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Heuristics for Mixed Strength Sensor Location Problems

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ABSTRACT

Location-detection problems are pervasive. Examples include the detection of faults in microprocessors, the identification of contaminants in ventilation systems, and the detection of illegal logging in rain forests. In each of these applications a network provides a convenient modelling paradigm. Sensors are placed at particular node locations that, by design, uniquely detect and locate issues in the network. Open locating-dominating (OLD) sets constrain a sensor's effectiveness by assuming that it is unable to detect problems originating from the sensor location. Sensor failures may be caused by extreme environmental conditions or by the act of a nefarious individual. Determining the minimum size OLD set in a network is computationally intractable, but can be modelled as an integer linear program. The focus of this work is the development and evaluation of heuristics for the minimum OLD set problem when sensors of varying strengths are allowed. Computational experience and solution quality are reported for geometric graphs of up to 150 nodes.

KEYWORDS

Diversification Strategy, Integer Linear Program, Mixed-Weight Open Locating-Dominating Sets, Open Locating-Dominating Sets, Tabu Search

INTRODUCTION

The merger of the locating and dominating requirements for a set of nodes on a network has led to a wealth of research activity in the last 20 years. The introduction of locating sets is generally attributed to Hakimi (1964), although Jordan (1869) and Hua (1962) both had earlier contributions. The origination of dominating sets is usually traced to Ore (1962). Combining the ideas of location and domination have led to a number of distinct problems including, locating-dominating sets (Colbourn, Slater, & Stewart, 1987; Slater 1987), identifying codes (Cohen, Honkala, Lobstein, & Zémor, 2001), open locating-dominating sets (Seo & Slater, 2010; Seo & Slater, 2011) and metric locating dominating sets (Henning & Oellermann, 2004).

The detection and location of anomalies in a network is of central concern in a wide variety of application arenas. In sensor applications, sensors are placed at particular node locations designed to detect and locate anomalies that arise in the network. One way to lower initial costs and long-term maintenance costs for a collection of sensors, is to minimize the number of sensors needed to appropriately monitor the network. Unfortunately, these location–domination problems are, in general, computationally intractable (Givens, Kincaid, Mao, & Yu, 2017; Sweigart, 2019). A finite, connected, simple graph $G(V, E)$ with node set V and edge set E provides a convenient model of a network or physical space. Nodes represent locations or regions and edges represent connections or

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communication ranges between those locations. The edges are assumed to be of unit length unless otherwise noted.

The focus of this paper is open locating dominating (OLD) sets that allow non-uniform weights (sensor strengths). However, it is helpful to consider three related problems to understand the difference between OLD sets and other location domination problems. Let an open neighborhood of a node v, $N(v)$, be the set of nodes adjacent to node v, excluding v. A closed neighborhood of a node v is denoted by $N[v]$ and is the union of $N(v)$ with v. Let $S \subseteq V$ denote a subset of nodes in a graph $G(V,E)$.

In (Seo & Slater, 2011) a set *S* is a Locating-Dominating (LD) set if every node not in *S* has at least one neighbor in *S* and there is no distinct pair of nodes not in S with the same set of neighbors in *S*. Hence, *S* is a LD set if both of the following conditions are met.

LD 1 For all $v \in V - S$, $N(v) \cap S \neq \varnothing$.
LD 2 For all $v, u \in V - S$ such that $v \neq u$, $(N(v) \cap S) \neq (N(u) \cap S)$.

The first requirement is a dominating constraint for *V* - *S*. while the second requirement is a locating constraint. The definition of Open Locating Dominating (OLD) Sets (Seo & Slater, 2010) is quite similar. A set S is an OLD set if every node in the graph has at least one neighbor in S and no two nodes in the graph have the same set of neighbors in S. That is,

OLD 1 For all $v \in V$, $N(v) \cap S \neq \varnothing$.
OLD 2 For all $v, u \in V$ such that $v \neq u$, $(N(v) \cap S) \neq (N(u) \cap S)$.

