.e. from the mathematical point of view, the teleoperated system is just a relationship between the velocities and forces of the two robots, i.e. a set of four signals, namely v_m (velocity of the human operator/master robot), v_s (velocity of the slave robot/remote environment), F_h (action force that the operator applies to the master robot), and F_e (reaction force from the remote environment). Sometimes also positions are used instead of velocities. However, these signals can relate to each other in terms of different matrices that define the teleoperator model [2]. Both the impedance and admittance matrix can be used to relate forces and velocities. Alternatively, the transmission matrix can be used to relate the master variables on one side and the slave variables on the other. It is also possible to transform from one representation to another. Though various forms of the two-port network teleoperator model exists any of the proposed matrices provides complete information or characterization of the teleoperated system. Nevertheless no one seems to be perfect in meaningful interpretation of the most interesting teleoperator physical features. However, the most popular matrix for the analysis of teleoperated systems is the hybrid *h*-matrix,

$$\begin{bmatrix} F_h \\ -V_s \end{bmatrix} = \boldsymbol{h} \begin{bmatrix} V_m \\ F_e \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} V_m \\ F_e \end{bmatrix} \tag{1}$$

which is a locally linearized version of the system dynamics around a selected operating point., and whose entries determine the following parameters:

$$h_{11} = \left. \frac{F_h}{V_m} \right|_{F_e = 0}$$

- h_{11} is related to the input impedance and gives the *unconstrained movement impedance* – the equivalent inertia and damping that the operator feels moving the master robot if the slave is unconstrained - which is desired to be as low as possible,

$$h_{21} = \left. \frac{-V_s}{V_m} \right|_{F_e = 0}$$

- h_{21} is the transfer function of the *velocity tracking* during unconstrained motion – the ability of the slave robot to copy the position of the master robot – which should tend to unity with infinite bandwidth,

$$h_{12} = \left. \frac{F_h}{F_e} \right|_{V_m = 0}$$

- h_{12} is related to the force reflection, i.e. tracking of forces in contact tasks,

$$h_{22} = \left. \frac{V_s}{F_e} \right|_{V_m = 0}$$

- h_{22} is output admittance, i.e. the contact admittance on the slave side.

Obviously, the all parameters of the hybrid matrix $\boldsymbol{h}$ do not have clear physical background. Therefore, also other models can be considered in order to characterize properties of a teleoperator, e.g. the force reflection index h_{12} is not very meaningful because it is defined when the operator keeps the master steady against the forces that the slave encounters which is quite unnatural operation mode; more natural is to observe the index of force tracking in hard contact that can be derived as $\left. \frac{F_h}{F_e} \right|_{V_s=0} = \frac{h_{12}h_{21} - h_{11}h_{22}}{h_{21}}$.

B. Human and environment models

One port network models can be applied to describe both human operator and remote environment. Nevertheless, the dynamics of a human operator is governed by

$$F_h = F_h^* - \boldsymbol{Z}_h V_m \qquad (2)$$

where Z_h denotes impedance of the operator arm and F_h^* is force generated by the operator and the environment response can be described by the following equation,

$$F_e = Z_e V_s \qquad (3)$$

where Z_e stands for environment impedance.

C. Transparency

In bilateral teleoperation, information about the task at the remote site is required to help a human operator to feel as they are physically present at the remote place. The operator moves a master device and its velocity v_m is transmitted to the slave device, which is forced to follow the master movement. The ideal motion tracking assumes that the slave will follow the commanded motion at every time instant, i.e. $v_s = v_m$. When the slave device contacts the remote environment, the environment reaction force F_e is transmitted back to the human operator who should sense $F_h = F_e$. The ideal behaviour for a bilateral teleoperation system is to provide a faithful transmission of signals between the master and slave to couple the operator as closely as possible to the remote task: i) the force that human operator applies to the master arm is matched to the force reflected from the environment in the steady state (this can help operators realize force sensation), ii) the slave position is matched with the master position in the steady state, iii) the teleoperator must remain stable. If positions, velocities, and forces of the master and slave device are matched, then a teleoperator provides complete *transparency* [4], that is an important performance measure or evaluation index in bilateral teleoperation. Ideally, the teleoperation system would be completely transparent, so that operators would feel that they are directly interacting with the remote task. The desired dynamic behaviour of the teleoperator is, therefore, close to a rigid rod with minimal inertia and maximal stiffness. Thus, the connection of the master and slave arms should have zero mass and infinite stiffness. When the slave robot performs a contact task, then the slave velocities and forces are not independent. They are related by impedance of the slave environment Z_e. If operators are to feel as if they are touching the task directly, then the operator's force on the master F_h and the master's motion V_m should have the same relationship, i.e., for the same forces $F_h = F_e$ the same motion is also desired $V_s = V_m$. This requires that the impedance that is felt by the operator, which means the impedance that is transmitted to the operator Z_t, and is defined by the relation between operator's force and master motion,

$$F_h = Z_t V_m \qquad (4)$$

satisfies the transparency condition

$$Z_t = Z_e \qquad (5)$$

Z_t is referred to as *transmitted impedance* and is property of a teleoperator. It can be derived from a teleoperator two-port matrix description. The solving of F_h and V_m in terms of V_s and F_e, using the remote environment impedance, and eliminating V_s yields the transmitted impedance. If the h-matrix is used, then it is given as

$$Z_t = h_{11} - \frac{h_{12}h_{21}}{Z_e^{-1} + h_{22}} \qquad (6)$$

Theoretically, the conditions for perfect transparency $Z_t \equiv Z_e$ can be determined: i) $h_{11} = 0$, ii) $h_{22} = 0$, iii) $h_{12}h_{21} = -1$. Furthermore, in order to assure kinematic correspondence, the following condition must also be fulfilled: iv) $h_{21} = -1$ and $h_{12} = 1$.

