A Decision Support System and Visualisation Tools for AHP-GDM[1]

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ABSTRACT

The Precise Consistency Consensus Matrix (PCCM) is a decisional tool for AHP-Group Decision Making (AHP-GDM). Based on the initial pairwise comparison matrices of the individuals, the PCCM constructs a consensus matrix for the group using the concept of consistency. This paper presents a decision support system (PRIOR-PCCM) that facilitates the construction of the PCCM in the context of AHP-GDM, and the calculus of four indicators that allows comparison of the behaviour of group consensus matrices. PRIOR-PCCM incorporates the possibility of considering different weights for the decision makers and includes a module that permits the extension of the initial PCCM which can achieve the minimum number of non-null entries required for deriving priorities or establishing a complete PCCM matrix. It also includes two cardinal indicators for measuring consistency and compatibility and two ordinal indicators for evaluating the number of violations of consistency and priority. The paper introduces some new visualisation tools that improve comprehension of the process followed for obtaining the PCCM matrix and allow the cognitive exploitation of the results. These original contributions are illustrated with a case study.

Keywords: Decision Support System (DSS), Analytic Hierarchy Process (AHP), Group Decision Making (GDM), Consensus, Consistency, Compatibility, Visualisation tools.

[1]This work was financed by the Spanish Ministry of Economy and Competitiveness and FEDER funding (ECO2015-66673-R).

INTRODUCTION

Consensus is fundamental concept in decision making with multiple actors (Moreno-Jiménez et al., 2005, 2008, 2016; Choudbury et al., 2006; Yu and Lai, 2011), and this is especially true in Group Decision Making (GDM). In the scientific literature on group decision making, the term consensus is commonly employed to reflect the idea of agreement or compatibility between individual and collective preferences (Chiclana et al., 2008; Alonso et al., 2010; Dong et al., 2010; Wu and Xu, 2012; Zhang et al., 2017).

Of the different multicriteria approaches followed for decision making, the Analytic Hierarchy Process (AHP) (Saaty, 1980) is recognised as one that best captures the two fundamental issues inherent in the Knowledge Society (multiple actors and the integration of intangible aspects). AHP allows the application of most perspectives (determinist, stochastic, fuzzy etc.) used in the scientific literature with regards to the search for consensus (Saaty, 1980; Jensen, 1986; Ramanathan and Ganesh, 1994; Bryson, 1996; Herrera et al., 1996; Forman and Peniwati, 1998; Yeh et al., 2001; Moreno-Jiménez et al., 2005, 2008; Srdjevic, 2007; Saaty and Peniwati, 2008; Dong et al., 2010; Zhang et al., 2012,2014; Wu and Xu, 2012; Ren et al., 2016).

Another relevant feature of the Analytic Hierarchy Process is the possibility of evaluating the consistency of the pairwise comparison matrices (PCMs) used in order to capture the preferences of the Decision Makers (DMs). The idea of using the concept of consistency in group decision making was first proposed by the authors in 2002 (Moreno-Jiménez et al., 2002, 2005) and it has been extensively utilised in the scientific literature (Dong et al., 2010; Wu and Xu, 2012; Zhang et al., 2012, 2014).

Following this line of research, the authors proposed the Precise Consistency Consensus Matrix (PCCM) (Aguarón et al., 2016; Escobar et al., 2015), a decisional tool for AHP multi-actor decision making whose main aim is the construction of a consensus matrix based on consistency. Each entry of the new consensus matrix - known as 'the Consistency Stability Interval Judgement Matrix - belongs to all the Consistency Stability Intervals (CSIs) associated to each decision maker. This guarantees that the modifications made in the initial matrix do not exceed the maximum permitted level of inconsistency.

This current work presents PRIOR-PCCM, a Decision Support System (DSS) designed for constructing the PCCM and calculating behavioural indicators. The work also introduces some visualisation tools that help to understand the process followed in the application of the algorithm (construction of the consensus matrix) and aid the cognitive exploitation of the results (Moreno-Jiménez et al., 2014; Yepes et al., 2015; Moreno-Jiménez and Vargas, 2018). The DSS offers the scientific community a non-elementary calculation procedure for obtaining collective priorities in AHP-GDM and it provides indicators that allow the comparison of procedures for the construction of collective consensus matrices in AHP-GDM. PRIOR-PCCM facilitates interactive exploitation which reveals the critical points and decisional opportunities of the resolution process. Through the visual analysis of the results, a better understanding and dissemination of the extracted knowledge can be achieved.

The rest of the paper is structured as follows: Section 2 (Background) reviews consistency and AHP group decision making whilst explaining the basics of the decision-making tool (PCCM) and the algorithm followed for its construction; Section 3 includes a description of the DSS designed for the construction of the PCCM and presents the visualisation tools that can provide greater knowledge of the procedure; Section 4 details a case study which illustrates the use of the DSS and visualisation tools; and Section 5 summarises the main conclusions and briefly indicates possibilities for future research.

BACKGROUND

The Analytic Hierarchy Process (AHP)

AHP (Saaty, 1980, 1994) is one of the most popular multicriteria approaches, both from theoretical and practical points of view. AHP has been criticised, but it is extensively employed (Moreno-Jiménez and Vargas, 2018) because: (1) it is intuitive and realistic in scientific decision making; (2) using hierarchies and clustering it can integrate the large and the small; (3) it is capable of combining tangible and intangible aspects of problems by means of absolute pairwise comparisons that yield relative ratio scales of priorities; (4) it is flexible enough to consider dependencies between levels in a hierarchy with the extension of the AHP known as ANP (Analytic Network Process); (5) in group decision making it allows decision makers to construct group welfare functions that do not violate Arrow's conditions; and, (6) it offers great strength in negotiations and learning/cognition (discussion, extraction and dissemination of knowledge).

Another important characteristic of the AHP is that it explicitly evaluates the consistency of the judgements elicited by the decision makers in order to incorporate their preferences. Saaty (1980) defines the consistency of PCMs as the cardinal transitivity between the judgments. Given a pairwise comparison matrix $A = (a_{ij})$, $i,j = 1,\ldots,n$, A is said to be consistent if $\forall i,j,k \; a_{ij} \cdot a_{jk} = a_{ik}$. When the Eigenvector (EGV) method is used to obtain the local priorities, Saaty (1980) proposed the Consistency Ratio for measuring the inconsistency of the judgment elicitation process.

