Fractional Order PIλDμ Control Design for a Class of Cyber-Physical Systems with Fractional Order Time-Delay models

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Fractional Order PI^{λ} and $PI^{\lambda}D^{\mu}$ Control Design for a Class of Fractional Order Time-Delay Systems

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ABSTRACT

In this paper, a fractional order PI^{λ} and $PI^{\lambda}D^{\mu}$ control design is investigated for a class of fractional order Time-Delay systems. The proposed control design approach is simple and efficient. The controller parameter's adjustment is achieved in two steps: first, the relay approach is used to compute satisfactory classical PID coefficients, namely k_p, T_i and T_d. Then, the fractional orders λ and μ are optimized using performance criteria. Simulation results show the efficiency of the proposed design technique and its ability to enhance the PID control performance.

Keywords: Fractional order control; Time-delay systems; Fractional PI^{λ} controller, Fractional $PI^{\lambda}D^{\mu}$ controller; Relay method; Parameters adjustment, Optimization, Performance.

1. INTRODUCTION

Fractional calculus (FC) is an old mathematical analysis concept which is attracting great interest nowadays (Ladaci et al. 2008; Ladaci and Charef 2012; Rabah et al. 2017). Initially introduced pure mathematics, recently, FC has invested various engineering domains (Ladaci and Charef 2006; Petráš 2011). Fractional order models have proven to better represent many physical systems, like dielectric polarization, semi-infinite transmission lines, viscoelasticity…etc. (Schmidt and Gaul 2002). In the automatic control engineering field, an important effort is done towards design and development and application of fractional order controllers with various control strategies (Khettab et al. 2019; Vafaei et al. 2019).

Recently, a great number of researchers focused their work on the stability analysis of fractional order time-delay systems (Lazarević 2011; Zhang et al. 2016; Li et al. 2014; He et al. 2018). Finite-time stability has been investigated in (Lazarević and Spasić 2009; Chen et al. 2014; Naifar et al. 2019), while different stability criteria for this class of systems have been proposed in literature (Hwang and Cheng 2006; Merrikh-Bayat and Karimi-Ghartemani 2009; Shi and Wang 2011; Fioravanti et al. 2012; Gao 2014). Stability of linear fractional order systems with delays has been studied in many papers (Busłowicz 2008), also fractional-order nonlinear systems with delay (Lazarević and Debeljković 2008; Wang and Li

2014), neutral fractional-delay systems (Bonnet and Partington 2002; Xu et al. 2016) and fractional-order time-varying delay systems (Boukal et al. 2016).
In the field of control engineering, the introduction of fractional operators and systems has proven certain ability for performance enhancement Li and Chen 2008). One of the pioneering works was developed by Podlubny in 1999 who has proposed a generalized form for PID controllers, namely $PI^{\lambda}D^{\mu}$ controllers, using an integrator of order λ and a differentiator of order μ (where λ and μ have real values) (Podlubny 1999). He showed that the fractional order PID control improves the control system performance in comparison with the classical PID controller because of the extra real parameters λ and μ involved. The implementation of this fractional controller needs these non-integer operators to be approximated, using frequency domain approximations such as Oustaloup's method (Oustaloup et al. 2000) and Charef's method (Charef et al. 1992)…
However the problem of PID controllers' tuning remains an important issue (Rabah et al. 2018; Bourouba et al. 2018). There exist many adjusting techniques that do not require any model of the plant to control. All that is needed to apply such rules is to have time response of the plant. Examples of such sets of rules are those due to Ziegler and Nichols, Cohen and Coon, and the Kappa-Tau rules (Valério and Costa 2006). In particular, the Ziegler-Nichols method (Ziegler et al. 1942) still remains popular, particularly in industry. An interesting similar modern approach is based on the relay auto tuning (Atherton et al. 2014).
In this paper, a fractional order PI^{λ} and $PI^{\lambda}D^{\mu}$ control design technique is proposed for a class of fractional order Time-Delay systems. The controller parameter's adjustment is achieved in two steps: first, the relay approach is used to compute satisfactory classical PID coefficients, namely k_p, T_i and T_d. Then, the fractional orders λ and μ are optimized using performance criteria.