However, maximum transmitable impedance, i.e. Z_t that is a teleoperator capable to present to the human operator at the local site, can be estimated by (7).

$$Z_t^{\max} = \left. \frac{F_h}{V_m} \right|_{V_s = 0} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}} \qquad (7)$$

D. Stability

The desired transparency of the bilateral teleoperator can be achieved only if *stability* of the bilateral teleoperation system can be assured. Thus, (robust) stability remains the most important goal in any bilateral controller design. The accurate closed-loop analysis with precise and sufficient condition for stability is extremely difficult to obtain since the system is multivariable in general and dependent on a particular human and environment characteristics. However, one can study stability properties by application of *absolute stability* condition that guarantees stability by assuring passivity of the one-port networks resulted from terminating master-slave two-port network by any passive environment and operator. By Llewellyn's absolute stability criterion one can state that the teleoperator two-port network LTI model (1) will be stable if and only if [3]:

- the hybrid parameters h_{11} and h_{22} have no poles on the right-half plane, and

- any poles of h_{11} and h_{22} on the imaginary axis are simple and have real and positive residues, and
- the inequalities below hold on $j\omega$ axis for all $\omega \geq 0$.

$$\Re\{h_{11}\} \geq 0$$

$$\eta_h(\omega) = -\frac{\Re\{h_{12}h_{21}\}}{|h_{12}h_{21}|} + 2\frac{\Re\{h_{11}\}\Re\{h_{22}\}}{|h_{12}h_{21}|} \geq 1$$

The stability criterion suggests that the transparency-optimized teleoperator can be only marginally stable as $\Re\{h_{11}\} \equiv 1$ and $\eta_h(\omega) \equiv 1$, thus stability is hard to achieve in case of perfect transparency. Transparency and robust stability are obviously conflicting design goals in bilateral teleoperation systems since good transparency usually implies strong coupling from master to slave, and in contrast, the sufficient conditions for stability results in conservative design criteria leading to poor transparency [4]. Thus, bilateral control design will be a compromise between stability and the teleoperator performance.

E. *Bilateral teleoperation control architectures*

A few basic control architectures can be designed for master-slave bilateral teleoperation:

- Position-Position (PP) architecture
- Force-Position (FP) architecture
- Four channel (4ch) architecture

PP architecture that is shown by Fig. 3 is practical and simple to implement. Thus, it can be found in a very early design of a bilateral teleoperation system [5]. The only information required is the position of the master and slave robots. Master position is passed as a command to the slave position controller. The slave will try to follow the master position. The control scheme is totally symmetric, which means that the slave position is returned to the master as a position command. This makes sense if the position controllers have good tracking capabilities, since the master and slave will closely follow each other. Force reflection is obtained as a result of the actuation produced by the master position controller when the error tracking grows due to the interaction between the slave robot and the remote environment. The master and slave are interconnected in a feedback loop, and the dynamics of the closed loop must also be considered. It has been observed that a highly accurate position control system on the master is *not* desirable. This makes the system feel "sluggish" in free space motion, since the lags between master and slave position movements cause large reaction forces to be supplied to the operator. Indeed, if one derives entries of the hybrid matrix, then it yields the unconstrained impedance h_{11},

$$h_{11} = Z_m + \frac{Z_s C_m}{Z_s + C_s} \approx Z_m + \frac{C_m}{C_s} Z_s \tag{8}$$

where Z_m, Z_s, C_m, C_s stand for dynamics of master robot, slave robot, master robot position controller, and slave robot position controller, respectively. Stiff position control loop on master robot means very high gains in C_m that in turn increase the unconstrained impedance. On the other hand, position tracking in the unconstrained motion will depend on the position control loop on the slave robot.

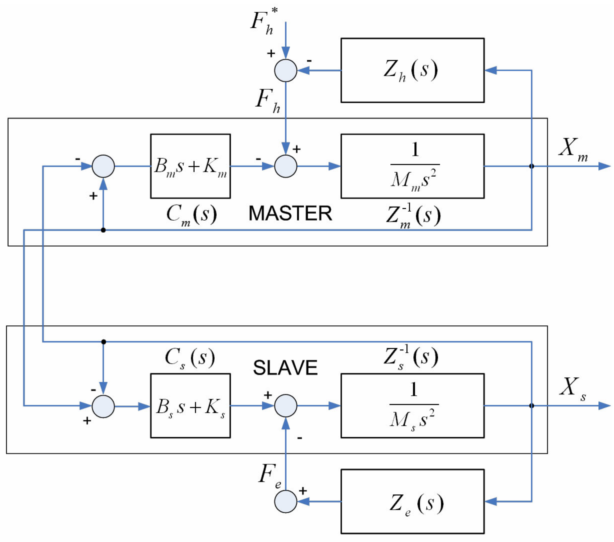

Figure 3. Position-position bilateral control architecture

$$h_{21} = -\frac{X_s}{X_m}\bigg|_{F_e=0} = -\frac{C_s}{Z_s + C_s} \approx -1 \tag{9}$$

Force tracking in hard contact task can be derived by the following index

$$\frac{F_h}{F_e}\bigg|_{V_s=0} = \frac{Z_m + C_m}{C_s} \approx \frac{C_m}{C_s} \tag{10}$$

and the transmitted impedance can be expressed as

$$Z_t = \frac{Z_m + C_m}{Z_s + C_s + Z_e} Z_e + \frac{Z_m Z_s + Z_m C_s + C_m Z_s}{Z_s + C_s + Z_e} \tag{11}$$

that in hard contact reduces to $Z_t^{\max} = Z_m + C_m \approx C_m$. This is the maximum transmittable impedance.

In the simplification above, quite large gains in the position controllers have been assumed. It can be demonstrated that if the system is linear and continuous, it will be stable for any gain of the PD controllers. This control scheme is intrinsically stable. Large unconstrained movement impedance makes this architecture useful only if a very low force reflection ratio is needed. Thus, in the context of industrial teleoperation (light masters and heavy slaves) it is unable to offer both large force-reflection ratios and light manoeuvring capability.