The PCCM uses the Row Geometric Mean (RGM) method to derive the local priorities and the Geometric Consistency Index (GCI) (Crawford and Williams, 1985; Aguarón and Moreno-Jiménez, 2003) to measure the inconsistency of a matrix. The index is defined as:

$$GCI = \frac{2}{(n-1)(n-2)} \sum_{1 \le i < j \le n} \log^2 e_{ij} \text{ with } e_{ij} = a_{ij} \cdot \frac{w_j}{w_i} \qquad (1)$$

where $w = (w_j)$, $j = 1,\ldots,n$, is the priority vector.

Aguarón and Moreno-Jiménez (2003) established the thresholds which, depending on the order of the matrix, allow an analogous interpretation of up to 10% for Saaty's Consistency Ratio; for the GCI, these values are:0.31 for $n = 3$; 0.35 for $n = 4$ and 0.37 for $n > 4$.

AHP-Group Decision Making

This section describes how the concept of consistency is used in the procedure (PCCM) followed for obtaining the collective PCM from individual PCMs (for the cognitive use of the PCCM, see Moreno-Jiménez et al 2014, Moreno-Jiménez and Vargas, 2018). This permits discussion and the extraction and dissemination of the knowledge derived from the scientific resolution of the problem.

The two classical approaches usually followed in AHP-GDM (Saaty, 1980; Ramanathan and Ganesh, 1994; Forman and Peniwati, 1998) are: (i) the *Aggregation of Individual Judgements* (AIJ), which constructs a PCM for the group from which the priority vector is calculated by following any of the existing prioritisation procedures; and (ii) the *Aggregation of Individual Priorities* (AIP), in which the group priorities are obtained by aggregating the individual priorities (the Weighted Geometric Mean is most commonly used as the aggregation procedure).

As previously mentioned, there is published research that uses the idea of consistency in AHP-GDM. Moreno-Jiménez et al. (2005, 2008) proposed a decisional tool, the Consistency Consensus Matrix (CCM), which identifies the core of consistency of the group decision using an interval matrix that may not be complete or connected. In 2014, the same authors refined this tool and introduced the PCCM (Aguarón et al., 2016), which selects a precise value for each interval judgement in such a way that the quantity of slack that remains free for successive algorithm iterations is the maximum possible.

Escobar et al., (2015) extended the PCCM to allow the assignment of different weights to the decision makers and to guarantee that the group consensus values were acceptable to the individuals in terms of inconsistency. In the same work, these authors put forward a number of methods for completing the PCCM matrix if it were incomplete. The improved version of the algorithm for constructing the PCCM can be seen in Escobar et al., (2015). A flowchart of this algorithm can be seen in Figure 1.

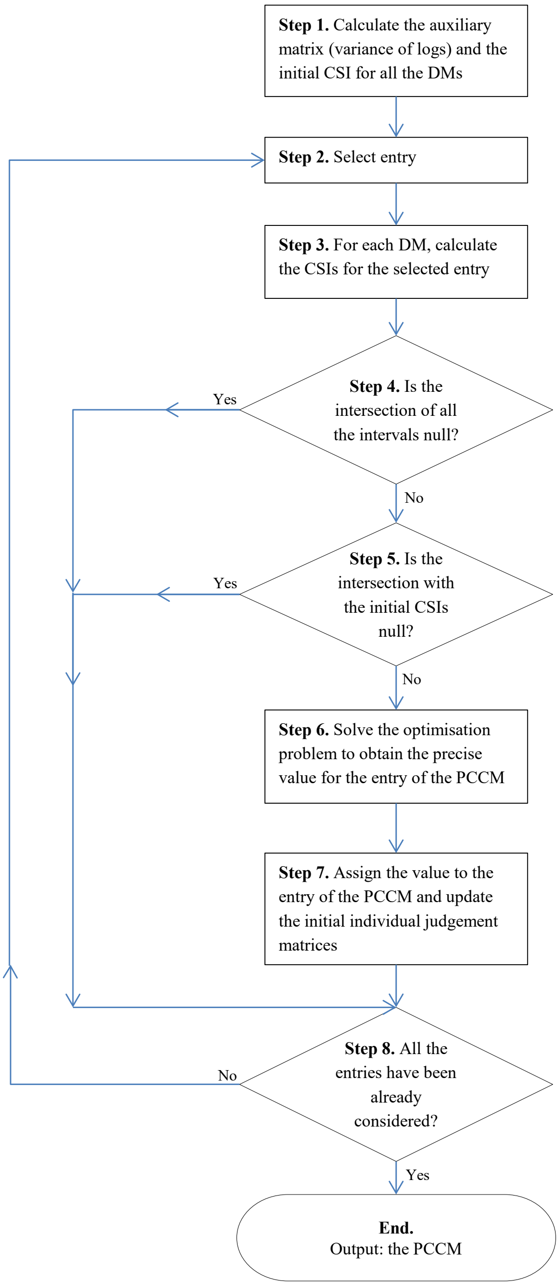


Figure 1: The PCCM algorithm

The Geometric Compatibility Index (GCOMPI) is used to evaluate the compatibility of the individual positions with respect to the collective position provided by the PCCM. The expression of the GCOMPI for a decision maker k is given by:

$$GCOMPI^{(k,G)} = \frac{2}{(n-1)(n-2)} \sum_{i<j} \log^2\left(a_{ij}^{(k)} w_j^{(G)} / w_i^{(G)}\right) \qquad (2)$$

The *GCOMPI* for the group is given by:

$$GCOMPI^{(G)} = \Sigma_k \alpha^{(k)} GCOMPI^{(k,G)} \qquad (3)$$

In addition to presenting a DSS for obtaining the collective PCM in an AHP-GDM context and four indicators for comparing the behaviour of group consensus matrices (Aguarón et al., 2016), this work also focuses on the development of visualisation tools that can be used in the discussion phase, in line with the cognitive perspective (Moreno-Jiménez and Vargas, 2018) proposed for the exploitation of the PCCM. The discussion phase is incorporated between two prioritisation rounds (Moreno-Jiménez et al., 2014), and it incorporates the arguments that support the different positions and decisions. The new visualisation tools facilitate better understanding of the process followed for obtaining the PCCM matrix.

