A Pragmatic Characterization of Concept Algebra

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A few formal remarks on Denotational Mathematics as seen by Y. Wang

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***Abstract* – Taking into account the framework of denotational mathematics as seen by Yingxu Wang, in this brief note we wish to implement a possible pragmatic dimension into the algebraic structure of concept algebra.**

Keywords – abstract concept; concept algebra; category; semantics; syntax; pragmatics; semiotics

I. INTRODUCTION

The history of science, technology and engineering shows that often new problems require new forms of mathematics. In the last years, informatics has seen a great enlargement of its framework with the institution of some new subdisciplines, amongst which are cognitive and natural informatics, natural intelligence, artificial intelligence, denotational mathematics, neural informatics. *Cognitive Informatics* (CI) is a new discipline, and the problems in it require new mathematical means that are descriptive and precise in expressing and denoting human and system actions and behaviors. Conventional analytic mathematics are unable to solve the fundamental problems inherited in cognitive informatics and related disciplines such as neuroscience, psychology, philosophy, computing, software engineering and knowledge theory. So, the new denotational mathematical structures and means beyond mathematical logic are yet to be sought. Although there are various ways to express facts, objects, notions, relations, actions, and behaviors in natural languages, it is found in CI that human and system behaviors may be classified into three main basic categories known as 'to be', 'to have', and 'to do'. All mathematical means and forms, in general, are an abstract and formal description of these three main categories of expressibility and their rules. Taking this view, mathematical logic may be perceived as the abstract means for describing 'to be', set theory describing 'to have', and algebras, particularly process algebra, describing 'to do'.

The utility of mathematics is the means and rules to express thought rigorously and generically at a higher level of abstraction. Investigation into the cognitive models of information and knowledge representation in the brain is perceived to be one of the fundamental research areas that help to unveil the mechanisms of the brain. The human brain and its information processing mechanisms are centered in cognitive informatics. A valuable cognitive informatics model of the brain, proposed by Yingxu Wang, tries to explain the natural intelligence via interactions between the inherent (subconscious) and acquired (conscious) life functions. The model mainly demonstrates that memory is the foundation for any natural intelligence. Formalism in forms of mathematics, logic, and rigorous treatment is introduced into the study of cognitive and neural psychology and natural informatics. Fundamental cognitive mechanisms of the brain, such as the architecture of the thinking engine, internal knowledge representation, long-term memory establishment, and roles of sleep in long-term memory development have been investigated. *Natural Intelligence* (NI) is the domain of cognitive informatics. Software and computer systems are recognized as a subset of intelligent behaviors of humans described by programmed instructive information. The relationship between *Artificial Intelligence* (AI) and natural intelligence can be described saying that the law of compatible intelligent capability states that artificial intelligence (AI) is always a subset of the natural intelligence (NI), that is AI $\subseteq$ NI. This statement indicates that AI is dominated by NI. Therefore, one should not expect a computer or a software system to solve a problem where humans cannot. In other words, no AI or computing system may be designed and/or implemented for a given problem where there is no solution being known by human beings.

Neural Informatics (NeI) is a new interdisciplinary enquiry regarding the biological and physiological representation of information and knowledge in the brain at the neuron level and their abstract mathematical models. NeI is therefore a branch of CI, where memory is recognized as the foundation and platform of any natural or artificial intelligence. *Denotational Mathematics* (DM) is a new scientific discipline of theoretical informatics, having useful and fertile applications in computer sciences, mainly sprung out of the works of Wang. Broadly speaking, denotational mathematics is a category of certain expressive mathematical structures and notions, that deals with high level mathematical entities beyond numbers, classes and sets, such as abstract objects, complex relations, behavioral information, concepts, knowledge, processes, and systems. Denotational mathematics is usually in the form of abstract algebra that is a branch of mathematics in which a system of abstract notations is adopted to denote relations of abstract mathematical entities, and their algebraic operations based on given axioms, rules and laws. The emergence of denotational mathematics has been motivated by the practical needs in cognitive informatics, intelligence science, software science, and knowledge science, because all of these modern disciplines study complex human and machine behaviors, as well as their rigorous treatments.