FP architecture [6] that is shown by Fig.4 is perhaps most intuitive. The idea here is also to send master's positions as commands for the slave to follow. However, the interaction force at the slave is sent back directly (scaled by the constant K_f) as a reaction force to the master, thus requiring a reaction force sensing device. Thus, force tracking in hard contact task can be now expressed as

$$\frac{F_h}{F_e}\bigg|_{V_s=0} = K_f + \frac{Z_m}{C_s} \tag{12}$$

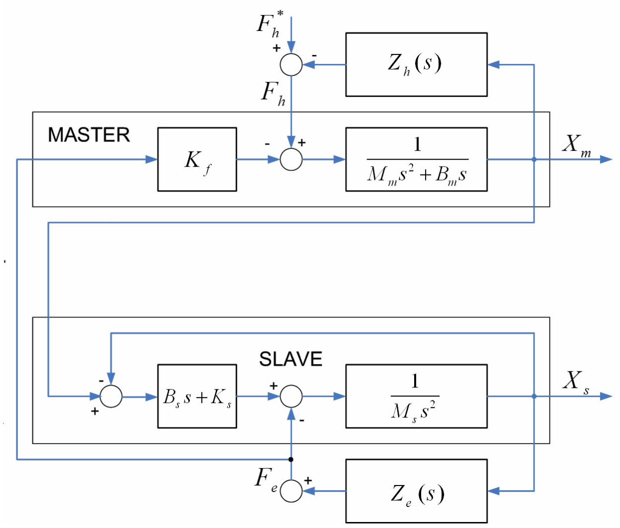

Figure 4. Force-position bilateral control architecture

If the slave faithfully reproduces the master motions and the master accurately feels the slave forces, the operator should experience the same interaction with the teleoperated task as would the slave. The transmitted impedance can be derived

$$Z_t = \frac{Z_m + K_f C_s}{Z_s + C_s + Z_e} Z_e + \frac{Z_m Z_s + Z_m C_s}{Z_s + C_s + Z_e} \tag{13}$$

which in hard contact means $Z_t^{\max} = Z_m + K_f C_s$, and in free motion reduces to unconstrained movement impedance

$$h_{11} = Z_m \tag{14}$$

Again, the static way of thinking does not address the dynamics of the interconnected system. It can be easily shown that theoretical limit for the positive force gain is $K_f \leq M_m / M_s$ otherwise the force feedback turns positive and destabilizes the system. Thus, in this force reflection architecture, stability can be often a problem unless the force feedback to the master is carefully attenuated. A notch filter can be used to improve the stability of the control system [7].

The 4-channel architecture is most general. As depicted by Fig.5, general multivariable system architecture is utilized which includes all four types of data transmission between master and slave: force and velocity in both directions.

Although not shown on the scheme, local force feedback loops may also be included to improve stability in contact tasks [8]. The 4-channel architecture requires position and force sensors in both robots, in order to feed data to the four communication channels. It has been shown that a proper use of all four channels is of critical importance in achieving high performance sense of accurate transmission of task impedances to the operator. In this algorithm, all the parameters are much coupled and it is not easy to predict how a change in any of them will affect the performance of the system.

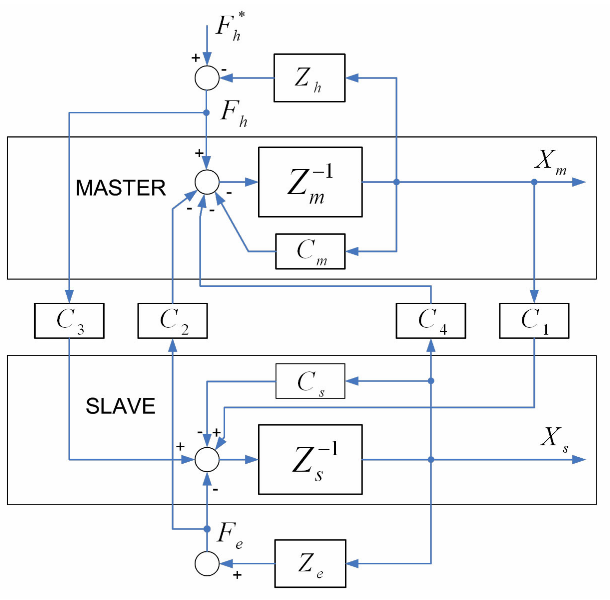

Figure 5. 4-channel bilateral control architecture

However, the analytical expressions obtained for this control scheme show its capability to achieve perfect transparency by tuning the transfer functions of the communication controllers as

$$\begin{gathered} C_1 = Z_s + C_s \\ C_4 = -\left(Z_m + C_m\right) \\ C_2 = C_3 = 1 \end{gathered} \tag{15}$$

The controller setup as given above leads to transparency optimized teleoperator with null unconstrained movement impedance, unitary force and position tracking with infinite bandwidth, and also unlimited maximum transmittable impedance - such teleoperator is therefore ideally transparent. The 4-channel controller seems to allow optimal employment of the information that the system generates in order to achieve excellent performance not only in free motion, but also in contact tasks, proving to be clearly superior to the other algorithms from any point of view. However, in order to achieve perfect transparency, the master and slave dynamics have to be cancelled out simultaneously and the forces fed forward have to match forces exerted by the operator or the environment exactly. Moreover, this selection of parameters requires the evaluation of accelerations that is usually not available in practice and therefore the architecture is hardly to provide necessary robust stability. Thus, acceleration is cancelled from the control signal that yields to the following master-slave governing dynamics equations:

$$\begin{gathered} Z_m X_m + C_m \left(X_m - X_s\right) = F_h - F_e \\ Z_s X_s + C_s \left(X_s - X_m\right) = F_h - F_e \end{gathered} \tag{16}$$

Such configuration reasonably impairs performance of the bilateral teleoperator. The derivation of the evaluation indexes yields the unconstrained movement impedance

$$h_{11} = \frac{Z_s + C_s}{Z_s + C_s + C_m} Z_m + \frac{C_m}{Z_s + C_s + C_m} Z_s \tag{17}$$

that can be approximated by

$$h_{11} \approx \frac{C_s}{C_s + C_m} Z_m + \frac{C_m}{C_s + C_m} Z_s \,,$$

and position tracking in the unconstrained motion is given by (18).

$$h_{21} = -\frac{Z_m + C_m + C_s}{Z_s + C_s + C_m} \approx -1 \tag{18}$$

Furthermore, the transmitted impedance is described by (19).

$$Z_t = h_{11} + \frac{Z_m + C_m + C_s}{Z_s + C_s + C_m} Z_e \approx h_{11} + Z_e \tag{19}$$

The indexes of force tracking in hard contact and maximum transmittable impedance remain as in the ideal 4channel bilateral control.