This paper is organized as follows: In Section 2, the design of both integer order PI and PID controllers with relay feedback method is given. Charef's approximation method for fractional order systems is introduced in Section 3, whereas the proposed design methodology for fractional order PI and PID controllers is given in section 4. In section 5, simulation examples about integer and fractional time delay systems are presented. Concluding remarks are given in Section 6.

2. DESIGN OF INTEGER ORDER PI AND PID CONTROLLERS FOR TIME-DELAY SYSTEMSD

The transfer function of an integer order PI controller and the integer order system are given respectively by,

$$C(s) = k_p\left(1+\frac{1}{T_i s}\right) \tag{1}$$

$$G(s) = \frac{z_m s^m + z_{m-1}s^{m-1} + \ldots + z_0}{q_n s^n + q_{n-1}s^{n-1} + \ldots + q_0} \tag{2}$$

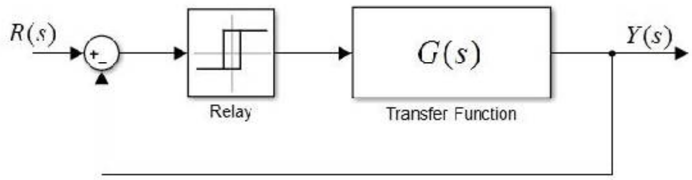
Where z_i, q_i are real parameters and m, n are integer positive numbers.
The open loop transfer function (L(s)) of the control system is given in (3),

$$L(s) = C(s)G(s) \tag{3}$$

The transfer function of integer order PID is given by

$$C(s) = k_p\left(1+\frac{1}{T_i s}+T_d s\right) \tag{4}$$

Figure 1. Relay feedback of integer plant.



The relay method is based on determining the critical gain K_u and the critical frequency w_u (Yüce et al. 2016) as illustrated in the Matlab Simulink diagram of Fig. 1. The relay input-output characteristic is given in Fig. 2 and the relay output is shown in Fig. 3.

Figure 2. Relay input-output characteristic.

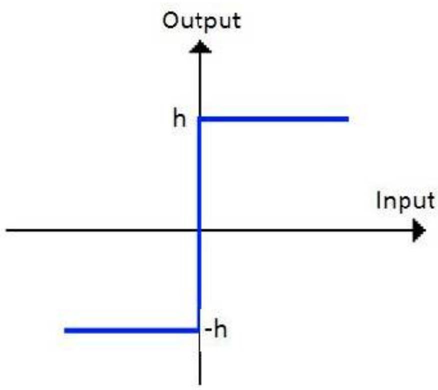


Figure 3. Relay output.

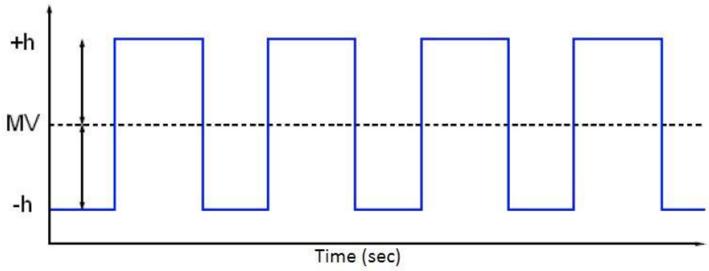
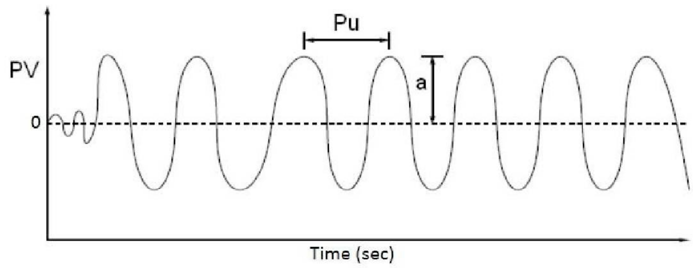


Figure 4. Output of closed loop control system with relay.



For a typical plant the oscillation obtained in the test occurring at the relay input is as shown in Figure 4. The critical frequency is taken as the measured frequency and the critical gain K_u is obtained by the formula,

$$K_u = \frac{4h}{\pi a} \tag{5}$$

Where h represents the amplitude of the relay actuation and a is the amplitude of the resulting oscillation.

From the values of K_u and the ultimate period P_u, the three parameters (k_p, T_i and T_d) for the P, PI, and the PID gains can be calculated from Table 1 using the closed-loop Z-N method (Yüce et al. 2016).