In both LD and OLD sets the set of neighbors is open but the domains are different, $V - S$ versus V. The open neighborhood distinguishes LD and OLD sets from node identifying codes (IC) in which a closed neighborhood is required. The set S is an identifying code if every node in the graph is dominated and no two nodes have the same set of neighbors. Specifically,

ID 1 For all $v \in V$, $N[v] \cap S \neq \varnothing$.
ID 2 For all $v, u \in V$ such that $v \neq u$, $(N[v] \cap S) \neq (N[u] \cap S)$.

The last related problem is the Metric Locating Dominating (MLD) set (Henning & Oellermann, 2004). The set *S* is called a resolving set if MLD2 is satisfied but MLD1 need not hold. The minimum cardinality resolving set is defined to be the metric dimension of G. *S* is a metric locating dominating (MLD) set if it is both a resolving set and a dominating set. That is, S is an MLD set if,

MLD 1 For all $v \in V - S$, $N(v) \cap S \neq \varnothing$.
MLD 2 For all $u \neq v \in V$, there is a $w \in S$ such that $d(u,w) \neq d(v,w)$.

The conditions OLD1 and ID1 are similar (closed versus open neighborhood) while LD1 and MLD1 are identical. The main difference between these two sets of definitions is the domain of the node *v*, either $V - S$ or V. Notice that the LD1 and MLD1 requirements do not change if $N[v]$ replaces $N(v)$ since the domain is restricted to $V - S$. It is only when the domain includes all of V that the distinction between $N(v)$ and $N[v]$ is important. It is easy to see that any set $S \subseteq V$

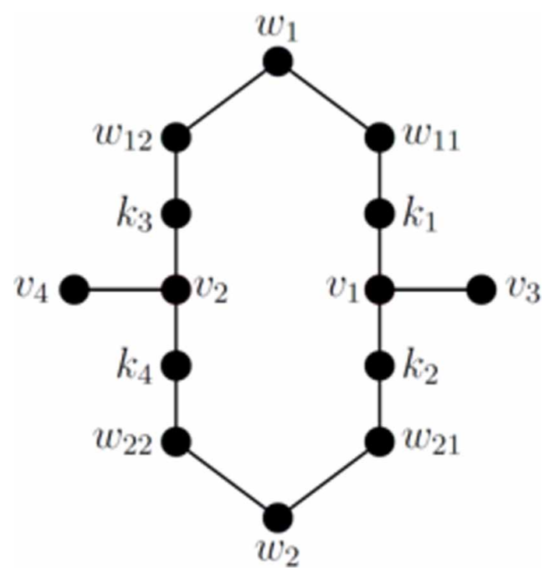
that satisfies the OLD1 condition also satisfies the LD1 and MLD1 conditions. When does the converse hold? That is, when does the condition $N(v) \cap S \neq \varnothing$ fail for $v \in S$. This failure can only occur if there is a node in S that is not adjacent to any other node in S.

Consider the graph in Figure 1. An MLD set is given by $S_{MLD} = \{v_1, v_2, w_1, w_2\}$ It is easy to see that S_{MLD} is a dominating set (MLD1) but tedious to check MLD2. An LD set is given by $S_{LD} = \{v_1, v_2, w_{12}, w_{22}, w_{11}, w_{21}\}$. Again, it is easy to see that S_{LD} is a dominating set. The locating constraint, LD2, only needs to be checked for nodes not in S_{LD}. Note that for any node $u \in S_{LD}$, $N(u) \cap S_{LD} = \varnothing$ due to the open neighborhood. Checking the nodes in $V - S_{LD}$ finds all of the intersections unique and non-empty. For example, $N(w_1) \cap S_{LD} = \{w_{11}, w_{12}\}$. An OLD set is given by the union of the previous two sets plus two additional nodes. That is, $S_{LD} = \{v_1, v_2, w_{12}, w_{22}, w_{11}, w_{21}, w_1, w_2, k_1, k_4\}$. Adding $\{w_1, w_2, k_1, k_4\}$ to S_{LD} forces the intersections of elements in S_{OLD} with itself to be non-empty and unique. For example, $N(v_1) \cap S_{OLD} = \{k_1\}$. Note that S_{LD} satisfies ID1. The use of the closed neighborhood in ID1 makes each of the $N[u] \cap S_{LD} = u$ for all $u \in S_{LD}$. However, S_{LD} fails to meet the ID2 condition since $N[v_3] \cap S_{LD} = N[v_1] \cap S_{LD} = \{v_1\}$. A set that meets ID1 and ID2 is $\{v_1, v_2, w_1, w_2, k_1, k_2, k_3, k_4\}$.