III. SLIDING MODE IMPEDANCE CONTROL

A. Robot Dynamics

The motion equation for a nonredundant robot dynamics with rotational joints can be written as [9]

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{c}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} - \boldsymbol{J}^T \boldsymbol{f}_{ext} \tag{20}$$

where $\boldsymbol{q}$, $\dot{\boldsymbol{q}}$, $\ddot{\boldsymbol{q}}$, $\boldsymbol{\tau}$ are *n*-dimensional vectors of joint position, velocity, acceleration and applied motor torque, respectively, and *n* denotes number of robot degrees-of-freedom. $\boldsymbol{f}_{ext}$ is in general a spatial 6D vector and denotes external force acting on the robot end effector due to contact with environment. $\boldsymbol{M}$ is *n*x*n* symmetric and positive definite matrix, called the joint-space inertia matrix. $\boldsymbol{c}$ determines effects of Coriolis and centrifugal forces expressed in joint space, $\boldsymbol{g}$ stands for the effect of gravitational field. The matrix $\boldsymbol{J}$ is the Jacobian of the robot end-efector that satisfies equation

$$\dot{\boldsymbol{x}} = \boldsymbol{J}(\boldsymbol{q})\dot{\boldsymbol{q}} \tag{21}$$

where $\dot{\boldsymbol{x}}$ is spatial 6D velocity vector of end effector and the Jacobian can be derived as $\boldsymbol{J}(\boldsymbol{q}) = \partial \boldsymbol{L}(\boldsymbol{q}) / \partial \boldsymbol{q}$ where $\boldsymbol{L}(\boldsymbol{q})$ denotes geometrical transformation of joint position vector to the task space robot end-efector spatial 6D vector.

$$\boldsymbol{x} = \boldsymbol{L}(\boldsymbol{q}) \tag{22}$$

Furthemore, if one define spatial 6D acceleration vector by

$$\ddot{\boldsymbol{x}} = \boldsymbol{J}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \dot{\boldsymbol{J}}(\boldsymbol{q})\dot{\boldsymbol{q}} \tag{23}$$

then the alternative form of robot dynamics can be expressed in the operational-space, i.e. in the robot task-space, which is the space in which the robot is commanded to operate.

$$\boldsymbol{M}_x(\boldsymbol{x}) \cdot \ddot{\boldsymbol{x}} + \boldsymbol{c}_x(\boldsymbol{x},\dot{\boldsymbol{x}}) + \boldsymbol{g}_x(\boldsymbol{x}) = \boldsymbol{f} - \boldsymbol{f}_{ext} \tag{24}$$

where the control input is related by motor toques as $\boldsymbol{\tau} = \boldsymbol{J}^T \boldsymbol{f}$, $\boldsymbol{M}_x$ is operational-space mass matrix, $\boldsymbol{c}_x$ contains Coriolis and centrifugal force terms whereas $\boldsymbol{g}_x$ contains gravity force terms.

B. Chattering-free Sliding Mode Control

Robotic mechanisms are generally characterized by multivariable input-output nonlinear dynamics and present a hard control problem. Model-based control techniques can be applied in order to linearize behavior. Inverse dynamics calculation requires the complete knowledge of robot dynamics. Consequently, such model-based control law techniques are sensitive to the structured and unstructured uncertainties, which always exist in the robot model, and the desired performance can not be achieved.

Theory of Variable System Structure provides a framework for robust Sliding Mode Control [10] that can be used for systems with bounded control input Sliding Mode Control if the uncertainties in the model structure are bounded In this case, full disturbance rejection is possible if so-called matching condition is fulfilled.

The robot dynamics (24) can be viewed as a set of interconnected SISO systems

$$m_{ii}(\boldsymbol{x}) \cdot \ddot{x}_i + \sum_{j=1, j\neq i}^{6} m_{ij}(\boldsymbol{x})\ddot{x}_j + c_i(\boldsymbol{x},\dot{\boldsymbol{x}}) + g_i(\boldsymbol{x}) = f - f_{ext} \tag{25}$$

where $i = 1...6$. If one defines the signal that contains system perturbation and external disturbance by

$$d_i = -\left(\Delta m_{ii}(\boldsymbol{x})\ddot{x}_i + \sum_{j=1, j\neq i}^{6} m_{ij}(\boldsymbol{x})\ddot{x}_j + c_i(\boldsymbol{x},\dot{\boldsymbol{x}}) + g_i(\boldsymbol{x}) + f_{ext}\right) \tag{26}$$

where $\Delta m_{ii}(\boldsymbol{x}) = m_{ii}(\boldsymbol{x}) - \hat{m}_i(x_i)$, and $\hat{m}_i$ is a nominal mass of the ***i***-th system, then the SISO (25) system can be abstracted by the following state space form:

$$\begin{aligned} \dot{z}_j &= z_{j+1} \\ \dot{z}_n &= f(\boldsymbol{z}) + b(\boldsymbol{z})u + d \end{aligned}$$

where $j = 1,...,n-1$, $\boldsymbol{z}^T = \left[z_1,...,z_n\right]$, u is scalar input, and $f(\boldsymbol{z})$, $b(\boldsymbol{z})$ are in general nonlinear functions of state. In the SMC approach, the goal of the control design is to find such control input u that restricts the motion of the system states $\boldsymbol{z}$ to a selected sliding manifold $\sigma(\boldsymbol{z},t) = 0$. Here, $\sigma(\boldsymbol{z},t)$ is so called switching function that can be often selected as a linear combination of system states and time-variant reference, i.e. $\sigma(\boldsymbol{z},t) = r(t) - \boldsymbol{G}\boldsymbol{z}$. It has been proven that control with discontinuities on the sliding manifold $\sigma(\boldsymbol{z},t) = 0$ such as