The utility of denotational mathematics serves as the means and rules to rigorously and explicitly express design notions and conceptual models of abstract architectures and interactive behaviors of certain complex systems at the highest level of abstraction, in order to deal with problems and of cognitive informatics and computational intelligence, which are characterized with large scales, complex architectures, and long chains of computing behaviors. Therefore, denotational mathematics is chiefly a system level mathematics, in which many detailed individual computing behaviors may still be modeled by conventional analytical mathematics at lower and simpler levels. Typical paradigms of denotational mathematics are known as *Concept Algebra*, *System Algebra*, *Real-Time Process Algebra*, and *Visual Semantic Algebra* (see [1]). In this note, we wish to outline some further formal remarks about the structure of the paradigm of concept algebra, trying to implement in it, a pragmatics component besides the syntactic and semantic ones.

II. The Notion of Concept Algebra

A. The notion of concept

A *concept* is a basic cognitive unit to identify and/or model a real-world concrete entity as well as a perceived-world abstract subject. Based on concepts and their possible relationships, meanings of real-world concrete entities may be represented, and semantics of abstract subjects may be embodied. Concepts can be classified into two main categories, known as the *concrete* and *abstract* concepts. The former are proper concepts that identify and/or model real-world entities such as the sun, a pen, and a computer, while the latter are virtual concepts that identify and/or model abstract subjects, which cannot be directly mapped to a real-world entity, such as the mind, a set, and an idea. The abstract concepts may be further classified into *collective* concepts such as collective nouns and complex concepts, or *attributive* concepts such as qualitative and quantitative adjectives. The concrete concepts are used to embody meanings of subjects in reasoning, while the abstract concepts are used as intermediate representatives or modifiers in reasoning. The *narrow sense* or the *exact semantics* of a concept is determined by the set of common attributes shared by all of its objects, while, on the contrary, the *broad sense* or the *fuzzy semantics* of a concept is referred to as the set of all attributes identified by any of its objects as defined below. The complete set of attributes of a concept, or the instant attributes denoted by all objects of the concept, is a closure of all objects' intensions.

A concept can be identified by its intension and extension. The *intension* of a concept is the attributes or properties that a concept intrinsically connotes, while the *extension* of a concept is the members or instances that the concept extrinsically denotes. For example, the intension of the concept 'pen' connotes the attributes of being a writing tool, with a nib, and with ink. The extension of the 'pen' denotes all kinds of pens that share the common attributes as specified in the intension of the concept, such as a ballpoint pen, a fountain pen, and a quill pen. In computing, a concept is an identifier or a name of a class. The intension of the class concerns with all operational attributes of the class, while the extension of the class concerns with all its instantiations or objects and derived classes. Roughly speaking, a *concept algebra* provides a rigorous mathematical model and a formal semantics for object-oriented class modeling and analyses. The formal modeling of computational classes as a dynamic concept with predesigned behaviors may be referred to *system algebra* (see [2]).

B. First basic notions of formal linguistics and computing

As has been said above, concepts are the basic unit of human knowledge and reasoning. The rigorous modeling and formal treatment of concepts are at the center of all the theories for knowledge presentation and manipulation. A *concept* in linguistics is simply a noun or a noun-phrase that serves as the subject of a 'to-be', 'to have' or 'to do' statement category. A new mathematical structure – i.e., that of concept algebra – is introduced below for a formal treatment of abstract concepts. Roughly, *semantics* is a basic domain of linguistics that studies the possible interpretations of words, utterances, phrases and sentences, analyzing their meanings. In linguistics, semantics deals with how the meaning of a sentence or utterance in a given language is obtained and comprehended. Studies and methods of semantics explore mechanisms in the understanding of language and the nature of meaning where basilar syntactic structures – belongings to *syntax*, i.e., the study of the arrangements of words and phrases to create well-formed sentences or utterances in a given language – play an important role in the interpreting sentences and utterances, as well as the intension and extension of word meanings. To sum up, semantics concerns therefore with the meaning of symbols, notations, concepts, functions, and behaviors, as well as their relations that can be deduced upon a set of predefined entities and/or known concepts (syntax).