- $S_{MLD} = \{v_1, v_2, w_1, w_2\}$
- $S_{LD} = \{v_1, v_2, w_{12}, w_{22}, w_{11}, w_{21}\}$
- $S_{ID} = \{v_1, v_2, w_1, w_2, k_1, k_2, k_3, k_4\}$
- $S_{OLD} = \{v_1, v_2, w_1, w_2, w_{12}, w_{22}, w_{11}, w_{21}, k_1, k_4\}$

A mixed-weight open locating-dominating set (mixed-weight OLD set), was introduced in Givens (2018) and Givens et al. (2017) extends the OLD set problem definition by allowing sensors of varying

Figure 1. Compare LD, OLD, ID, and MLD solutions



strengths (see also (Kincaid & Givens, 2019)). The varying sensor strengths are modelled through the placement of weights on the nodes of the graph. Just as an increased sensor strength expands the reach of a sensor throughout a region, an increase in weight on a node increases the reach of a node by an equal number of edges. Multiple types of sensors are often used in a wireless sensor network (WSN), such as those that monitor natural habitats (Mainwaring, Culler, Polastre, Szewczyk. & Anderson, 2002). WSNs have been used to study a variety of environmental areas such as glaciers (Martinez, Padhy, Riddoch, Ong, & Hart, 2005), marine pollution (Akyildiz, Pompili, & Melodia, 2005), animal behavior and welfare (Mainwaring, Culler, Polastre, Szewczyk. & Anderson, 2002), and the effect of climate change on farming (Di Palma et al., 2010).

The paper is organized in the following way. Section 2 provides an integer linear programming formulation of the unweighted OLD set problem and identifies a connection to the uniform cost set covering problem. Sections 3 and 4 are dedicated to mixed weight OLD sets. Section 3 provides an example of a graph that does not admit an OLD set unless mixed weights are allowed, while Section 4 summarizes the computational results for several heuristics. Concluding remarks are made in Section 5.

INTEGER LINEAR PROGRAM FOR MINIMUM OLD SETS

A natural goal, for each of the four problems identified in the previous section, is to seek a minimum number of node locations. One way to do this is to formulate the problem as an optimization model. The minimum OLD set problem was formulated as an integer linear program (ILP) for the first time in Sweigart, Presnell, & Kincaid (2014). The ILP assumes that the adjacency matrix A is available (note that $A_{i,j} = 1$ if nodes i and j are adjacenct and zero otherwise). The adjacency matrix serves to identify the open neighborhood of a node. From the adjacency matrix the shortest path distance matrix D can be computed (in polynomial time). Let $d(i,j)$ denote the shortest path distance between nodes i and j. A binary matrix E is formed by letting $E_{i,j} = 1$ if $d(i,j) \le 2$ and zero otherwise. If $d(i,j) > 2$ for any pair of nodes then the nodes, i and j, cannot share any common neighbors. The resulting minimum OLD set ILP is given by

Minimize: $\sum_{j \in V} x_j$

Subject To: $\sum_{j \in V} A_{i,j} x_j \ge 1 \quad \forall i \in V$

$$\sum_{k \in V} \left(A_{i,k} - A_{j,k}\right)^2 x_k \ge E_{i,j} \quad \forall i,j \in V$$

$$x_j \in \{0,1\} \quad \forall j \in V$$

It is quite likely that no feasible solution may exist for the OLD set ILP. For simple, connected graphs this occurs when there is at least one pair of nodes with the same set of neighbors. Graphs with no such nodes are called *twin-free*, (Foucaud, Henning, Lowenstein, & Sasse, 2016). If the solution to the ILP for the OLD set problem has no feasible solution then it has a twin. Note that the OLD set ILP formulation is equivalent to the uniform cost set covering problem (SCP). The SCP is NP-complete but has been extensively studied, (Yelbay, Birbil, & Bulbul, 2015). There are many heuristic search procedures that are known to generate high quality feasible solutions to the SCP.