A DSS AND VISUALISATION TOOLS

The Software

PRIOR-PCCM is a DSS developed for constructing the PCCM. The software was programmed in Delphi and completes other modules that were previously incorporated into the PRIOR software (Aguarón et al., 2010; Turón et al., 2010; Turón et al., 2013). PRIOR-PCCM includes several modules for: (i) the calculation of the CSIs; (ii) the resolution of the optimisation problem; (iii) the derivation of priorities in the case of incomplete matrices; and, (iv) the evaluation of consistency and compatibility. The architecture of PRIOR-PCCM is shown in Figure 2.

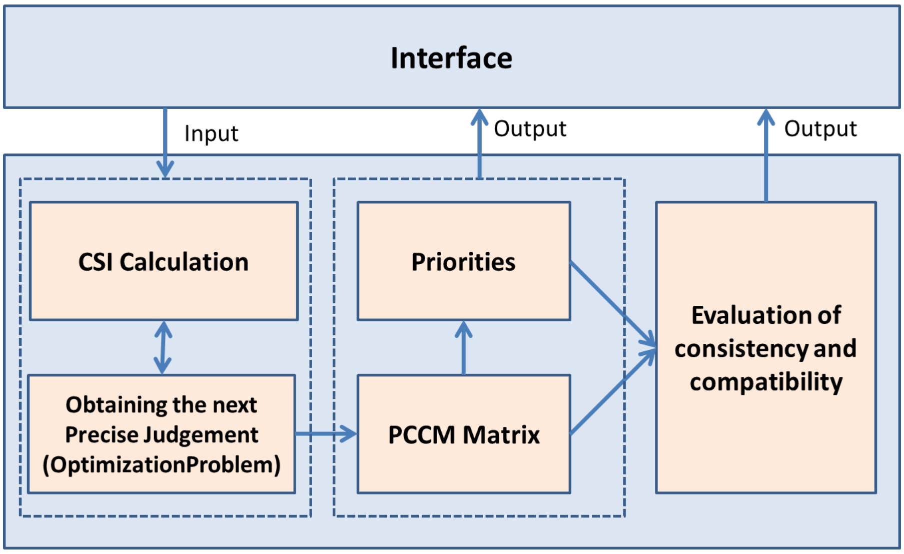


Figure 2: The architecture of the PRIOR-PCCM DSS

The main window of the DSS (see Figure 3) offers options to: introduce a new problem (*Parameters* and *Judgments*); read a previously resolved problem (*Load Data*); calculate the individual priorities (*Priorities*); calculate the Initial Stability Intervals Matrix (*Stability Intervals*); calculate the AIJ and PCCM matrices (*AIJ Matrix* and *PCCM*). It also provides the values of four indicators: two are cardinal (GCI and GCOMPI) and measure the consistency and compatibility of the consensus PCM, two are ordinal (CVN – Consistency Violation Number - and PVN – Priority Violation Number) and evaluate the number of violations in consistency and priority (Aguarón et al., 2016). In addition, the Consensus Matrix may be edited (*Edit Consensus Matrix*), an option that is particularly useful if the resulting PCCM is incomplete.

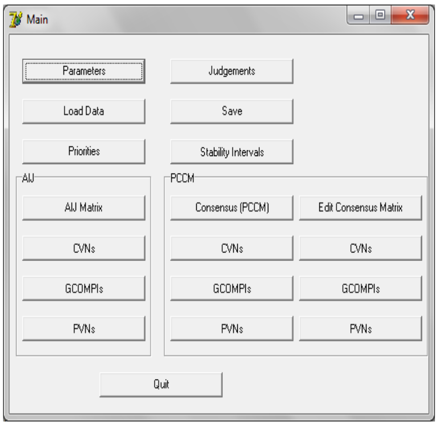


Figure 3: Main window of PRIOR-PCCM

When the PCCM matrix is obtained, the corresponding priority vector and GCI are also calculated (see Figure 4). A minimum of n-1 connected entries are needed to be able to obtain the corresponding priority vector, which can be achieved from this same window. If the resulting matrix (PCCM) is not complete, the DSS allows us to edit the consensus matrix and add new judgements (see Figure 5). For the different options, see Escobar et al., (2015). The user-defined matrices can be saved in order to retrieve them as and when required.

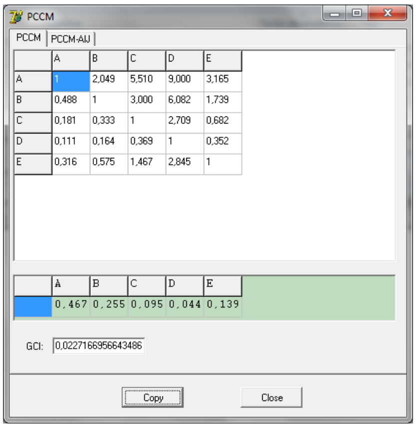


Figure 4: PCCM, priority vector and GCI

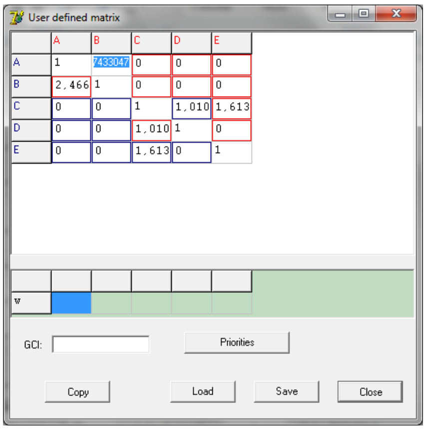
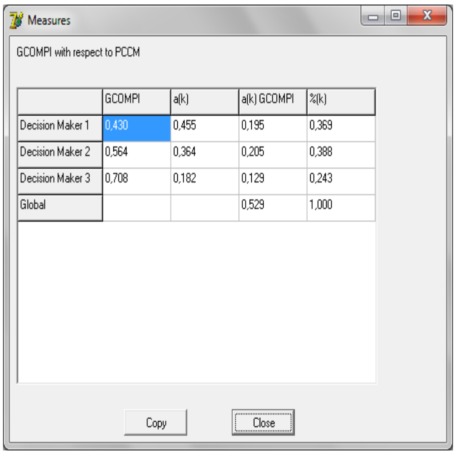


Figure 5: PCCM edit when incomplete

Once the PCCM has been obtained, the DSS calculates other indicators (CVN, GCOMPI and PVN) that measure the compatibility of the individual positions with the group position. It also compares the two AHP-GDM procedures currently implemented in the software: the AIJ and the PCCM (see Figure 6). All the windows that show results can be copied for use outside the DSS (Excel, etc.).