Semantic computing is an emerging computational methodology that models and implements computational structures and behaviors at semantic or knowledge level beyond that of symbolic data. In semantic computing, formal semantics can be mainly classified into the categories of 'to be', 'to have', and 'to do' semantics. Semantic analysis and comprehension are a deductive cognitive process. According to the *Object-Attribute-Relation* (OAR) model for internal knowledge representation, the semantics of a sentence in a natural language L may be considered to be understood when: (a) the logical relations of parts of the sentence are clarified; and (b) all parts of a sentence are reduced to the terminal entities, which are either a real-world image or a primitive abstract concept. Within OAR, a concept is therefore a cognitive unit by which the meanings and semantics of a real world or an abstract entity may be represented and embodied based just on the OAR model. Semantics can be classified into the categories of 'to be', 'to have', and 'to do' semantics. A 'to be' semantics infers the meaning of an equivalent relation between an unknown and known entities or concepts; a 'to have' semantics provides the meaning of a structure or a composite entity; and, a 'to do' semantics provides the meaning of an action or behavior of a system or a human. Based on the abstract concept model, any real-world and concrete concept can be rigorously modeled.

C. The Object-Attribute-Relation (OAR) model

The OAR model describes human memory, with particular attention to the long-term memory, by using the relational metaphor rather than the traditional container metaphor that used to be adopted in psychology, computing, and information science. The OAR model shows that human memory and knowledge are represented by relations, that is, connections of synapses between neurons, rather than by the neurons themselves as the traditional container metaphor described. The OAR model can be used above all to explain a wide range of human information processing mechanisms and cognitive processes. According to the functional model of the brain as formulated in [3], genomes may only explain things at the level of *inherited* life functions (genotype), rather than that of *acquired* life functions (phenotype), because the latter cannot be directly represented in genomes in order to be inherited. Therefore, high-level cognitive functional models of the brain are yet to be sought to explain the fundamental mechanisms of the abstract intelligence. In recent genome research people expect that the decoding and probing of human genomes will solve almost all problems and answer almost all questions about the myths of the natural intelligence. Although the aim is important and encouraging, computer and software scientists would doubt this promising prediction. This is based on the basic reductionism of science and the following observations. Albeit the details of computer circuitry are fully observable at the bottom level, i.e., at the gate or even the molecular level, seeing computers only as the low-level structures would not help explaining the mechanisms of computing rather than get lost in an extremely large number of interconnected similar elements, if the high-level functional architectures and logical mechanisms of computers were unknown.

The capacity of the associative cerebral cortex, the main physiological organ of long-term memory (LTM), had been increased dramatically as humans evolved from animals to primates and so forth. An important cognitive model of LTM is just the OAR model which explains how information or knowledge is represented in LTM. It is found that, at the fundamental level, the brain moulds two elemental artifacts, the object and the relation, where

1) *Object* is the abstraction of an external entity and internal concept.

2) *Attribute* is a sub-object that is used to denote detailed properties and characteristics of an object.

3) *Relation* is a connection or inter-relationship between pairs of object-object (R^i, R^o), object-attribute (R^i, R^o), and attribute-attribute (R^c).