In the set covering problem, as well as in the OLD set problem, all customers, located at the nodes, must be served. However, if the objects that are to be located have a cost associated with them then there may be an additional budget constraint limiting the number of objects to be located, i.e. $\sum_{j \in V} x_j = k$ where k is a positive integer. When this constraint is added to the above formulation there will be values of k for which not all of the customers can be dominated and our new objective is to seek a solution that minimizes the number of customers not covered (not dominated). In the OLD set context this is easiest to see for the OLD1 dominating constraints. The OLD2 constraints might then only be enforced for those nodes that are dominated in the OLD1 constraints. In any case, when node weights other than 1 are added to the formulation this version of the problem has additional interest since some nodes are now more important than others. Maximum covering location models for OLD sets have been proposed and studied in Sweigart (2019) and Sweigart and Kincaid (2017).

DEFINING MIXED WEIGHT OLD SETS

Allowing OLD set nodes to have non-uniform weights results in a mixed weight OLD set problem. Without loss of generality, assume that OLD set locations are for sensors and that the graph represents a sensor network. In Givens (2018) and Givens et al. (2017) a mixed weight OLD set problem is defined in which each sensor has weight 1 or 2. Moreover, the potential weights for each node location is known apriori. If a weight 2 sensor is located at a node then a signal may be sensed up to 2 edges away. That is, the range of the antenna associated with the sensor is doubled. For the unweighted OLD set problem the OLD1 and OLD2 requirements rely solely on the neighborhood of a node and its interaction with the OLD set S. For weight 2 nodes the antenna strength allows not only the neighborhood to be served but also the neighbors of the neighbors. To capture this effect define, for each node v, $B^{out}\left(v\right)$, the set of nodes that can be reached from v, as well $B^{in}\left(v\right)$, the set of nodes that can reach v. Let the outgoing ball $B^{out}\left(v\right)$ be the set $\left\{u \vee u \in V \wedge d\left(v,u\right) \leq w\left(v\right)\right\}$ the incoming ball $B^{in}\left(v\right) = \left\{u \vee u \in V \wedge d\left(u,v\right) \leq w\left(u\right)\right\}$. When all of the weights are 1, $N\left(v\right) = B^{out}\left(v\right) = B^{in}\left(v\right)$ for all $v \in V$. For the sensor location problem, $S \subseteq V$ is a mixed-weight OLD set if

MW-OLD 1 For all $v \in V$, $B^{in}\left(v\right) \cap S \neq \varnothing$.
MW-OLD 2 For all $v,u \in V$ such that $v \neq u$, $\left(B^{in}\left(v\right) \cap S\right) \neq \left(B^{in}\left(u\right) \cap S\right)$.

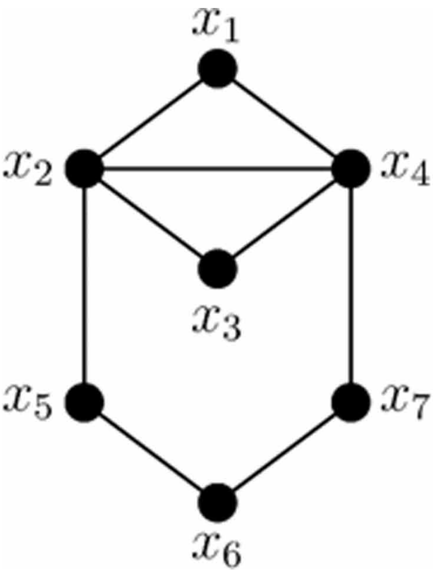
Note that an equivalent mixed-weight OLD set problem can be defined by replacing the role of $B^{in}\left(v\right)$ with $B^{out}\left(v\right)$ **.** In this version of the problem the locations send out a signal rather than receive a signal.