	GCOMPI	a(k)	a(k) GCOMPI	%(k)
Decision Maker 1	0,430	0,455	0,195	0,369
Decision Maker 2	0,564	0,364	0,205	0,388
Decision Maker 3	0,708	0,182	0,129	0,243
Global			0,529	1,000

Figure 6: Results for the GCOMPI indicator

Visualisation Tools

A library of visualisation tools has been included in PRIOR-PCCM to help the user interpret and understand the results and visualise the evolution of the indicators calculated by the DSS. These tools have been codified as R functions, so that they can make use of existing interactive three-dimensional visualisation packages (Adler and Murdoch, 2017). After running the analysis, the visualisations can be applied to facilitate the exploitation of the results from a cognitive orientation.

PRIOR-PCCM offers a number of multidimensional visualisations:

- *Evolutionary MDS diagrams*: based on multidimensional scaling, these show the process followed for obtaining the PCCM. The tool visualises the relative positions of the decision makers' priorities throughout the iterations of the algorithm utilised for the construction of the consensus matrix (Figures 7 and 8).
- *Evolutionary CGI diagrams*: a line chart illustrates the evolution of the CGI values throughout the iterative process (Figure 9).
- *GCOMPI weights*: depict the contribution of each DM to the value of the GCOMPI (Figure 10).

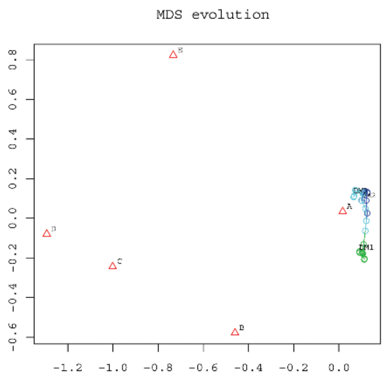


Figure 7: Evolution of the relative positions of the DMs and the alternatives

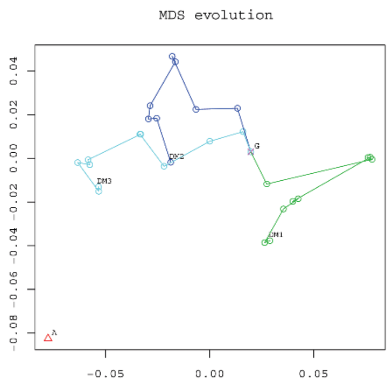


Figure 8: Zoom on the evolution of the relative positions of the DMs

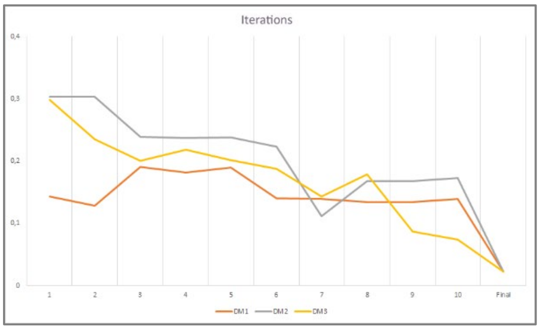


Figure 9: Evolution of the GCI

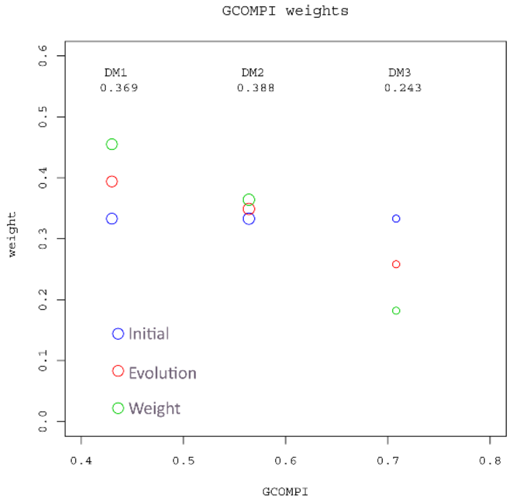


Figure 10: Contribution of the DMs to the GCOMPI

CASE STUDY

The DSS has been applied to the case study used by Aguarón et al., (2016) and Escobar et al., (2015). It concerns a real-life, public investment project for the restoration of the historical and cultural heritage of the village of Monreal del Campo (Spain). The project was known as 'HistoPark'. The investment (including the technical plans, technical work and the acquisition and urbanisation of approximately 100,000m^2 of land for the development of a tourist complex) was to be around 24 million Euros. It was hoped that the park would receive some 130,000 visitors per year: 65,000 direct tourists and another 65,000 excursionists or passers-by.

Moreno-Jiménez et al. (2009) proposed a methodology for the evaluation of the viability of public investment projects based on use values (direct, indirect and potential) and non-use values (existence and bequest) for the assessment of social and environmental aspects (usually considered as intangible) in economic terms. In the case of the HistoPark project, environmental aspects were not considered relevant so viability was exclusively focused on the economic and social aspects, using the following five values (for more details, see Moreno-Jiménez et al. 2009): Direct Use Value (DUV); Indirect Use Value (IUV); Potential Use Value (PUV); Existence Value (EV) and Bequest Value (BV).

The five values were assessed by means of a questionnaire which was sent to the spokespeople of the three political parties represented on the municipal council: the *Partido Socialista Obrero Español* (PSOE); the *Partido Popular* (PP) and the *Partido Aragonés* (PAR). Table 1 shows the initial judgement matrices. Different weights were associated to the decision makers in proportion to the number of councillors on the city council (PSOE: 5; PP: 4; and PAR: 2).

Table 1: Pairwise comparison matrices for the three decision makers

DM1	DUV	IUV	PUV	EV	BV
DUV	1	3	5	8	6
IUV		1	3	5	4
PUV			1	3	2
EV				1	1/3
BV					1

DM2	DUV	IUV	PUV	EV	BV
DUV	1	3	7	9	5
IUV		1	3	7	1
PUV			1	5	1/5
EV				1	1/5
BV					1

DM3	DUV	IUV	PUV	EV	BV
DUV	1	5	7	7	5
IUV		1	1	5	1
PUV			1	5	1/3
EV				1	1/5
BV					1

Table 2 gives the resulting priorities using the RGMM for each of the three individual matrices and their corresponding rankings. It can be seen that the three political parties gave the most relative importance to the DUV and the least relative importance to the EV.