Table 1. PID Parameters Tuning using Ziegler-Nichols method.

	k_p	T_i	T_d
P	$K_u/2$		
PI	$K_u/2.2$	$P_u/1.2$	
PID	$K_u/1.7$	$P_u/2$	$P_u/8$

3. CHAREF'S APPROXIMATION METHOD FOR FRACTIONAL ORDER SYSTEMS

This method proposed by Charef et al. (1992), is based on the approximation of a function of the form:

$$H(s) = \frac{1}{\left(1+\frac{s}{p_T}\right)^{\alpha}} \tag{6}$$

Where $1/P_T$ is the relaxation time constant and $0 < \alpha < 1$.

$$H(s)=\frac{\prod_{i=0}^{n-1}\left(1+\frac{s}{z_i}\right)}{\prod_{i=0}^{n}\left(1+\frac{s}{p_i}\right)} \tag{7}$$

We define:

$$a=10^{\left(\frac{y}{10(\alpha-1)}\right)},b=10^{\left(\frac{y}{10\alpha}\right)},ab=10^{\left(\frac{y}{10\alpha(\alpha-1)}\right)} \tag{8}$$

Therefore, we can obtain the distribution of poles and zeros as:

$$p_0=p_T\sqrt{b},\ p_i=p_0\left(ab\right)^i,\ z_i=ap_0\left(ab\right)^i \tag{9}$$

A. Fractional order $PI^{\lambda}D^{\mu}$ controller realization

The transfer function of the fractional order $PI^{\lambda}D^{\mu}$ controller is given in the frequency domain by the following irrational function,

$$C(s)=K_p+\frac{T_I}{s^{\lambda}}+T_Ds^{\mu} \tag{10}$$

where K_p is the proportional constant, T_I is the integration constant, T_D is the differentiation constant and λ and μ are positive real numbers.

In general, these real numbers are such that $1<\lambda<2$ and $1<\mu<2$. Hence, (10) can be written as:

$$C(s)=K_p+\left[\frac{T_I}{s}\right]\left(\frac{1}{s^{m1}}\right)+\left[T_Ds\right]\left(s^{m0}\right) \tag{11}$$

Where (T_I/s) is a first-order integrator, $(1/s^m)$ is a fractional order integrator with $0<m_1<1$, (T_Ds) is a first-order differentiator and (s^{m0}) is a fractional-order differentiator with $0<m_D<1$.

In order to represent the fractional $PI^{\lambda}D^{\mu}$ controller of (10) by a linear time-invariant system model, it is necessary to approximate its irrational transfer function by a rational one. Hence, in a given frequency band of practical interest (w_L,w_H), the fractional-order integrator can be modeled by an FPP and the fractional-order differentiator by an FPZ.

$$\begin{aligned}C(s)=K_p&+\left[\frac{T_I}{s}\right]\left(K_I\frac{\prod_{i=0}^{N_I-1}\left(1+\left(s/z_{I_i}\right)\right)}{\prod_{i=0}^{N_I}\left(1+\left(s/p_{I_i}\right)\right)}\right)\\&+\left[T_Ds\right]\left(\left(K_D\frac{\prod_{i=0}^{N_D-1}\left(1+\left(s/z_{D_i}\right)\right)}{\prod_{i=0}^{N_D}\left(1+\left(s/p_{D_i}\right)\right)}\right)\right)\end{aligned} \tag{12}$$

It has also been shown how the FPP and the FPZ can be approximated by rational functions. Hence, (10) becomes (12).

4. FRACTIONAL ORDER PID CONTROL DESIN STRATEGY

Fractional order PI^{λ} and $PI^{\lambda}D^{\mu}$ parameters adjustment remains one of the major issues in fractional order control domain (Shaha and Agashe, 2016). This control design technique is proposed for a class of fractional order Time-Delay systems. The controller parameter's adjustment is achieved in two steps:

- First, the relay approach is used to compute satisfactory classical PID coefficients, namely k_p, k_i and k_d.
- Then, the fractional orders λ and μ are optimized using performance specifications like the rise time, the peak time, the settling time, and the overshoot and the following and quadratic error performance criteria,

$$J = \sqrt{\int_0^{t_f} e(t)^2 dt} \tag{13}$$

The cost function J is minimized relatively to the fractional integral and derivative orders in order to find the best controller tuning.