Based on these definitions, the OAR model of memory is simply formalized as follows. The OAR model of LTM can be described as a triple, say OAR $= (\mathcal{O}, \mathcal{A}, \mathcal{R})$, where $\mathcal{O}$ is a finite set of objects identified by symbolic names, $\mathcal{A}$ is a finite set of attributes for characterizing an object, and $\mathcal{R}$ is a finite set of relations between an object and other objects or attributes of them, as defined in 3). As we shall see later, an OAR model is nothing but a semantic environment or context Θ. Semantic analysis and comprehension are a deductive cognitive process. According to the OAR model for internal knowledge representation, the semantics of a sentence or an utterance in a natural language L may be considered having been understood when: a) the logical relations between parts of the sentence or utterance are clarified; and b) all parts of a sentence or utterance are reduced to the terminal entities, which are either a real-world image or a primitive abstract concept. *Comprehension* is an higher cognitive process of the brain that searches relations between a given concept and a set of attribute (A), object (O), and/or relations (R) in long-term memory (LTM) in order to establish a representative OAR model for the concept by connecting it with appropriate clusters of the LTM. In cognitive psychology, comprehension involves constructing an internal representation based on existing knowledge previously gained in the brain. The formation of a concept by indentifying its intention (attributes) and extension (objects) is the fundamental cognitive approach to concept comprehension. Higher level comprehension at sentence and article levels can be manipulated by concept relational and compositional operations as defined in concept algebra.

The well-known mathematician George Pólya observed that when solving a problem one goes through the following four phases: a) understanding (comprehending) the problem; b) generating one or more related hypotheses; c) testing such hypotheses, and d) checking the result so obtained. In Pólya's generic problem solving model, problem comprehension is the first and important step toward problem solving. In cognitive psychology, comprehension involves constructing an internal representation according to Margaret W. Matlin. Both Pólya and Matlin agree that comprehension has an intrinsic relation with the given comprehender's background knowledge. This indicates that whatever one is trying to comprehend one relies on one's existing knowledge previously gained in the brain. In a few words, the mathematical model of comprehension is a mapping from a given concept to a concept network expressed by the OAR model, which involves concept establishment, OAR interpretation, OAR updating, and memorization. Then, on the basis of the explanation of the comprehension process based on the abstract concept and the OAR models as developed previously, a rigorous description of the cognitive process of comprehension can be formally modeled using a particular structure called *Real-Time Process Algebra* (RTPA) (see [3], [4], [5], [6]).

D. The notion of concept algebra, and other

If $\mathcal{O}$ denotes a finite or infinite nonempty set of *objects*, and $\mathcal{A}$ is a finite or infinite nonempty set of *attributes*, then a *semantic environment* or *context* Θ is a triple, i.e., $(\mathcal{O}, \mathcal{A}, \mathcal{R})$, where $\mathcal{R}$ is a set of relations between $\mathcal{O}$ and $\mathcal{A}$, and vice versa. Concepts in denotational mathematics are an abstract structure that carries certain meaning in almost all cognitive processes such as thinking, learning, and reasoning. According to the Object-Attribute-Relation (OAR) model, the three essences in Θ can be defined as follows. An object o is an instantiation of a concrete entity and/or an abstract concept. In a narrow sense, an object is the identifier of a given instantiation of a concept. An attribute a is a subconcept that is used to characterize the properties of a given concept by more specific or precise concepts in the abstract hierarchy. In a narrow sense, an attribute is the identifier of a sub-concept of the given concept. A relation r is an association between any pair of object-object, object-attribute, attribute-object, and/or attribute-attribute. On the basis of OAR and Θ, an abstract concept is a composition of the above three elements as given below. An *abstract concept* c on the semantic environment or context Θ is 5-tuple $c_\Theta = (\mathcal{O}, \mathcal{A}, R^c, R^i, R^o)$ where $\mathcal{O}$ is a nonempty set of objects of the concept, $\mathcal{O} = \{o_1, \dots, o_m\} \subseteq \mathcal{P}(O)$ (= power set of $\mathcal{O}$); $\mathcal{A}$ is a nonempty set of attributes, $\mathcal{A} = \{a_1, \dots, a_n\} \subseteq \mathcal{P}(\mathcal{A})$; $R^c = \mathcal{O} \times \mathcal{A}$ is a set of internal relations, $R^i \subseteq C' \times C$, is a set of input relations, and $R^0 \subseteq C' \times C$ is a set of output relations, where C, C' are sets of external concepts. Amongst other things, in the algebraic structure of concept algebra, it is possible to descry an underlying categorical structure (above all related to $\mathcal{R}$ of the context Θ).