Table 2. Individual priority vectors and rankings

Priorities	**PSOE**	**PP**	**PAR**
DUV	0.513	0.520	0.560
IUV	0.251	0.195	0.135
PUV	0.115	0.072	0.101
EV	0.042	0.030	0.035
BV	0.079	0.182	0.168
Rankings	1-2-3-5-4	1-2-5-3-4	1-5-2-3-4

The application of the PCCM algorithm implemented in the software to these data is as follows:

Step 1: some initial calculations are made. First, the auxiliary matrix that contains the variance of the logarithms of the judgements is computed (Table 3). In order to calculate these values, the normalised weights are needed:

$$\alpha^{(1)} = \frac{5}{11}, \quad \alpha^{(2)} = \frac{4}{11}, \quad \alpha^{(3)} = \frac{2}{11}.$$

These values are used (Table 4) to determine, from the minimum to the maximum, the order in which the judgements will be considered in the algorithm (one judgement per iteration).

Table 3. Variance-Covariance matrix

Σ	DUV	IUV	PUV	EV	BV
DUV	0	0.038817	0.028069	0.007942	0.008241
IUV		0	0.179546	0.026198	0.476482
PUV			0	0.064696	1.158921
EV				0	0.064696
BV					0

Table 4. Entrance order

Order	DUV	IUV	PUV	EV	BV
DUV		5	4	1	2
IUV			8	3	9
PUV				6	10
EV					7
BV					

Next, the CSIs for each decision maker are calculated. From these, the intersection of the CSIs for each judgement is obtained. The resulting matrix is known as the Group Consistency Stability Interval (GCSI) matrix (Table 5). The values of the PCCM will never exceed these intervals.

Finally, the value of t is initialised: $t = 0$.

Table 5. GCSI matrix

	DUV	IUV	PUV	EV	BV
DUV		[1.481, 5.715]	[3.261, 12.028]	[6.834, 83.558]	[1.508, 6.796]
IUV			[1.102, 4.274]	[2.654, 8.448]	[0.573, 1.747]
PUV				[0.587, 6.385]	[0.243, 2.502]
EV					[0.134, 0.352]
BV					

Iteration 1.

Step 2, the first entry is selected. This represents (1.4) the judgements that have the smallest variance of the logarithms (see Tables 3 and 4).

Table 6 shows the values that should be considered for the calculations required in steps 3, 4, 5 and 6.

Table 6. Steps 3 to 6 of the algorithm for $a_{14}^{(k)}$

	$a_{14}^{(k)}$	$GCI^{0,(k)}$	$\Delta^{0(k)}$	$[\underline{a}_{14}^{0,(k)}, \overline{a}_{14}^{0,(k)}]$	$e_{14}^{(k)}$	$\varepsilon_{14} = \log e_{14}^{(k)}$	$\alpha_{14} = \log a_{14}^{(k)}$
PSOE	8	0.143	0.227	[3.041, 83.558]	0.661	-0.414	2.079
PP	9	0.303	0.067	[6.834, 101.888]	0.524	-0.645	2.197
PAR	7	0.298	0.072	[5.510, 141.952]	0.436	-0.831	1.946

The intersection of the stability intervals corresponding to this iteration (*Step 4*) is: $\cap_k[\underline{a}_{14}^{0,(k)}, \overline{a}_{14}^{0,(k)}]$ = [6.834, 83.558]. The intersection of the initial stability intervals is: $\cap_k[\underline{a}_{14}^{0,(k)}, \overline{a}_{14}^{0,(k)}]$= [6.834, 83.558]. In this case, it is the same as the previous one, as it is the first iteration. Finally, the intersection of the previous interval with Saaty's range for the judgements [1/9, 9] is (*Step 5*): $[\underline{a}_{rs}^{0}, \overline{a}_{rs}^{0}]$ = [6.834, 83.558] $\cap$ [1/9, 9] = [6.834, 9]. This is the interval in which the optimisation problem will seek the solution in the following step. The optimisation problem (*Step 6*) for this iteration is:

$$Min_{\alpha_{rs}} Max_k GCI^{(k)} + \frac{2}{n(n-1)}\left[\left(\alpha_{rs} - \alpha_{rs}^{(k)}\right)^2 + \frac{2n}{n-2}\left(\alpha_{rs} - \alpha_{rs}^{(k)}\right)\varepsilon_{rs}^{(k)}\right] =$$

$$Min_{\alpha_{rs}} Max \left\{\frac{5}{11}\left(0.143 + \frac{2}{20}\left[(\alpha_{rs} - 2.079)^2 + \frac{10}{3}(\alpha_{rs} - 2.079)(-0.414)\right]\right), \frac{4}{11}\left(0.303 + \frac{2}{20}\left[(\alpha_{rs} - 2.197)^2 + \frac{10}{3}(\alpha_{rs} - 2.197)(-0.645)\right]\right), \frac{2}{11}\left(0.298 + \frac{2}{20}\left[(\alpha_{rs} - 1.946)^2 + \frac{10}{3}(\alpha_{rs} - 1.946)(-0.831)\right]\right)\right\}$$

with $\alpha_{rs} \in [\log 6.834, \log 9]$

The solution of the optimisation problem is: $\alpha_{14} = \log 9$, therefore $a_{14} = 9$.

*Step 7*obtains the matrices for each decision maker $A^{1,(k)}$ (Table 7).

Table 7. Updated matrices after iteration 1

	DM 1 (PSOE)					DM 2 (PP)					DM 3 (PAR)				
	DUV	IUV	PUV	EV	BV	DUV	IUV	PUV	EV	BV	DUV	IUV	PUV	EV	BV
DUV	1	3	5	9	6	1	3	7	9	5	1	5	7	9	5
IUV	1/3	1	3	5	4	1/3	1	3	7	1	1/5	1	1	5	1
PUV	1/5	1/3	1	3	2	1/7	1/3	1	5	1/5	1/7	1	1	5	1/3
EV	1/9	1/5	1/3	1	1/3	1/9	1/7	1/5	1	1/5	1/9	1/5	1/5	1	1/5
BV	1/6	1/4	1/2	3	1	1/5	1	5	5	1	1/5	1	3	5	1

Step 8 checks that there are entries which have not already been considered, then, the value of t is increased (t = 1) and we return to Step 2 (iteration 2).