5. SIMULATION EXAMPLES

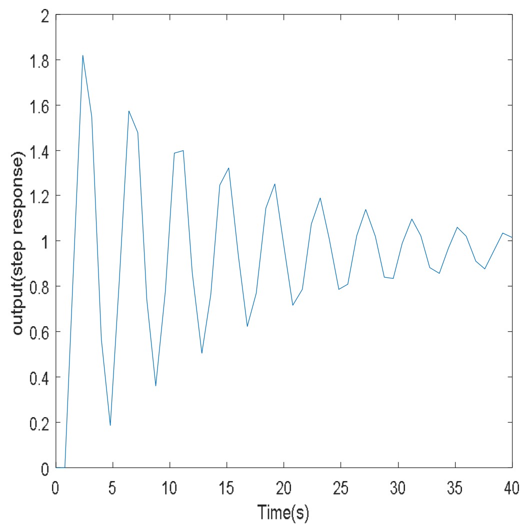
Example 1.

Consider the first order transfer function with time delay as fellows (Yüce et al. 2016),

$$G(s) = \frac{26e^{-s}}{16.2s+1} \tag{14}$$

The step response of the closed loop system with unity feedback is shown in Fig. 5. which is an oscillating response.

Figure 5. Step response of the closed loop system with unit feedback.



From the Simulink diagram shown in Figure1, critical values are calculated. And from Figure 6 we obtain the parameters shown in table 2.

Figure 6. Output of closed loop of control system with relay.

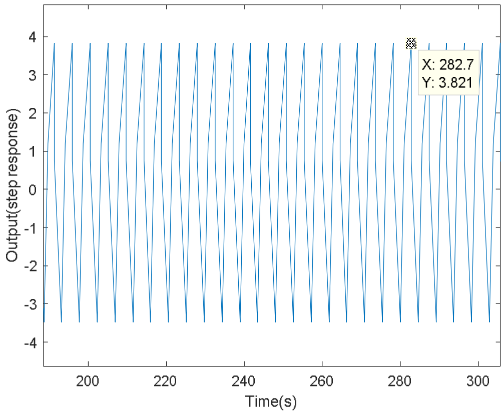


Table 2. Critical values using the Relay method

Amplitude (a)	5.881
Oscillating period (Pu)	8.9
Critical gain (Ku)	0.43

Design of PI controller

The design of PI controller using table 1, is given by:

$$k_p = \frac{K_u}{2.2} = \frac{0.43}{2.2} = 0.195 \tag{15}$$

$$T_i = \frac{P_u}{1.2} = \frac{8.9}{1.2} = 7.417 \tag{16}$$

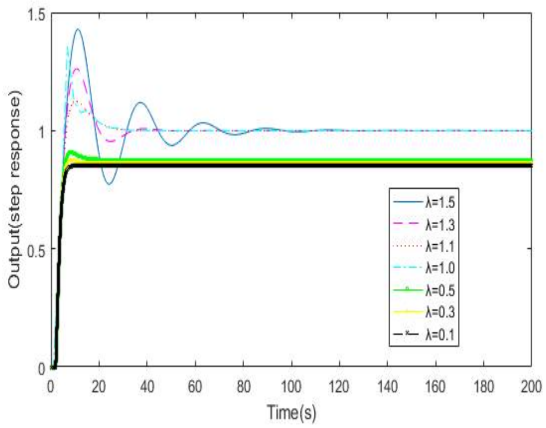
From these parameters, we obtain both, integer and fractional order PI controllers as follow:

$$C(s) = 0.195(1 + \frac{0.135}{s}) \tag{17}$$

$$C_F(s) = 0.195(1 + \frac{0.135}{s^{\lambda}}) \tag{18}$$

The step response of closed loop control system with both integer and fractional PI controllers are shown in Figure 7.

Figure 7. Step responses of closed loop control system with fractional PI controller for different values of λ.



Specification values for the step responses for different λ are given in table 3.