Not all connected, simple, undirected graphs admit an OLD set. Consider the following example. In the graph of Figure 2 all the node weights are 1. No OLD set solution exists since both x_1 and x_3 have the same incoming ball $B^{in}\left(x_1\right) = B^{in}\left(x_3\right) = \left\{x_2, x_4\right\}$ **.** That is, $N\left(x_1\right) = N\left(x_3\right)$ **.** However in Figure 3, by allowing the weight at node x_3 to be 2, a mixed-weight OLD set solution can be found since all of the incoming balls, $B^{in}\left(v\right)$**,** are unique for all $v \in V$ **.** For convenience, directed edges are added from the weight 2 node to all nodes two edges away, as is pictured in Figure 3.

HEURISTIC SEARCH FOR MIXED WEIGHT OLD SET

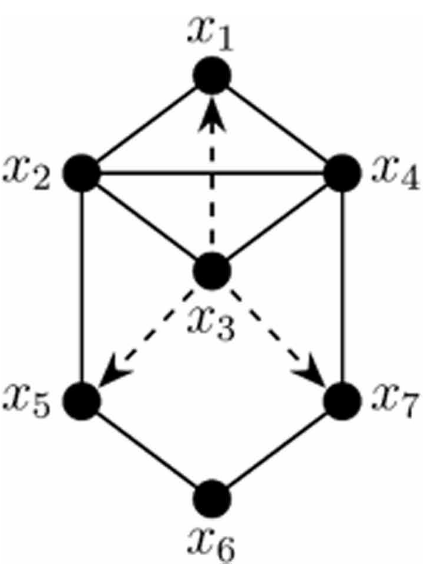
The computational experiments in this section make use of a geometric graph test bed found in (Givens, 2018). Several heuristics were proposed and tested for constructing solutions to the mixed weight OLD set problem (weight 1 and 2 node locations given) on randomly generated geometric graphs with 50, 100 and 150 nodes in (Givens, 2018). Geometric graphs are constructed by randomly generating an ordered pair of coordinates, (x,y), on the unit square. That is, both x and y are uniformly generated on the interval (0,1). Then, an edge is drawn between any pair of coordinates $\left(x_i, y_i\right)$ and $\left(x_j, y_j\right)$ if the distance between the points is less than or equal to a predetermined distance threshold

Figure 2. All node weights 1 ; x_1 and x_3 same incoming-ball $\{x_2, x_4\}$



x	$N(x) = B^{in}(x)$
x_1	$\{x_2, x_4\}$
x_2	$\{x_1, x_3, x_4, x_5\}$
x_3	$\{x_2, x_4\}$
x_4	$\{x_1, x_2, x_3, x_7\}$
x_5	$\{x_2, x_6\}$
x_6	$\{x_5, x_7\}$
x_7	$\{x_4, x_6\}$

Figure 3. If $w(x_3) = 2$ then $B^{in}(x_i)$ is unique $\forall x_i \in V$



x	$N(x) = B^{in}(x)$
x_1	$\{x_2, x_3, x_4\}$
x_2	$\{x_1, x_3, x_4, x_5\}$
x_3	$\{x_2, x_4\}$
x_4	$\{x_1, x_2, x_3, x_7\}$
x_5	$\{x_2, x_3, x_6\}$
x_6	$\{x_5, x_7\}$
x_7	$\{x_3, x_4, x_6\}$

r . 500 graphs are generated for each size, for a total of 4500 graphs. The node weights (1 or 2) are also randomly assigned with the probability of a weight 2 node, ρ , chosen from {0.25,0.50,0.75}. The distance r which governs the assignment of edges is also chosen from {0.25,0.50,0.75}. The resulting geometric graphs may be a single connected component or multiple connected components.

The heuristic which performed the best in Givens (2018) is a stingy heuristic, see Algorithm 1. If a graph admits an OLD set then $S = V$ is an OLD set. The stingy heuristic deletes a node from the current OLD set, checks to see if the reduced set of nodes satisfies MW-OLD1 and MW-OLD2,

Algorithm 1. Stingy Mixed Weight OLD set Heuristic

```
procedure STINGY(B^out, V, S)
    S:= V                                  # initial mixed weight OLD set S
    V_sort:= sort V by largest B^out       # reorder V, largest to smallest
    for v∈V_sort do
        if S−v is a mixed weight OLD set then
            S:= S−v
    return S                               # mixed weight OLD set
```

and continues until no further reduction can be made (a local optima). There are number of ways to determine the order in which to delete nodes in the OLD set. In Givens (2018) a number of possible orders were tested. As a result of these tests, the largest decrease in OLD set size was found by deleting OLD set in decreasing order of largest out going ball size, $B^{out}\left(v\right)$.