Iteration 2.

Step 2: the following judgement is selected, it corresponds to entry (1,5).

Step 3: the Consistency Stability Intervals for the judgement (1,5) are calculated, they are given in Table 8, along with the other values needed in steps 3, 4, 5 and 6.

Table 8. Steps 3 to 6 of the algorithm for$a_{15}^{(k)}$

	$a_{15}^{(k)}$	$GCI^{1,(k)}$	$\Delta^{1,(k)}$	$[\underline{a}_{15}^{1,(k)}, \overline{a}_{15}^{1,(k)}]$	$e_{15}^{(k)}$	$\varepsilon_{15} = \log e_{15}^{(k)}$	$\alpha_{15} = \log a_{15}^{(k)}$
PSOE	6	0.128	0.242	[1.489, 33.999]	0.903	-0.102	1.792
PP	5	0.303	0.067	[0.567, 6.796]	1.753	0.561	1.609
PAR	5	0.235	0.135	[0.747, 10.189]	1.429	0.357	1.609

Step 4 obtains the intersection of the stability intervals corresponding to this iteration: $\cap_k[\underline{a}_{rs}^{t,(k)}, \overline{a}_{rs}^{t,(k)}]$ = [1.489, 6.796].

Step 5 obtains the intersection of the previous interval with the intersection of the initial stability intervals for this same judgement, $\cap_k[\underline{a}_{rs}^{0,(k)}, \overline{a}_{rs}^{0,(k)}]$ = [1.508, 6.796] and Saaty's range for the judgements [1/9, 9]: $[\underline{a}_{rs}^{1}, \overline{a}_{rs}^{1}]$ = [1.489, 6.796] $\cap$ [1.508, 6.796] $\cap$ [1/9, 9] = [1.508, 6.796].

The optimisation problem solved in *Step 6* is the following:

$$Min_{\alpha_{rs}} Max \ \left\{ \frac{5}{11}\left(0.128 + \frac{2}{20}\left[(\alpha_{rs} - 1.792)^2 + \frac{10}{3}(\alpha_{rs} - 1.792)(-0.102)\right]\right), \frac{4}{11}\left(0.303 + \frac{2}{20}\left[(\alpha_{rs} - 1.609)^2 + \frac{10}{3}(\alpha_{rs} - 1.609)(0.561)\right]\right), \frac{2}{11}\left(0.235 + \frac{2}{20}\left[(\alpha_{rs} - 1.609)^2 + \frac{10}{3}(\alpha_{rs} - 1.609)(0.357)\right]\right)\right\}$$

with$\alpha_{rs} \in [\log 1.508, \log 6.796]$

The solution of this problem is $\alpha_{15} = 1.1522$ so the value of entry (1,5) in the PCCM will be $a_{15} = 3.165$.

Step 7 obtains the matrices $A^{2,(k)}$ (Table 9):

Table 9. Updated matrices for t=2

	DM 1 (PSOE)					DM 2 (PP)					DM 3 (PAR)				
	DUV	IUV	PUV	EV	BV	DUV	IUV	PUV	EV	BV	DUV	IUV	PUV	EV	BV
DUV	1	3	5	9	3.16	1	3	7	9	3.16	1	5	7	9	3.16
IUV	1/3	1	3	5	4	1/3	1	3	7	1	1/5	1	1	5	1
PUV	1/5	1/3	1	3	2	1/7	1/3	1	5	1/5	1/7	1	1	5	1/3
EV	1/9	1/5	1/3	1	1/3	1/9	1/7	1/5	1	1/5	1/9	1/5	1/5	1	1/5
BV	1/3.16	1/4	1/2	3	1	1/3.16	1	5	5	1	1/3.16	1	3	5	1

The last step of this iteration (*Step 8*) again checks that there are entries that have not been considered. The value of t is then increased (t = 2) and we return to *Step 2* to begin Iteration 3. As the first two iterations have been explained in detail, for the following iterations the most important values that appear in each one are summarised. The values are shown in Table 10.

The result of all the processes followed in the application of the algorithm is that the final PCCM is obtained by using the DSS. Figure 4 shows the corresponding priority vector and the value of the GCI (0.023). It can be observed, in this particular case, that the PCCM is complete. Figure 7 shows the evolution of the relative positions of the DMs and the alternatives throughout the iterations of the algorithm. The DMs have quite similar opinions and there are no big modifications in their relative positions. Figure 8 shows a zoom of this graph with the evolution of DMs in more detail. Figure 9 illustrates the evolution of the GCI for the three DMs throughout the iterations. It is clear that whilst the values of the GCI for some intermediate iterations have increased, the final GCI is notably better than the initial individual GCIs.

Figure 6 shows the values of the GCOMPI, calculated by the DSS, for the example. The values of this indicator for each of the three decision makers are given in the first column. The second column contains the weights associated to each decision maker. The third column calculates the contribution of each decision maker to the value of the GCOMPI for the group ($GCOMPI^{(G)}$ = 0.529) and the last column contains this contribution in relative terms. Figure 10 shows a graphic presentation of all these values; although decision-maker 3 (PAR) has the greatest GCOMPI value, the contribution of this decision-maker to the group GCOMPI is the lowest, as its weight is the smallest. The second decision-maker (PP) made the greatest contribution to the total group GCOMPI.

Discussion, Improvements and Limits

The development of the PRIOR-PCCM DSS fosters the non-trivial calculus of the group consensus matrix known as the PCCM and the associated group priorities, both in the case of complete and incomplete consensus matrices and with the same or different weights for the actors involved in the resolution process.

The procedure to obtain the PCCM requires the resolution (for all the iterations of the algorithm) of an optimisation problem to select the judgement that provides the greatest slack for the most inconsistent matrix. The consideration of different weights for the decision makers has notably increased the difficulty of this optimisation model.

The DSS tool also provides the values of four indicators employed for measuring the performance of any consensus matrix: two are cardinal (GCI and GCOMPI) and measure the consistency and compatibility of the consensus PCM; two are ordinal (CVN and PVN) and evaluate the number of violations in consistency and priority (Aguarón et al., 2016). Obtaining the CVN indicator is not easy and, as far as we know, this is the first time that a DSS tool has been developed that is able do this. The use of these indicators and the possibility for editing the consensus matrix mean that the behaviour of group matrices obtained by any existing AHP-GDM procedure can be compared.