Widely speaking, concept algebra deals with the algebraic relations and associational rules of abstract concepts. The various associations of concepts give rise to a foundation to denote complicated relations between concepts in knowledge representation. In turn, the associations amongst concepts can be classified into nine categories, that is to say, inheritance, extension, tailoring, substitute, composition, decomposition, aggregation, specification, and instantiation. All these nine associations describe composing rules among concepts, except instantiation which is a relation between a concept and a specific object. An arbitrary structural concept model, say $c_\Theta = (\mathcal{O}, \mathcal{A}, R^c, R^i, R^o)$, may be also usefully illustrated with a suitable categorial diagram making mainly use of the basic elements c_Θ, $\mathcal{A}$, $\mathcal{R} = \{R^c, R^i, R^o\}$, which denote respectively the concept, its attributes, the objects, and the set of internal/external relationships. Concept algebra is a new mathematical structure for the formal treatment of abstract concepts and their algebraic relations, operations, and associative rules for composing complex concepts and knowledge. Concept algebra also deals with the algebraic relations and associational rules of abstract concepts.

The associations of concepts constitute a foundation for denoting complicated relations between concepts in knowledge representation. A *generic knowledge* K is an n-nary relation R_k among a set of n multiple concepts in C, that is[1]

$$K := R_k : \prod_{i=1}^{n} C_i \to C$$

where $C = \bigcup_{i=1}^{n} C_i$, $R_k \in \mathfrak{R}$, and $\mathfrak{R}$ is the set of the nine concept operations in concept algebra which serve for explicating knowledge composing rules. Based on concept algebra, a knowledge system is formally modeled as a concept network. Moreover, case studies demonstrate that concept algebra provides a powerful denotational mathematical means for manipulating complicated and variegated abstract and concrete knowledge. A *concept network* (CN) is roughly a hierarchical network of concepts interlinked by the set of nine composing rules $\mathfrak{R}$ in concept algebra. The (free-context) *identification* of a new concept, say $c = (O, A, R^c, R^i, R^o)$, is nothing but the elicitation of its objects O, attributes A and internal relations R^c, from any possible semantic environment $\Theta = (\mathcal{O}, \mathcal{A}, \mathcal{R})$, that is to say, it is a structure of the following type $c = \{A, O, R^c, R^i, R^o;\ O \subset \mathcal{O},\ A \subset \mathcal{A},\ R^c = O \times A,\ R^i = \emptyset,\ R^o = \emptyset\}$. The two empty conditions $R^i = R^o = \emptyset$ simply denote the fact that the identification operation is an initialization of a newly created concept where the input and output relations may be determined later when it is put into comparison with the non-empty subsets of input-output relations belonging to $\mathcal{R}$ of a certain context of knowledge Θ, so giving rise to that structure said to be *concept algebra* $C_\Theta = (O, A, R^c, R^i, R^o, \mathfrak{R}, \Theta)$, where $\mathfrak{R}$ is the set of relational and compositional operators between concepts as defined above. Concept algebra provides a new and powerful denotational mathematical means for algebraic manipulations of abstract concepts. Concept algebra can be used to model, specify, and manipulate generic 'to be' and 'to have' type problems, particularly system architectures, knowledge bases, and detail-level system designs, for instance in cognitive informatics, intelligence science, computational intelligence, computing science, software science, and knowledge science (see [7], [8], [9]).

[1]The product should be meant as a Cartesian product of concepts.