$$u = \begin{cases} u^+, & \sigma(z,t) > 0 \\ u^-, & \sigma(z,t) < 0 \end{cases}$$

can enforce sliding mode if u^+ and u^- are selected such that the derivative of Lyapunov function candidate $v = \sigma^2/2$ is negative definite. By application of the equivalent control method u^+ and u^- can be selected such that $u^+ > u_{eq} > u^-$ are continuous functions of the system states and the equivalent control u_{eq} is solution of $\dot{\sigma}|_{\sigma=0}=0$. The dynamics of the closed loop system in sliding mode are fully rejecting model perturbations and disturbances if the matching conditions are fullfiled. The SMC also reduces the order of the closed loop system. However, the discontinuous control has disadvantages related to the bang-bang control action that in mechanical systems may be hard to realize since forces are continuous function, and in addition it may excite high frequency dynamical terms. Therefore, such discontinues control must be strictly avoided in mechanical systems, since it causes well known chattering that may lead to increased wear of the actuators and to excitation of the high order unmodeled dynamics which can cause damage on mechanical parts and break the servo system.

Smooth control signal solution can be found by augmenting the original system with an additional system state in order to eliminate discontinuities on the control signal. It yields

$$\dot{z}_j = z_{j+1}$$
$$\dot{z}_n = z_{n+1}$$
$$\dot{z}_{n+1} = g(z,u) + b(z)\dot{u} + \dot{d}$$

where $g(z,u) = \dot{f}(z) + \dot{b}(z)u$. The derivative of v can have the form $\dot{v} = -D\sigma^2$, $D > 0$ is arbitrary chosen control gain, which guarantees asymptotical reaching law. The switching function may have the form $\sigma(z) = z_{n+1} + \boldsymbol{G}z$. From condition $\sigma\dot{\sigma} = -D\sigma^2$ and by application of the equivalent control method one can derive control u

$$u = \int_0^t \dot{u}\, d\upsilon$$
$$\dot{u} = \dot{u}_{eq} + D\sigma$$

that assures invariant system motion in sliding mode. The system is said to be robust to system perturbation and external disturbance that comply with the matching conditions.

The derivation of the continues SM control law applies calculation the equivalent control signal that requires complete information about systems dynamics which is hardly to be exactly known in practice and thus it is not practical for implementation. Hence, the equivalent control signal is replaced with its estimated value which utilizes nominal model and the rest is considered unknown system perturbation and disturbance. Thus, the control is implemented accordingly by

$$u = \hat{u}_{eq} + D\int_0^t \sigma\, d\upsilon$$

The control law has two components. One is representing estimation of the equivalent control which is based on the available model knowledge that is worthless to be neglected. Another can be referred to as a robust controller that is representing the disturbance estimation and the convergence to the selected sliding mode manifold. The block diagram of the SMC with smooth chattering-free control is shown by Fig.6.

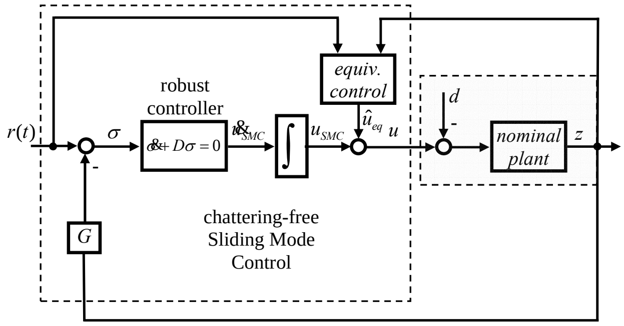

Figure 6. Principal control scheme for chattering-free SMC

The application of the Lyapunov function analysis shows that $\dot{v} = -D\sigma^2 + \sigma\dot{d}$. In order to guarantee invariant asymptotically stable solution $\sigma = 0$ the disturbance should satisfy the requirement $\dot{d} = 0$, or in other words, it should be constant. However, if disturbance changes relatively slowly, then $\dot{d} \approx 0$, and by high value of gain D the control can keep the system states in the vicinity of the sliding mode manifold ($\sigma \approx 0$). By proper tuning of the D one can desensitize the system from the disturbance. Obviously, in steady state zero control error is assured. Thus, it is possible to satisfy control requirements given by the definition of the switching function and simultaneously perform smooth and fair robust control.

C. *Sliding Mode Impedance Control*

When a robot is commanded to perform a task by interaction with its environment then pure position control fails and the contact force must be regulated. One approach to provide a proper contact of the robot with its environment is impedance control [11]. In the impedance control a desired dynamic relationship between position or velocity vs. contact force is enforced, i.e. to be obtained in a closed-loop by properly designed control. Simple impedance dynamics of second order for a single DOF can be given by

$$M\ddot{x} + B\dot{x} + Kx = -f_{ext} \tag{27}$$

where M, B, and K are desired mass, damping and stiffness of the mechanism, respectively. x is the position and f_{ext} is contact force between the mechanism and the environment. The mass, damping, and stiffness, are in general nonnegative constants to define asymptotically stable linear response dynamics of the mechanism while in the contact.