PRIOR-PCCM DSS includes some visualisation tools that reflect the evolution of the GCI and the GCOMPI, and the relative position of DMs and alternatives, through the iterative solution of the problem. From a cognitive perspective, this information, in conjunction with the discussion stage, can be used to identify the critical points and decisional opportunities of the resolution process.

Although PRIOR-PCCM DSS allows the evaluation of the four indicators for any group consensus matrix, at present, it only implements the calculation of the consensus matrix obtained by the AIJ and PCCM procedures. In the immediate future, other methods such as that proposed by Dong (Dong et al., 2010) will be incorporated, along with other visualisation tools associated with the PVN and CVN.

CONCLUSIONS

This paper presents a DSS (PRIOR-PCCM) that implements the PCCM: a decisional tool for AHP-GDM that, based on the concept of consistency, provides a consensus matrix for the group. It searches for a consensus between the different decision-makers in which the modifications of the individual judgements should preserve, as much as possible, the initial positions. In this case, the new judgements are inside the range of values accepted for a given inconsistency level.

PRIOR-PCCM also calculates four indicators (GCI, GCOMPI, PVN and CVN) that allow the comparison of the behaviour of the group consensus matrices obtained by means of different procedures in the AHP-GDM context.

The paper introduces new visualisation tools that facilitate the exploitation of the model from a cognitive orientation. They improve the presentation of the intermediate and final results and provide a deeper knowledge of the process followed for obtaining the PCCM. These new tools and the implementation of other AHP-GDM consensus methods are currently being integrated into the DSS software.

Future lines of research include the comparison of the behaviour of the PCCM and other consensus methods used in AHP-GDM and the extension of the analysis to other prioritisation procedures.

ACKNOWLEDGMENTS

This paper is an extension of the work presented at the International Conference on Decision Support Systems Technology (ICDSST) 2017 (Namur, May 2017). The research was financed by the Spanish Ministry of Economy and Competitiveness and FEDER funding (Project ECO2015-66673-R). The authors would like to thank English translation professional David Jones for his collaboration in the preparation of the manuscript.

Table 10. Summary of the iterations of the algorithm used to obtain the PCCM

# Iterat.	Selec. Judg.	Original Judg.			Initial GCI			Consistency Stability Intervals			Intersect. of the initial CSIs	Range Judg.	Common Interval	Precise Value
		PSOE	PP	PAR	PSOE	PP	PAR	I_{PSOE}	I_{PP}	I_{PAR}				
1	(1,4)	8	9	7	0.143	0.303	0.298	[3.041, 83.558]	[6.834, 101.888]	[5.510, 141.952]	[6.834, 83.558]	[1/9, 9]	[6.834, 9]	9
2	(1,5)	6	5	5	0.128	0.303	0.235	[1.489, 33.999]	[0.567, 6.796]	[0.747, 10.189]	[1.508, 6.796]	[1/9, 9]	[1.508, 6.796]	3.165
3	(2,4)	5	7	5	0.191	0.239	0.201	[1.736, 27.178]	[1.916, 19.303]	[0.929, 13.626]	[2.654, 8.448]	[1/9, 9]	[2.654, 8.448]	6.082
4	(1,3)	5	7	7	0.182	0.237	0.218	[0.848, 14.407]	[1.994, 20.244]	[1.182, 16.416]	[3.261, 12.028]	[1/9, 9]	[3.261, 9]	5.510
5	(1,2)	3	3	5	0.19	0.238	0.202	[0.265, 6.293]	[0.610, 6.883]	[0.730, 11.972]	[1.481, 5.715]	[1/9, 9]	[1.481, 5.715)	2.049
6	(3,4)	3	5	5	0.14	0.223	0.188	[0.593, 12.368]	[0.269, 8.263]	[0.564, 11.526]	[0.587, 6.386]	[1/9, 9]	[0.593, 6.386]	2.709
7	(4,5)	1/3	1/5	1/5	0.139	0.112	0.143	[0.112, 2.753]	[0.032, 0.819]	[0.043, 0.869]	[0.134, 0.352]	[1/9, 9]	[0.134, 0.352]	0.352
8	(2,3)	3	3	1	0.134	0.168	0.179	[0.483, 10.905]	[0.751, 12.916]	[0.486, 14.181]	[1.102, 4.274]	[1/9, 9]	[1.102, 4.274]	3
9	(2,5)	4	1	1	0.134	0.168	0.087	[0.555, 13.197]	[0.298, 5.303]	[0.265, 8.380]	[0.573, 1.747]	[1/9, 9]	[0.573, 1.747]	1.739
10	(3,5)	2	1/5	1/3	0.139	0.173	0.074	[0.106, 4.396]	[0.106, 4.396]	[0.106, 4.396]	[0.243, 2.502]	[1/9, 9]	[0.243, 2.502]	0.682

REFERENCES

Adler, D., & Murdoch, D. (2017): 3D Visualization Using OpenGL. Web page: [https://cran.r-project.org/web/packages/rgl/rgl.pdf].

Aguarón, J., & Moreno-Jiménez, J.M. (2003). The geometric consistency index: approximate thresholds. *European Journal of Operational Research,* 147(1), 137-145.

Aguarón, J., Escobar, M.T., & Moreno-Jiménez, J.M. (2016): The precise consistency consensus matrix in a local AHP-group decision making context. *Annals of Operations Research*, 245 (1-2), 245-259.

Aguarón, J., Escobar, M.T., Moreno-Jiménez, J.M., & Turón, A. (2010).PRIOR-WEB: A discrete multicriteria prioritization tool for the global economy context. In A. Respicio et al. (eds.), *Bridging the Socio-technical Gap in Decision Support Systems* (pp. 249-258). Amsterdam, Netherland: IOS Press.

Alonso, S., Herrera-Viedma, E., Chiclana, F., & Herrera, F. (2010).A web based consensus support system for group decision making problems and incomplete preferences. *Information Sciences*, 180, 4477-4495.

Bryson, N. (1996). Group Decision Making and the analytic hierarchy process: exploring the consensus-relevant information content. *Computers and Operations Research*, 23, 27-35.

Chiclana, F., Mata, F., Martínez, L., Herrera-Viedma, E., & Alonso, S. (2008). Integration of a consistency control module within a consensus decision making model. *International Journal of Uncertainty, Fuzziness and Knowledge Based Systems*, 16(1), 35-53.