III. On Syntax, Semantics and Pragmatics

A. The main domains of natural and formal languages

Linguists commonly agree there is a universal underlying language structure for all humans, known as the *universal grammar*. However, a grammar may be precise and explicit as in formal languages, or ambiguous and implied as in natural languages. Although every language string is symbolically constructed and for reading sequentially, all natural languages have however the so-called *metalinguistic* ability to reference themselves out of the sequences themselves, that is, the ability to construct strings that refer to other strings in the language. From a linguistic viewpoint, software and computing sciences are above all the application of information technologies in communicating between a certain variety of stake holders in computing, such as professionals and customers, architects and software engineers, programmers and computers, as well as computing systems and their environments. Therefore, linguistics and formal language theories play important roles in computing theories, without them computing and software engineering theories would not be considered as complete. On the other hand, it is noteworthy that, historically, language-centered programming had been the dominate methodology in computing and software engineering. However, this should not be taken as the only and unique approach allowed for software development, because the expressive power of programming languages is inadequate to deal with complicated software systems. In addition, the extent of rigorousness and the level of abstraction of programming languages are too low to model the architectures and behaviors of software systems. This is why bridges in mechanical engineering or buildings in civil engineering were not modeled or described by natural or artificial languages. This remark leads to the recognition of the need for mathematical modeling of both software system architectures and static/dynamic behaviors, supplement with the support of automatic code generation systems (see [10]).

Syntax deals with relations and combinational rules of words in sentences and utterances, while *semantics* embody the meaning of words, sentences and utterances. Syntax and semantics of natural languages, are the foundation of the universal language processing model and of the deductive grammar of a language. The syntactic rules of languages that underlie natural languages form the domains of formal linguistics and grammars. One of the most influential linguistic framework, known as the theory of *universal grammar* (UG), was proposed by Noam Chomsky. Universal grammar and its modern version, the *government and binding theory* (GBT), have become a basic linguistic premise on grammatical analyses in linguistics. A syntax is a domain of linguistics that studies sentence/utterance formation and structures. An *abstract syntax* is the abstract description of a syntax system where concrete strings of tokens and their grammatical relations are symbolically represented and analyzed. As has been said above, all semantic relations of sentences in natural languages can be rigorously treated by means of a Real-Time Process Algebra (RTPA) structure. The semantic relations of sentences, say R_S, in natural languages are a finite set of semantic connectors which obey the formal semantics of RTPA. The semantic relations of sentences R_S are a set of important connectors, which formally models phrase and sentence compositions and their joint meaning in complex sentence/utterance structures. Syntactic and semantic analyses in linguistics rely on a set of explicitly described rules which form the *grammar* of a language. Therefore, contemporary linguistic analyses focus on the study of grammars, which is mainly centered in language acquisition, understanding, and interpretation. The grammar of a language is roughly a set of common rules which integrates among them phonetics, phonology, morphology, syntax, and semantics of the language. Moreover, the grammar governs the articulation, perception, and patterning of speech sounds, the formation of phrases and sentences, and the interpretation of utterance (see [10]). In the next section, we shall introduce a further domain of grammar, after C.W. Morris known as *pragmatics*.