The stingy heuristic for the mixed weight OLD set (Givens, 2018) is improved upon in two ways– adding a diversification-based restart and embedding stingy within a tabu search, see Algorithms 2 and 3, respectively. The goal of the diversification strategy is to provide a new OLD set input for the stingy heuristic that is as different (diverse) as possible from the current OLD set. The OLD set constructed by the stingy heuristic is received as input for the diversification scheme. Next, all pairwise swaps between nodes in S and V − S are examined. After each swap, the resulting set is checked to see if MW-OLD1 and MW-OLD2 are satisfied. If true, then the swap is accepted. Otherwise the swap is rejected. After all potential swaps have been examined, the resulting mixed weight OLD set will be the same size as the one constructed by the stingy heuristic (an alternate local optima). Next, the stingy heuristic is invoked with the diversified solution as the initial solution.

Tabu search and, more generally, adaptive memory programming continue to have success in efficiently generating high-quality solutions to difficult practical optimization problems. Glover (1996) provides forty-two vignettes each of which describes a different application of tabu search by researchers and practitioners. Constructing a simple tabu search heuristic requires defining a move set or neighborhood, the tabu tenure of an accepted move, an aspiration criterion and the maximum number of neighborhoods, max_it to be examined. One iteration of a simple tabu search examines the neighborhood of the current solution and either selects the first-improving neighbor or the least nonimproving (non-tabu) neighbor. For the OLD set problem a move is the exchange of any node currently in the OLD set with a node that is not in the OLD set. Each move generates a neighbor of the current OLD set solution. A move is tabu if the exchange has previously been accepted and fewer

Algorithm 2. Recency-Based Diversification Strategy

```
procedure DIVERSIFY(V, S)
    for i∈S do
        for j∈V−S do
            form S−i+j and V−j+i           # swap i and j
            if S−i+j is a mixed weight OLD set then
                S:= S−i+j                  # make swap permanent
                V:= V−j+i
    return S                               # mixed weight OLD set
```

Algorithm 3. Tabu Search for Mixed Weight OLD sets

```
procedure TABU SEARCH(V, S_1, S^ℓ)
    Call Stingy(B^out, V, S_1)
    Call Diversify(V, S_1)
    Call Stingy(B^out, V, S_1)                 # construct initial OLD set S_1
    S^ℓ := S_1                                 # initialize best solution
    for i:=1 to max_it do
        repeat                                 # for neighbors S' of S_i
            S':= neighbor of S_i
            if S' is a mixed weight OLD set then
                Call Stingy(B^out, V, S')
                if S' improving and move is NOT Tabu then
                    S_i:= S'
                    Make move tabu
                else if move Tabu but S' beats S^ℓ then      # aspiration
                    S_i:= S'
                    S^ℓ:= S'
                    Make move tabu
            else S' is NOT a mixed weight OLD set
                increase size of S'
                if S' is a mixed weight OLD set then
                    Call Stingy(B^out, V, S')
                    S_i:= S'                   # improving or uphill move
            if S_i beats S^ℓ then
                S^ℓ:= S_i
        until S' improving OR no more neighbors
```

than tabu tenure iterations have transpired. The search proceeds by first checking if the neighbor is an OLD set. If so, then the stingy heuristic is applied to the neighbor to see if the OLD set size can be decreased. If the move is not tabu and the OLD set size is decreased then the move is accepted and the OLD set is updated.

A tabu move may be accepted if the OLD set size is smaller than any previously discovered OLD set (e.g. meets the aspiration criterion). If a non-tabu move does not result in an OLD set then the node that was removed from the OLD set is appended. If the resulting (larger) set of nodes is an OLD set then the solution is considered as a candidate for the least non-improving OLD set. At any iteration the first-improving move (decrease in OLD set size) is accepted and the iteration terminates. If no improving move is found among the neighbors, then the least uphill non-tabu solution is accepted.