Let describe the system dynamics by

$$m\ddot{x} = f - f_{ext} - f_{dist} \tag{28}$$

where m, and f denotes mass, and force control signal respectively, and f_{dist} is disturbance force. If the desired impedance is given by (27) then the switching function can be defined by

$$\sigma = \ddot{x} + k_v\dot{x} + k_p x + \tfrac{f_{ext}}{M} \tag{29}$$

where $k_v = \frac{B}{M}$ and $k_v = \frac{K}{M}$. Following the SMC design procedure above one can constitute the control signal as given by

$$f = m\left(\ddot{x}^{c} - D\int_0^t \sigma d\upsilon \right) \qquad (30)$$

where $\ddot{x}^{c} = -\left(k_v \dot{x} + k_p x\right) - \frac{f_{ext}}{M}$. Note that the control is simple and practical for implementation since acceleration signal which most often extremely noisy in practice is not required in calculation though implicitly present in (30) due to the switching function definition (29), i.e. $\sigma = \ddot{x}^{c} - \ddot{x}$ and thus $\int_0^t \sigma d\upsilon = \int_0^t \ddot{x}^{c} d\upsilon - \dot{x}$. However, the projection of system motion in the σ-space is governed by $\sigma = -D\int_0^t \sigma d\upsilon - \frac{f_{ext} + f_{dist}}{m}$ which can be written as

$$\dot{\sigma} + D\sigma = -\frac{\dot{f}_{ext} + \dot{f}_{dist}}{m} \qquad (31)$$

This guarantees that for constant disturbance and contact force the system motion will converge to the manifold defined by (29). Since in contact the disturbance can be assumed to change slowly, so that the control law can keep the system states close to the sliding manifold ($\sigma \approx 0$) if relatively slow changing contact force is to be applied.

IV. SLIDING MODE BILATERAL CONTROL

A. *Dynamics of bilateral master-slave teleoperator*

The dynamics of single DOF master-slave system may be modeled as a pure mass system

$$\begin{aligned} m_m \ddot{x}_m &= f_m + f_h \\ m_s \ddot{x}_s &= f_s - f_e \end{aligned} \qquad (32)$$

where m_i represents mass, $\ddot{x}_i$ denotes acceleration, and f_i is control signal (with index $i = m, s$ denoting master and slave, respectively). f_h and f_e are action force applied by the human operator at the master device and the force exerted on the slave by the environment.

Following the review of basic bilateral teleoperation control architectures fundamentally is to enforce impedance dynamics to each system in the master-slave structure. Best transparency can be achieved by the 4-channel architecture in which all contact forces and positions are communicating between the master and the slave. Therefore, the desired impedance relationship that is to be enforced for the master and the slave can be described by (33).

$$\begin{aligned} M_m \ddot{x}_m + B_m(\dot{x}_m - \dot{x}_s) + K_m(x_m - x_s) &= f_h - f_e \\ M_s \ddot{x}_s + B_s(\dot{x}_s - \dot{x}_m) + K_s(x_s - x_m) &= f_h - f_e \end{aligned} \qquad (33)$$

B. *Derivation of Sliding Mode Bilateral Control*

The SMC design procedure can be used for robust bilateral control. First step in the derivation of SMC is to select sliding manifold on which the desired system dynamics will be enforced and thus one should define the switching function. Though, the master-slave system is evidently MIMO system, as shown above, it is possible to design two separate controllers for master and slave, respectively, and simultaneously ensure stability of the overall system. Thus, in this paper, the bilateral SMC design is applied by the definition of two switching functions formed on the basis of (33). It yields

$$\begin{aligned} \sigma_m &= \ddot{x}_m + \tfrac{k_v}{2}(\dot{x}_m - \dot{x}_s) + \tfrac{k_p}{2}(x_m - x_s) - \tfrac{f_h - f_e}{M_m} \\ \sigma_s &= \ddot{x}_s + \tfrac{k_v}{2}(\dot{x}_s - \dot{x}_m) + \tfrac{k_p}{2}(x_s - x_m) - \tfrac{f_h - f_e}{M_s} \end{aligned} \qquad (34)$$

where index m refers to the master and index s refers to the slave part of the interconnected system. If both the master and slave dynamics can be described in the form of (32) then following the SMC design procedure presented in the section above it is easy to derive the bilateral control that yields (35)

$$\begin{aligned} f_m &= m_m\left(\ddot{x}_m^{c} - D_m \int_0^t \sigma_m d\upsilon \right) \\ f_s &= m_s\left(\ddot{x}_s^{c} - D_s \int_0^t \sigma_s d\upsilon \right) \end{aligned} \qquad (35)$$

and provides asymptotical stable response and disturbance sensitivity which can be given by (36),

$$\begin{aligned} \dot{\sigma}_m + D_m \sigma_m &= \frac{\dot{f}_h}{m_m} \\ \dot{\sigma}_s + D_s \sigma_s &= -\frac{\dot{f}_e}{m_s} \end{aligned} \qquad (36)$$

where D_m and D_s are positive constant control gains with arbitrary chosen values, respectively, and

$$\begin{aligned} \ddot{x}_m^{c} &= -\tfrac{k_v}{2}(\dot{x}_m - \dot{x}_s) - \tfrac{k_p}{2}(x_m - x_s) + \tfrac{f_h - f_e}{M_m} \\ \ddot{x}_s^{c} &= -\tfrac{k_v}{2}(\dot{x}_s - \dot{x}_m) - \tfrac{k_p}{2}(x_s - x_m) + \tfrac{f_h - f_e}{M_s} \end{aligned} . \qquad (37)$$

Note that where the calculation of the robust terms in (35) can be implemented as $\int_0^t \sigma_i d\upsilon = \int_0^t \ddot{x}_i^{c} d\upsilon - \dot{x}_i,\ i = m, s$.

C. *Bilateral Control In Virtual Space*

Obviously, the master-slave bilateral system is intercoupled since both the master and slave systems states are present in both the master and slave motion dynamics equations. Though good transparency is an important performance criteria, (robust) stability of the system remains the main control design criteria and it can be easily examined by an alternative description of the closed-loop dynamics of such 4ch bilateral master slave system, i.e. by observing virtual *differential* and *common* mode dynamics, which are decoupled since the particular system states are limited to a single motion dynamics equation. Thus, design of the bilateral control parameters is easy in the virtual modes. If the impedance characteristics are chosen such that $\frac{1}{2}k_v = \frac{B_m}{M_m} = \frac{B_s}{M_s}$, $\frac{1}{2}k_p = \frac{K_m}{M_m} = \frac{K_s}{M_s}$, then position and force tracking dynamics can be described by

$$\begin{aligned} \ddot{e} + k_v \dot{e} + k_p e &= (\tfrac{1}{M_m} - \tfrac{1}{M_s})\mathrm{E} \\ \ddot{S} &= (\tfrac{1}{M_m} + \tfrac{1}{M_s})\mathrm{E} \end{aligned} \qquad (38)$$

where $e = x_m - x_s$, and $\mathrm{E} = f_h - f_e$ denote position and force error, respectively, and $s = x_m + x_s$. Moreover, if identical master and slave impedance dynamics can be selected such that $M_m = M_s = 2M$ then

$$\begin{aligned} &\ddot{e} + k_v\dot{e} + k_p e = 0 \\ &M\ddot{s} = \mathrm{E} \end{aligned} \tag{39}$$