Choudhury, A.K., R. Shankar, M.K. & Tiwari (2006): Consensus-based intelligent group decision-making model for the selection of advanced technology. *Decision Support Systems*, 42, 1776-1799.

Crawford G., & Williams C. (1985). A Note on the Analysis of Subjective Judgement Matrices. *Journal of Mathematical Psychology*, 29, 387-405.

Dong, Y., Zhang, G., Hong, W.H, & Xu, Y. (2010). Consensus models for AHP group decision making under row geometric mean prioritization method. *Decision Support Systems,* 49, 281-289.

Escobar, M.T., Aguarón, J., & Moreno-Jiménez, J.M. (2015): Some Extensions of the Precise Consistency Consensus Matrix. *Decision Support Systems,* 74, 67-77.

Forman, E., & Peniwati, K. (1998). Aggregating individual judgements and priorities with the Analytic Hierarchy Process. *European Journal of Operational Research*, 108, 165-169.

Herrera, F., Herrera-Viedma, E., &Verdegay, J.L. (1996).A model of consensus in group decision making under linguistic assessments. *Fuzzy Sets and Systems*, 78, 73-87.

Jensen, R.E. (1986). Comparison of consensus methods for priority ranking problems. *Decision Sciences*, 17, 195-211.

Moreno-Jiménez, J.M., Aguarón, J., & Escobar, M.T. (2002). *Decisional Tools for Consensus Building in AHP-Group Decision Making*. Paper presented at the *12th Mini Euro Conference*. Brussels, Belgium.

Moreno-Jiménez, J.M., Aguarón, J., & Escobar, M.T. (2008). The Core of Consistency in AHP-Group Decision Making. *Group Decision and Negotiation,* 17(3), 249-265.

Moreno-Jiménez, J.M., Aguarón, J., Raluy, A., & Turón, A. (2005).A Spreadsheet Module for consistent consensus building in AHP-Group Decision Making. *Group Decision and Negotiation,* 14(2), 89-108.

Moreno-Jiménez, J.M., Gómez-Bahillo, C., &Sanaú, J. (2009).Viabilidad Integral de Proyectos de Inversión Pública. Valoración económica de los aspectos sociales. In J. R. Pires & J.D. Monteiro (Eds.), *Anales de Economía Aplicada XXIII*, 2551-2562. Madrid, Spain: Delta.

Moreno-Jiménez, J.M., Pérez Espés, C., & Velázquez, M. (2014). E-Cognocracy and the Design of Public Policies. *Government Information Quarterly,* 31, 185-194.

Moreno-Jiménez, J.M., Salvador, M., Gargallo, P., & Altuzarra, A. (2016).Systemic Decision making in AHP: A Bayesian approach. *Annals of Operations Research*, 245(1-2), 261-284.

Moreno-Jiménez, J.M., & Vargas, L.G. (2018). Cognitive Multiple Criteria Decision Making and the Legacy of the Analytic Hierarchy Process. Forthcoming in *Estudios de Economía Aplicada,* 36(1), xx-xx.

Ramanathan, R., & Ganesh, L.S. (1994). Group Preference Aggregation Methods employed in AHP: An Evaluation and an Intrinsic Process for Deriving Members' Weightages. *European Journal of Operational Research*, 79, 249-265.

Ren, P., Xu, Z., & Liao, H. (2016), Intuitionistic multiplicative analytic hierarchy process in group decision making. *Computers & Industrial Engineering,* 101, 513-524.

Saaty, T.L. (1980). *Multicriteria Decision Making: The Analytic Hierarchy Process*. New York: Mc Graw-Hill.

Saaty, T.L. (1994). *Fundamentals of Decision Making and Priority Theory with the Analytic Hierarchy Process*. Pittsburg, PA: RWS Publications.

Saaty, T.L., & Peniwati, K. (2008). *Group Decision Making: Drawing Out and Reconciling Differences*. Pittsburgh, PA: RWS Publications.

Srdjevic, B. (2007): Linking analytic hierarchy process and social choice methods to support group decision-making in water management. *Decision Support Systems*, 42, 2261-2273.

Turón, A., Aguarón, J., Cardeñosa, J., Escobar, M.T., Moreno-Jiménez, J.M., Ruiz-Mas, J., & Toncovich, A. (2013). PRIOR-W&K: A Collaborative Tool for Decision Making in the Knowledge Society. In M. D. Litras et al. (eds.), Information Systems, E-learning, and Knowledge Management Research. *Communications in Computer and Information Sciences* (CCIS) 278, 155-161. Berlin, Germany: Springer Heildelberg.

Turón, A., Aguarón, J., Escobar, M.T., Gallardo, C., Moreno-Jiménez, J.M., & Salazar, J.L. (2010): PRIOR-WK&E: Social Software for Policy Making in the Knowledge Society. In M.D. Lytras et al. (Eds.) *Communications in Computer and Information Sciences* (CCIS) 111, 139-149. Berlin, Germany: Springer Heildelberg.

Wu, Z.B., & Xu, J.P. (2012). A consistency and consensus based decision support model for group decision making with multiplicative preference relations. *Decision Support Systems,* 52, 757-767.

Yeh, J.M., Kreng, B., & Lin, C.H. (2001). A consensus approach for synthesizing the elements of comparison matrix in the Analytic Hierarchy Process. *International Journal of Systems Science*, 32, 1353-1363.

Yepes, V., García-Segura, T.,& Moreno-Jiménez, J.M. (2015).A cognitive approach for the multi-objective optimization of RC structural problems. *Archives of Civil and Mechanical Engineering*, 15(4), 1024-1036.

Yu, L., K.K., & Lai (2011): A distance-based group decision-making methodology for multiperson multi-criteria emergency decision support. *Decision Support Systems*, 51, 307-315.

Zhang, G.Q., Dong, Y.C., & Xu, Y.F. (2012). Linear optimization modelling of consistency issues in group decision making based on fuzzy preference relations. *Expert Systems with Applications,* 39, 2415-2420.

Zhang, G.Q., Dong, Y.C., & Xu, Y.F. (2014). Consistency and consensus measures for linguistic preference relations based on distribution assessments. *Information Fusion,* 17, 46-55.

Zhang, Y., Xu, Z., & Liao, H. (2017): A consensus process for group decision making with probabilistic linguistic preference relations. *Information Sciences,* 414, 260-275.