Tables 1-4 catalog the performance of the stand alone stingy heuristic, a diversification scheme followed by the stingy heuristic and a tabu search on 4,500 geometric graph instances. Optimal solutions were found by solving the integer linear programming (ILP) formulation via branch and bound. The ILP linear programming relaxation solution is also recorded for comparison purposes. For tabu search, the tabu tenure was set to one half the number of nodes and the maximum number of iterations, max_it, (neighborhoods examined) was 50 for both the 50, 100 and 150 node cases in 1 and 2. The 150 node case required about 50 minutes per geometric graph for tabu search on an iMac with a 3.2 GHz Intel Core i5 processor and 16 GB of 1867 MHz DDR3 memory. (In comparison, the computation time for the 100 node cases was about 6.3 minutes for each geometric graph and 15 seconds for each 50 node geometric graph.) As a result, tabu search was not tested on all 4,500 of the 150 node geometric graphs. Instead, one of the more difficult 500 geometric graph instances (r = 0.25 and ρ = 0.75) was used as the testbed. Tabu search results for these experiments also set the tabu tenure to one half the number of nodes, but set the maximum number of iterations, max_it to the number of nodes (see Table 5).

Table 1 records the average optimal OLD set sizes (column 2) as well as the average OLD set sizes for each of the heuristics (columns 4-6) for the 4,500 geometric graphs. Column 3 lists the average ILP relaxation objective values which provides an upper bound on the optimal value, but does not provide an OLD set (feasible solution). Table 2 records the average relative error, (heuristic value - optimal value)/(optimal value), for the same set of graphs. All three heuristics generate OLD sets with significantly better values than the ILP relaxation bound. Moreover, tabu search, which requires the most computational effort, provides the highest quality solutions.

Results for the 4,500 geometric graphs are shown for each grouping of 500 in Table 3 and Table 4. When $r = 0.75$ the resulting geometric graphs have the most edges of any of the graphs generated. The results show that these cases are the easiest ones to find a high-quality OLD set. In fact, when $r = 0.75$ and $\rho = 0.75$ the optimal solution is uncovered by the stingy heuristic for all 500 graphs. The most challenging cases are when $r = 0.25$. These graphs have the fewest edges. For the 3,000 geometric graphs with $r = 0.25$ or $r = 0.50$ the trend is the same, tabu search outperforms diversification followed by stingy which outperforms stingy. For example, the relative errors for the 500 geometric graphs generated with $r = 0.25$ and $\rho = 0.75$ were 31.8% for the stingy heuristic, 21.2% for diversification followed by stingy, and 13.6% for tabu search.

Table 5 catalogs the performance of tabu search for the OLD set problem on 500 geometric graphs. In Table 1 nine families of 500 geometric graphs were tested. The family of 500 geometric graphs tested had an edge between two points on the unit square if the distance between the points was less than 0.25 and the probability of a weight 2 vertex was 0.75. Tabu tenure was set to one half the number of nodes and the maximum number of iterations (neighborhoods examined) was set to the number of nodes. Notice that the solution quality improved as the number of nodes increased

Table 1. Average OLD set sizes for 4500 geometric graphs

n	Optimal	ILP-Relax	Stingy	Diverse + Stingy	Tabu
50	21.4	27.3	24.1	23.1	22.2
100	34.6	46.5	39.5	37.1	35.5
150	45.7	63.0	53.7	49.7	----

Table 2. Average relative error, (Heuristic-Opt)/optimal; 4500 geometric graphs

n	Optimal	ILP-Relax	Stingy	Diverse + Stingy	Tabu
50	21.4	27.6%	12.6%	8.1%	3.7%
100	34.6	34.3%	14.2%	7.2%	2.5%
150	45.7	37.9%	17.4%	8.9%	----

