Technology Development through Knowledge Assimilation and Innovation: A European Perspective

Debora Di Caprio, Department of Mathematics and Statistics, York University, Toronto, Canada

Francisco J. Santos-Arteaga, Departamento de Economía Aplicada II, Universidad Complutense de Madrid, Pozuelo, Spain

Madjid Tavana, Business Systems and Analytics Department, La Salle University, Philadelphia, PA, USA & Business Information Systems Department, University of Paderborn, Paderborn, Germany

ABSTRACT

The current paper studies an endogenous growth model driven by the technological development level of a country, which conditions both its factor productivity and the financial investment decisions of agents. Heterogeneity in the level of technological development among countries may not only lead to temporal divergences in income and productivity levels but also to divergent growth paths and poverty traps for identical available technologies. The authors illustrate the structural instability resulting from differences in the technological development level of countries and the subsequent financial constraints arising from such differences. Consequently, a strong national system of innovation should prove vital to ameliorate the negative real effects that follow from a severe financial shock. The obvious and imminent implications regarding the expected evolution of the European Monetary Union are derived both formally and numerically.

Keywords: *Divergence, Economic Growth, European Stability, Innovation, Knowledge Assimilation, Technological Development*

1. INTRODUCTION

The existence of a direct and reciprocal relationship between the real and financial sectors of an economic system should be understood and axiomatized by policy makers and managers at all levels within any organization, from a firm to a country. As a matter of fact, the existing link between finance and innovation was already recognized by Schumpeter in 1934. Similarly, Dosi (1990) and Aghion et al. (2005) have emphasized the considerable importance that the financial sector has for the technological development of countries. It should therefore

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be clearly understood that a robust financial system might play a fundamental role in smoothing macroeconomic shocks – by providing firms with the funds required to finance their innovation activities – when financial constraints become particularly relevant, i.e. during recessions, see O'Sullivan (2005). It seems plausible to assume that the flow of funds to a given country is conditioned by the repayment probabilities of the firms located within it. That is, private financial flows should be guided by a profit maximization motive on the side of investors that depends on the expected evolution of firms, which, at the same time, is determined by their level of technological development and its expected evolution. The explicit recognition of such a relationship and an analysis of its immediate consequences for a country should take place before any political or managerial decision affecting the innovation (and, as such, repayment) capacity of any country is implemented.

Consider the real side of an economy from a structural perspective. The economic growth literature has illustrated how the assimilation of the most advanced technological capital by less developed countries constitutes a growth mechanism requiring important amounts of both physical and human capital investment, see Aghion and Howitt (2005). In particular, the cumulative nature of the process governing the acquisition and assimilation of technological knowledge is widely recognized, see Mukoyama (2003), and the costs of learning a technology are known to be considerable, as shown by Jovanovic (1997). Education, while an important factor for growth, has been displaced by differences in total factor productivity among countries, for which education is only partially responsible, see Howitt and Mayer-Foulkes (2005) for a review of the literature on this topic. Even in the standard economic growth textbooks, see Aghion and Howitt (1999), Barro and Sala-i-Martin (2003) and Acemoglu (2008), knowledge is not assumed to flow immediately among agents and countries, but to diffuse following an endogenously determined sigmoid accumulation function. The complex nature of technology and its diffusion dynamics have already been studied by, to cite a few, Silverberg, Dosi, and Orsenigo (1988), Chiaromonte, Dosi, and Orsenigo (1993), and Patel and Pavitt (1998), who described the existence of divergent technological gaps among developed countries. At the same time, the business cycle literature has highlighted the importance of investment specific technological innovations as the main source of output growth through the cycle, see Greenwood, Hercowitz and Krusell (1997). Such a phenomenon implies the need for specialized capital in production, which builds on the knowledge already existing in the innovator country. Thus, imitation requires highly specialized physical and human capital in order to achieve the same factor productivity as the innovator country.