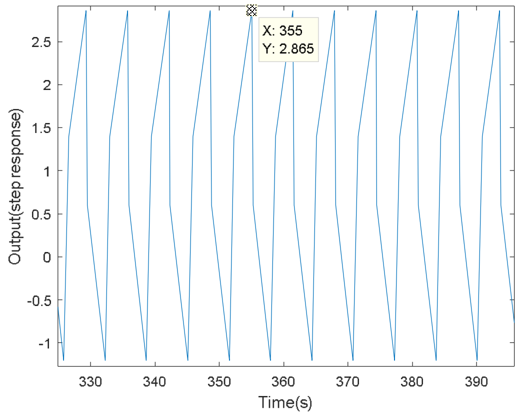
Table 3. Step Response Performance Values for Different Values of λ.

λ	t_r(s)	t_p(s)	t_s(s) 2%	t_s(s) 5%	M_p(%)	J
1.5	5.38	11.19	67.23	52.95	44.20	3.71
1.3	5.32	10.64	19.13	18.06	25.95	2.54
1.1	3.16	10.47	22.52	17.89	13.07	2.20
1.0	5.06	6.78	22.32	18.56	36.30	2.28
0.5	5.04	8.23	11.64	5.48	3.65	∞
0.3	5.07	8.32	6.10	5.60	1.53	∞
0.1	5.25	-	6.93	5.99	-	∞

From table 3 we can conclude that with decreasing the value of λ, values of rise time, peak time, settling time, overshoot and quadratic (square) error, decrease. For the interval [1-1.5] and from the previous results the best value of λ for the fractional order PI controller is equal to 1.1 for our system.
We can also see that the quadratic error tends to zero in steady state. On the other hand, we get infinitive values in the range of [0.1-0.5].

Design of PID controller

Figure 8. Relay output.



The design of the PID controller using table 1, and Figure 8 is given by:

$$k_p = \frac{K_u}{1.7} = \frac{0.43}{1.7} = 0.25 \tag{19}$$

$$T_i = \frac{P_u}{2} = \frac{8.9}{2} = 4.45 \tag{20}$$

$$T_d = \frac{P_u}{8} = \frac{P_u}{8} = 1.11 \tag{21}$$

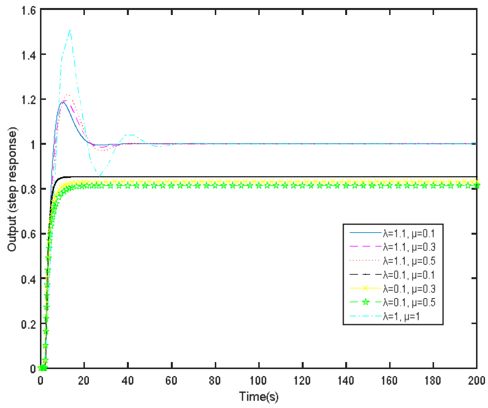
From these parameters, we obtain both, integer and fractional order PID controllers as follow:

$$C(s) = 0.25(1 + \frac{0.23}{s} + 1.11s) \tag{22}$$

$$C_F(s) = 0.25(1 + \frac{0.23}{s^{\lambda}} + 1.11s^{\mu}) \tag{23}$$

The step responses of closed loop control system with integer and fractional order PID controllers are shown in Figure 9.

Figure 9. Step responses of closed loop control system with fractional PID controllers for different values of λ and μ.



Specification values for the step responses for different values of λ and μ are given in table 4.

TABLE 4. Step Response Performance Values for Different Values of λ and μ

λ	**μ**	**t_r(s)**	**t_p(s)**	**t_s(s) 2%**	**t_s(s) 5%**	**M_p(%)**	**J**
1.1	1.5	6.14	12.06	32.20	20.33	22.84	2.45
1.1	1.3	5.74	11.59	21.42	19.44	19.88	2.32
1.1	1.1	5.29	9.97	20.00	17.59	18.45	2.24
1.0	1.0	6.74	13.47	45.72	32.80	50.76	3.73
0.1	0.5	6.84	-	11.44	8.81	-	∞
0.1	0.3	6.02	-	10.44	7.80	-	∞
0.1	0.1	5.17	-	7.218	6.03	-	∞

Table 4 shows that, when the value of λ is fixed at 1.1 in the range (1.1-1.5) for values of μ, values of rise time, peak time, settling time and overshoot decrease with decreasing the value of μ, with a finite quadratic error at steady state.
And the best value of μ which give us the best performances is equal to 1.1.
Otherwise, choosing values of λ and μ less than zero give an infinitive quadratic error in steady state which means an unstable system despite the good values of other performances.

Example 2.