The dynamics of the state variables *e* and *s* are connected to differential and common mode, respectively. The design of bilateral control in the virtual space enables that position and force control are considered separately. Obviously, force servoing in common mode should be provided by $\ddot{s} = 0$ while position regulation in differential mode should guarantee that $e \to 0$. It means that stiff coupling between master and slave manipulator will be obtained by such bilateral controller. However, from (38) it is evident that the chosen dynamics of both master and slave impedance (33) can provide asymptotically stable dynamics with the zero steady state values $e = 0$, $\mathrm{E} = 0$.

D. Disturbance Sensitivity In Virtual Space

The decoupled system dynamics can be observed via the couple [$\Delta\sigma$, $\Sigma\sigma$],

$$\begin{aligned} \Delta\sigma &= \ddot{e} + k_v\dot{e} + k_p e - (\tfrac{1}{M_m} - \tfrac{1}{M_s})\mathrm{E} \\ \Sigma\sigma &= \ddot{s} - (\tfrac{1}{M_m} + \tfrac{1}{M_s})\mathrm{E} \end{aligned} \tag{40}$$

where $\Delta\sigma = \sigma_m - \sigma_s$ and $\Sigma\sigma = \sigma_m + \sigma_s$. Let $D_s = D_m = D$, then one can derive the closed-loop dynamics equations

$$\begin{aligned} &\ddot{e} + k_v\dot{e} + k_p e - (\tfrac{1}{M_m} - \tfrac{1}{M_s})\mathrm{E} = \Delta\sigma \\ &\Delta\dot{\sigma} + D\Delta\sigma = \tfrac{1}{2}\left\{\left(\tfrac{1}{m_m} - \tfrac{1}{m_s}\right)\dot{\mathrm{E}} + \left(\tfrac{1}{m_m} + \tfrac{1}{m_s}\right)\dot{\Sigma}\right\} \end{aligned} \tag{41}$$

and

$$\begin{aligned} &\ddot{s} - (\tfrac{1}{M_m} + \tfrac{1}{M_s})\mathrm{E} = \Sigma\sigma \\ &\Sigma\dot{\sigma} + D\Sigma\sigma = \tfrac{1}{2}\left\{\left(\tfrac{1}{m_m} + \tfrac{1}{m_s}\right)\dot{\mathrm{E}} + \left(\tfrac{1}{m_m} - \tfrac{1}{m_s}\right)\dot{\Sigma}\right\} \end{aligned} \tag{42}$$

where $\Sigma = f_h + f_e$. It is clear, that the proposed bilateral control (35), (34) can provide fair robust stability of the master-slave system given by (32) and the desired impedance characteristics (33) under consideration.

V. EXPERIMENTAL RESULTS

A. Experimental system

The efficiency of the proposed SM bilateral control scheme was demonstrated by the experimental system shown in Fig.7. It consisted of two identical 1DOF robot manipulators with a handle connected to the motor axis via a planetary gearhead. The handles can either be used to operate the system by a human on master side or to provide the environment contact on the slave side. The handle rotational plane was in horizontal orientation that provides zero gravitational effects and thus friction present in the motors as well as the gearheads is the main disturbance effect.

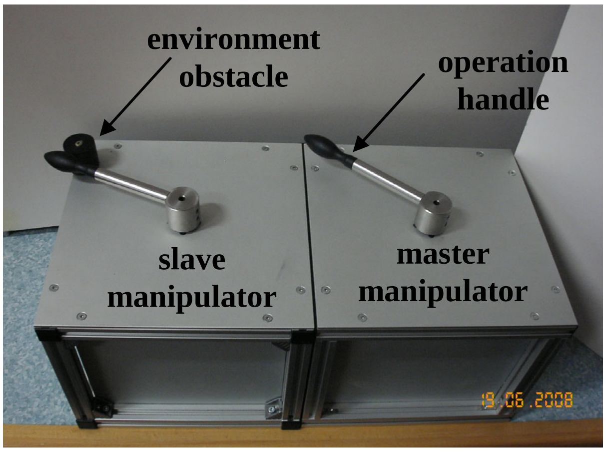

Figure 7. Experimental bilateral system

The master-slave manipulator system output positions that were measured by encoders mounted on the motor axes, while the velocity was obtained by traditional discrete differentiation and filtering of the position signals within the computer controller which processes the data and set the analog voltage reference value to the servo drivers. The reference value represents signal of desired current that the servodrivers were injected to the BLDC servomotors. The motor torque is considered to be proportional to the motor current.

B. The bilateral controller setup

The master and slave controllers were implemented as described by the bilateral control scheme given by (35) and (37) with $D_s = D_m = D$ and $M_m = M_s = 2M$; *M* was chosen so that matches total inertia in the manipulator. Although external force information was also necessary for the 4-channel bilateral architecture, force sensors were not used within this system. Instead, the external force observer [12] was applied to estimate external force. By application of the external force observer, the estimated force signal can be described by

$$\hat{f}_{ext} = \tfrac{g}{s+g} f_{ext}$$

where *g* denotes the cut-off frequency of the force observer. Table I shows the manipulators parameters and the control parameters are shown in Table II.