The current paper studies an endogenous growth model of technology assimilation defined within a quality ladders structural environment. Countries assimilate innovations through an adaptive learning process that determines the accumulation of technological knowledge and infrastructures. The real side of the economy follows from a simplified version of the model introduced in López et al. (2011). At the same time, a financial sector will be incorporated to the world economy. The financial sector accounts for the optimal purchase of firm assets, whose value is determined by the firms' innovation capacity, by rational decision makers. As already stated, the existing relationship between the innovation capacity of firms and their financial evolution follows directly from Schumpeter's 'Theory of Economic Development', see O'Sullivan (2005) for additional insights on this subject and a review of the literature. Moreover, as illustrated by Santos-Arteaga (2009) and López et al. (2011), heterogeneity in the level of technological development among different countries may not only lead to temporal divergences in income and productivity levels, due to differences in capacity utilization, but also to multiple diverging growth paths for identical available technologies. In the current setting, differences in the level of technological development among countries would also affect the asset value of their respective firms and, consequently, their capacity to collect funds and innovate.

Innovation and manufacturing processes, as well as the assimilation of new technologies, require the existence of a skilled labor base and a technological infrastructure that must be developed within the country, i.e. a national system of innovation. The technological infrastructure of a given country, which is determined – among many others – by the existence of techno-economic webs and industrial districts, limits its ability to innovate and learn through manufacturing, since it must develop simultaneously to the knowledge acquired by local workers in order to be implemented efficiently, see Furman et al. (2002), Fores and Camison (2011), Ghili et al. (2011), and López et al. (2011). Such a constraint implies that innovation and manufacturing require both a continuous learning process among workers as well as the existence of a solid technological base. That is, if agents stop learning the innovative activities of a country will eventually cease in spite of the state of its technological base. Similarly, this constraint emphasizes the redundancy of human capital if a technological infrastructure is not available to exploit it. In other words, innovating or efficiently manufacturing a given technology requires a developed technological base available to be used by highly skilled agents.

In the current setting, the accumulation of technological knowledge and infrastructures will determine the total manufacturing productivity of newly acquired technology, as well as the ability of firms to obtain additional knowledge and develop new innovations. That is, the technological development level of a country determines the learning abilities of its workers as well as the innovation probability and corresponding asset value of its firms, i.e. their capacity to finance future innovations. As a result, total factor productivity and the stochastic process governing the arrival rate of innovations will both be defined by the technological development level of a country. Given the level of technological development attained by a country, consumers/investors, who behave as perfectly rational utility maximizers in the same way as those defined in the quality ladders literature by Grossman and Helpman (1991), will distribute their available funds optimally among the firms composing the global financial system.

The results derived from our model are clear and simple. If, as argued by the Schumpeterian and neo-Schumpeterian literatures, see Perez (2002), Hanusch and Pyka (2004) and Perez (2004), technology and the technological development level of a country, i.e. its national, regional, sectoral and technological systems of innovation, play an important role in shaping its economic evolution, both in real and financial terms, then divergent growth patterns and poverty traps become a highly plausible possibility among technological laggards. Considering these results together with the structural instability triggered within a currency area whose members are not fit or able to assimilate asymmetric shocks to their real sectors, see Mundell (1961), we derive both formally and numerically the obvious and imminent implications regarding the expected evolution of the European Monetary Union. It should be noted that the theoretical environment of the paper will be designed to generate a convergence-prone scenario, where technological laggards may actually correct their divergent course through the cycle and try to catch up with the innovator, at least in technological development terms. In this regard, the model remains normative in nature and its recommendations contrast with the observed policies followed by some laggard countries, such as Spain, whose investment in R&D has actually been severely reduced in the latter years, see, for example, Moro-Martín (2012).

1.1. Managing the Global Flow of Information

We concentrate in this paper on the substantial amount of technological knowledge available to firms and countries, which are however constrained by their limited resources to assimilate and exploit it. At the micro level, economists refer to the technological knowledge inherent to a given technology together with the human capital required to extract it, while managers concentrate on adapting the resources of the firm to exploit the flow of information received. At the macro level, the national innovation system of a country is conditioned by its capacity to absorb and distribute the information flow received. In this regard, the management of the information assimilation capacities at the national level should aim at complementing those of the firms and workers at the industry level.