B. On The semiotic contribution of Charles W. Morris

The linguist Charles W. Morris developed, in the early 1940s, a behavioral theory of the sign – i.e., *semiotics* – as an attempt to partly unify amongst them logical positivism, behavioral empiricism, and pragmatism. The attempts of unification of these three philosophical perspectives provided useful further features of symbols explicated through three main types of relations with objects, persons, and with other symbols. Morris later called these respectively semantics, pragmatics and syntactics relations. Morris' semiotic is mainly concerned with the explanation of the reciprocal relationship between *syntactics*, *semantics*, and *pragmatics* in a binary way, different from the semiotics of Charles S. Peirce. The term *pragmatics* itself was coined in 1938 by Morris as a tribute to the philosophy of Peirce, i.e. *pragmatism* (or *pragmaticism*, as Peirce called it). All three terms etymologically derive from the Greek root *pragma* which means 'action' or 'activity'. In agreement with some hints already present in Peirce's semiotics, as said above Morris introduced the new terms *syntactics*, (henceforth, *syntax*), *semantics* and *pragmatics* to denote the three basic components of a semiotic, i.e. the description and the theory of a certain system of signs. Syntax was to be the most abstract study of signs disregarding their denotata and use. Semantics was to be more concrete including both syntax and the study of denotation, but not the use. Pragmatics finally was to be the full-bodied study of language use including both syntax and semantics. To be historically more accurate, in Morris' semantics was the study of relations between signs and objects and pragmatics the study of relations between signs and interpreters. According to Morris, however, semantics is the study of signification in all possible modes of signifying, while pragmatics is the study of the origin, use and effects of signs (see [11]). Therefore, pragmatics and semantics are quite different disciplines: roughly speaking, semantics deals with the main question of meaning, while pragmatics deals with questions of use. A typical semantic question is as follows: is a sentence S true? While a typical pragmatic question is as follows: is it appropriate so utter this sentence S in a given situation? informatics, intelligence science, computational intelligence, computing science, software science, and knowledge science (see [7], [8], [9]).

IV. A Possible Pragmatic Dimension of Concept Algebra: A First Attempt

In the structure of concept algebra, a both semantic and syntactic dimension is already implemented, as we have seen above. On the other hand, just as the semantics of a language is based upon its syntax, the pragmatics is based upon both the syntactic and semantic analyses (or, in the celebrated C.L. Hamblin's phrase, "it complements syntax and semantics"). The simplicity with which we can state the formal pragmatics rules for our concern is based upon this ability to have both the syntax and the semantics at hand upon which to build a theory of pragmatics linked with semantics and syntax as according to Charles W. Morris. We believe that a general theory of natural language pragmatics for a language L would involve a definition something like the following:

$$PI_L(\alpha) = f_{semiotic}(\mathcal{R}_{sy}(\alpha), \mathcal{R}_{se}(\alpha), C_1, \dots, C_q)$$

where $PI_L(\alpha)$, that is to say, the *pragmatic interpretation* of α in the language L, is a semiotic function with the following arguments:

1. $\mathcal{R}_{sy}(\alpha)$ provides the operational syntactic derivation of the sentence/utterance α, i.e. "how" the sentence/utterance was phrased in L;

2. $\mathcal{R}_{se}(\alpha)$ is the semantics of the sentence/utterance α, its denotation with respect to a Θ; essentially this indicates the objects referred to by the sentence/utterance;

3. each of the C_i's would represent various aspects, yet to be formalized, inside the given context Θ in which the sentence/utterance has been considered, including discourse elements, objects in view of the speaker and the hearer, participants in the discussion, belief models, and so forth, within OAR model (see [12]).

Therefore, if we consider, for instance, an arbitrary concept algebra $C_\Theta = (O, A, R^c, R^i, R^o, \mathfrak{R}, \Theta)$, in the context Θ, then we may implement in it a pragmatic dimension considering the set of relations $\{\mathcal{R}_{sy}, \mathcal{R}_{se}, f_{semiotic}\}$ plus the *pragmatic operator*

PI_L with respect to the natural language L, so obtaining the following extended structure of concept algebra

$$C_{\Theta}^{semiotic} = (O, A, R^c, R^i, R^o, \Re, \{\mathcal{R}_{sy}, \mathcal{R}_{se}, f_{semiotic}, PI_L\}, \Theta),$$

in which the class of operators and relations $\mathcal{R}_L^{semiotic} = \{\mathcal{R}_{sy}, \mathcal{R}_{se}, f_{semiotic}, PI_L\}$ provides a further possible pragmatic characterization of a concept algebra. In such a manner, we have worked out a wider formal structure in denotational mathematics, that we might call *semiotic concept algebra*, because it takes into account just the semiotic dimension of language.

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