TABLE I.
MANIPULATOR PARAMETERS

Motor torque	*Nm*	0.355
Motor power	*W*	80
Motor rotor inertia	*gcm²*	20
Encoder resolution	*lines/rev*	500
Gearhead reduction ratio		14:1
Gearhead mass inertia	*gcm²*	0.8
Max. torque at gear output	*Nm*	3
Handle inertia	*kgcm²*	5.5
Handle length	*cm*	13.5

TABLE II.
BILATERAL CONTROL PARAMETERS

velocity filter cutoff freq.	*rad/s*	250
external force observer cutoff freq. *g*	*rad/s*	125

robust gain *D*	*1/s*	125
position gain k_p	$1/s^2$	5000
velocity gain k_v	*1/s*	100
control rate	*Hz*	2000

C. Results

Experimental results are given by Fig.8-10. The top diagrams depict position response determined by handles angle trajectories of master and slave, respectively. The bottom diagrams depict force response indicated by the handles torque and estimated by the master and slave external torque observer, respectively.

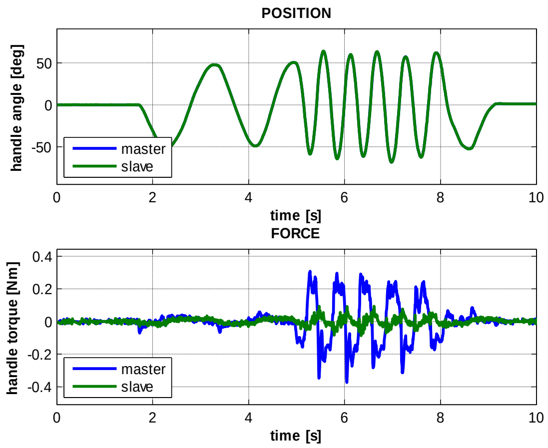

Figure 8. Bilateral teleoperation experiment in free motion

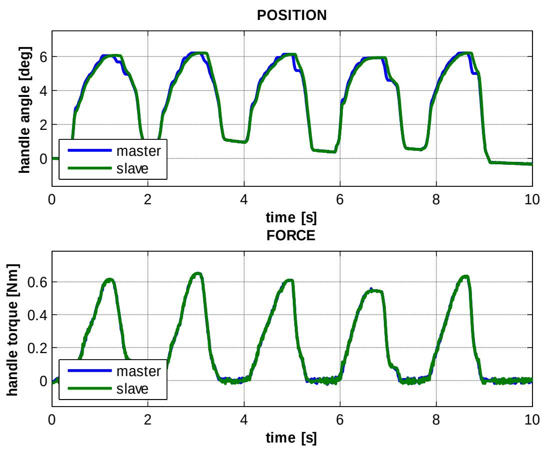

Figure 9. Bilateral teleoperation experiment in soft contact motion

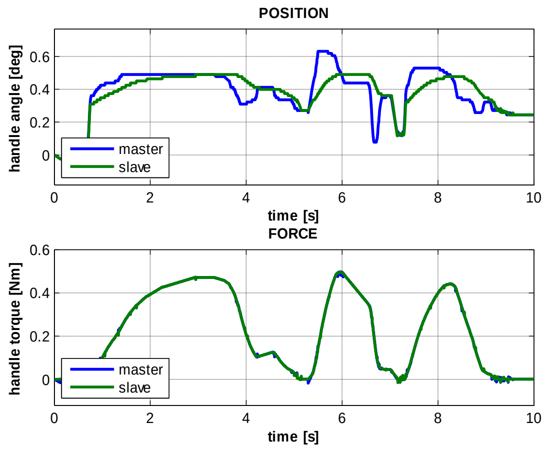

Figure 10. Bilateral teleoperation experiment in hard contact motion

Fig.8 shows free motion experiment. In free motion only human operater holds the master handle, while the slave handle is not constrained by environment. Thus, in slow motion only low force at the master handle was required, whereas for high speed oscillating the human driving force was significantly increased to compensate for the inertial force within the experimental teleoperator. However, in both regimes almost perfect position tracking was observed.

Fig.9 shows the experiment in which the slave handle was in contact with soft obstacle (sponge). Simultaneous position and force tracking was observed. More specific, force tracking was almost perfect, whereas position tracking was slightly deteriorated due to the fast decreasing of contact force. However, position tracking error significantly increased in case of hard contact as shown by Fig.10, while force tracking remained well. In contact with hard environment only small changes in position caused high changes in contact force. Nevertheless, steady state position error was zeroed though the contact with hard environment.

D. Discussion

The experimental results are closely related to the aforementioned indexes of the implemented 4ch bilateral scheme and the derived equations of the decoupled closed-loop dynamics in the virtual mode space. Note that position error dynamics according to the equations (41) turns to $\ddot{e}+k_v\dot{e}+k_p e=\frac{1}{m}\frac{p}{p+D}\Sigma$, however, it can explain high position tracking error due to rapid changing forces that appeared in the hard contact experiment. The disturbance rejection capability of the bilateral control scheme is strongly related to the values of the feedback gains – the equation suggests that higher values provide stronger disturbance rejection and consequently better performance. In the free motion and soft contact experiment the characteristics of the disturbance signals are far beyond the rejection capability of the control setup. In case of the hard contact experiment the disturbance signals fall into the domain in which robustness is drastically impaired. The robustness could be improved with higher values of the robust gain *D*, however, this is limited due to the noise/signal ratio of the measured signals and implemented control rate. The proposed control scheme is significantly dependent on the quality of the velocity signal. Note that position resolution of the experimental system reflected to the gearhead output was 0.013deg and sampling interval was 0.5ms. Therefore the the bilateral control performance could be improved by higher resolution of the measured signals.

VI. SUMMARY AND CONCLUSIONS

The paper overviews basic bilateral control modes and shows the derivation of the bilateral control scheme. Although the SMC design is used to obtain the control scheme, smooth continuous control has been derived that is required in motion control if forces or torques are considered. The control scheme for master and slave robot is based on impedance control approach, respectively, and is simple and easy to implement with the potential to provide high robustness against the disturbance. The proposed bilateral control scheme has been briefly examined by the closed-loop dynamics in the virtual space of the differential and common modes. Furthermore, it was experimentally validated for a 1DOF master-slave teleoperator. The experiments showed that high quality of measurement signals and acceleration control are extremely important in achievement of high-performance force reflection bilateral teleoperation.

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