Table 3. Average OLD set sizes; 9 sets of 500 graphs with 50 nodes

r, ρ	Stingy	Diverse + Stingy	Tabu	Optimal
0.25,0.25	24.78	22.62	21.15	18.45
0.25,0.50	24.47	22.29	20.77	17.74
0.25,0.75	24.28	22.29	20.90	18.40
0.50,0.25	19.23	17.58	15.62	15.50
0.50,0.50	19.01	18.29	17.29	17.24
0.50,0.75	22.70	22.52	22.26	22.25
0.75,0.25	23.82	23.61	23.12	23.10
0.75,0.50	26.00	25.89	25.73	25.72
0.75,0.75	33.13	33.13	33.13	33.13

Table 4. Average OLD set sizes; 9 sets of 500 graphs with 50 nodes

r, ρ	Stingy	Diverse + Stingy	Tabu	Optimal
0.25,0.25	36.43	32.04	28.45	27.69
0.25,0.50	35.99	31.40	28.09	27.11
0.25,0.75	36.48	31.24	28.34	27.61
0.50,0.25	33.62	29.65	25.34	24.73
0.50,0.50	32.45	30.65	28.18	27.81
0.50,0.75	37.66	37.23	36.55	36.48
0.75,0.25	40.77	40.21	38.71	38.48
0.75,0.50	44.32	44.07	43.56	43.51
0.75,0.75	57.78	57.76	57.74	57.74

Table 5. *Tabu Search (500 geometric graphs with $r = 0.25$ and $\rho = 0.75$)*

n	Optimal	Tabu Search	Rel Error	Itrs	Asp
50	18.4	20.9	13.6%	6.0	0.05
100	27.6	28.2	2.2%	23.7	0.34
150	35.8	36.4	1.7%	50.5	0.62

and the number of instances in which the aspiration criterion was invoked also increased with the number of nodes as well. For the 150 node graphs tabu search uncovered a solution that satisfied the aspiration criterion in 310 out of the 500 instances.

For the 4,500 geometric graphs tested, the weight associated with each node is fixed, apriori. Deciding which weight, 1 or 2, to assign to each node doubles the number of decision variables in the ILP. In Sweigart (2019) and Sweigart & Kincaid (2017) a mixed weight OLD set ILP is formulated that allows sensors of any integer strength from $1, 2, \ldots R$ to be considered.

CONCLUSION

The main contribution in this paper is the development and testing of two heuristics for the mixed-weight OLD set problem. The first imposes a diversification scheme upon the solution generated by a stingy heuristic and then re-applies the stingy heuristic to the diversified solution. The second is a tabu search heuristic which makes use of a short-term memory function (tabu tenure) in an attempt to drive the solution away from locally optimal solutions. In addition, an aspiration criterion is included to allow tabu moves to be accepted if they lead to solutions that have not yet been uncovered. Both heuristics led to better OLD sets than those found by the stand alone stingy heuristic for the 4,500 geometric graphs tested. It is expected that the same performance would be observed if all the weights on the nodes are uniform.

As was noted, the computational time for the tabu search when 150 nodes are considered is roughly 50 minutes per 50 iterations and does not scale well for larger graphs. Pursuit of strategies to address the excessive computing time are of interest. For example, a candidate list strategy, in which a small sample of the neighborhood is examined at each iteration may be implemented. Such candidate list strategies have proven successful in other application venues. Identifying good candidate lists for OLD set problems is a topic for future research.

As was noted in Figure 1, MLD sets contain many fewer nodes than OLD sets. In addition, MLD sets arise in a variety of applications. Three applications of MLD sets and resolving sets are summarized in Mladenovic, Kratica, and Cangalovic (2012)—network discovery (Beerliova et al., 2006), comparing chemical compounds (Chartrand, Eroha, & Oellermann, 2000) and robot navigation (Khuller & Rosenfield, 1996). Consider a robot navigating a space described by a network. The robot seeks information on its current position in the network by sending out a signal to a set of landmarks and determines its distance from them. The problem is to compute the minimum number of landmarks as well as their locations so that the robot is always able to uniquely determine its current position. As was the case for the OLD set problem, it is straight forward to extend an optimization model (see Mladenovic et al., 2012) for the MLD set problem to a maximum covering location problem when a budget constraint forces MLD1 or MLD2 to be violated. To the best of our knowledge this problem has not been studied.

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