Consider the fractional order system with time delay:

$$G_F(s) = \frac{26e^{-s}}{(16.2s+1)^{\beta}} \tag{24}$$

β is the fractional order of the first order system (24).
In our example $\beta = 0.7$.

Before calculating critical values of the control system, we start by an approximation of the fractional order system using charef's approximating method.
And from Figure 10 we can get parameters shown in table 5.

Figure 10 Output relay of the fractional system.

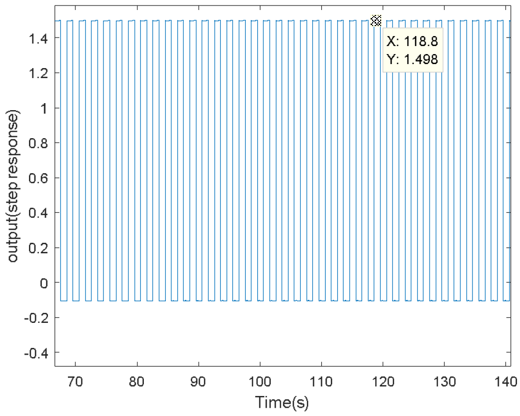


Table 5. Critical values of fractional system

Amplitude (a)	1.5
Oscillating period (Pu)	2
Critical gain (Ku)	0.17

Design of PI controller for fractional order system

The design of PI controller using table 1, is given by:

$$k_p = \frac{K_u}{2.2} = \frac{0.17}{2.2} = 0.08 \tag{25}$$

$$T_i = \frac{P_u}{1.2} = \frac{2}{1.2} = 1.67 \tag{26}$$

From these parameters, we can obtain both, integer and fractional order PI controllers as follow:

$$C(s) = 0.08(1 + \frac{0.6}{s}) \tag{27}$$

$$C_F(s) = 0.08(1 + \frac{0.6}{s^{\lambda}}) \tag{28}$$

The step responses of closed loop control system with fractional PI controller are shown in Figure 11.

Figure 11. Step responses of closed loop control system (fractional delay system with fractional PI controller) for different values of λ.

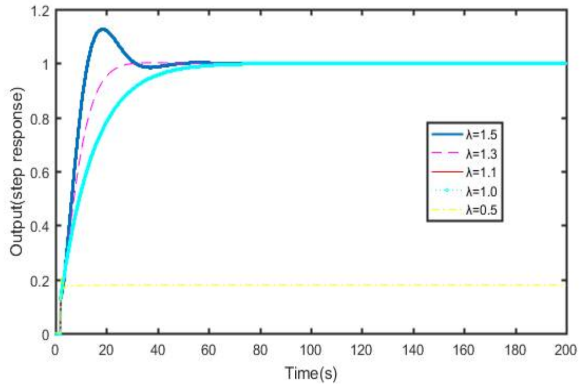


Table 4. Step Response Performance for Different Values of λ for Fractional Order System.

λ	**t_r(s)**	**t_p(s)**	**t_s(s)**		**M_p(%)**	**J**
			2%	**5%**		
1.5	10.9	18.48	28.75	26.07	13.06	3.8
1.3	16.57	-	23.48	19.80	-	4.4
1.1	29.47	-	48.86	38.35	-	5.9
1.0	30.23	-	50.05	40.53	-	6.2
0.5	8.00	-	16.13	11.43	-	∞

Table 6 shows that with decreasing the value of λ, both of rise time and quadratic error increase, otherwise peak time and overshoot decrease. However settling time is variant. The best response of the control system is given by tow values of λ : 1.1 and 1.0. the value of λ less than 1 give an unstable response.
Comparing with the integer system, fractional one is more slow but its overshoot decease with decreasing λ to be zero in the range 1.3-1, which present a good performance. And in spite of the complexity of fractional control systems, fractional PI controller could stabilize the system with good performances.

6. CONCLUSION

In this paper, a simple and efficient methodology for fractional order PID control design for a class of fractional order systems with delay is proposed.
The controller parameter's adjustment is achieved in two steps: first, the relay approach is used to compute satisfactory classical PID coefficients, namely k_p, T_i and T_d. Then, the fractional orders λ and μ are optimized using performance criteria. Simulation results show the efficiency of the proposed design technique and its ability to enhance the fractional order PID control performance.

Future research will focus on design of fractional order adaptive control laws for the class of fractional order systems with delays.

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