Thus, the model introduced in this paper could be understood in managerial terms as one where firms are not the only actors that must be accounted for when considering the optimal allocation of their technological resources. In the current setting, the technological development level of the country where firms operate determines their success and capacity to finance their investment requirements. Moreover, the capacity of firms to manage optimally their technological knowledge is acknowledged by the consumers/investors within each country when deciding how to distribute their financial resources between firms and countries.

The literature on information management accounts for the constrains faced by firms in terms of the assimilation capacity of the country where they operate and suggests different measures based on the potential stage of development where the country is located, see Loh et al. (1998). In particular, this literature emphasizes the costs incurred when acquiring the more developed technology as a potential shortcoming for the transmission and assimilation of information and communication technologies (ICTs). This choice of technology is based on the recent evolution of ICTs together with their considerable effect on productivity and the global knowledge connections that they provide, see N'Da et al. (2009). For example, when considering the appropriateness of advanced ICTs for developing nations, Loh et al. (1998 pg. 7-8) summarize the approach introduced by Schumacher (1973), relating to the management of the technological knowledge both explicit and inherent to these technologies.

Schumacher (1973) disagreed with the employment of high technologies in developing economies on several accounts. He felt that the level of technology may far exceed the ability of the population in these developing economies to operate and maintain them. In addition to adding to the foreign debt, the purchase of such "high" technologies might further increase the dependence of the developing economies on their industrialized debtors for assistance in their operation and maintenance.

We illustrate how the *free availability* of global technological knowledge does not necessarily solve the problem described by Schumacher, with substantial transfers of capital being exogenously required to ameliorate the expansion of the technological and productivity gap forming between differently developed countries. However, these standard redistributive policies may not suffice to solve the problem unless technological knowledge is managed at the national level while accounting for its interconnections at the global, in this case European, one.

Linking to the Schumpeterian branch of the economic literature described in the introduction, the information systems one also studies the effect of (information and communication) technology on growth. In this regard, the information management branch considers growth enhancing factors ranging from the standard financial, technological, political, and geographical ones (Mbarika et al. 2002) to the freedom of the economic system (N'Da et al. 2009). These papers highlight the key role played by the development level of ICT infrastructures required to interact with technology in order to generate economic growth.

At the same time, this literature complements the evidence illustrating a significant and positive relationship between technology and growth but requiring a minimum level of investments and infrastructure (Oliner and Sichel, 1994, Osei-Bryson and Ko, 2004) while being subject to a given technological learning curve (Dedrick et al. 2003). These requirements are such that the link between technology and economic growth weakens significantly among developing countries (Dewan and Kraemer, 2000; Lee et al. 2005). A similar conclusion follows from the international business literature when considering the assimilation of information and technological spillovers from multinational corporations (Singh, 2007; Alvarez, and Marin, 2010). This conclusion is also reached by Zhang and Lee (2007) when studying ICT spillovers within the information management branch.

Thus, the current paper synthesizes the main elements from the literature described above and relates them to the capacity of laggard economies to acquire the necessary funds to converge to the level of technological development exhibited by the technological leader. In this regard, we assume that countries dedicate a proportion of their productive resources to innovative activities, with the capacity of their human capital and infrastructures to accumulate knowledge being limited by their level of technological development.

At the same time, economic agents (consumers/investors) finance the innovative activities of firms and countries while maximizing the expected returns obtained from their investments. In this case, the capacity of firms to generate innovations determines the expected returns obtained by these agents and, therefore, the resulting incentives regarding how to distribute their funds.

The main variables defining the interactions described above among countries, firms and investors are summarized in Table 1 below:

Table 1. Main variables of the model.

λ^c	Level of technological development achieved by a country.
λ^*	Level of technological development necessary to generate the latest productivity improving innovation.
$\xi = \dfrac{\lambda^c}{\lambda^*}$	Relative level of technological development: technological distance between countries within a technological paradigm.
Γ	Increment in total factor productivity derived from an innovation, with $\Gamma > 1$.
n_{sn}	Proportion of skilled labor dedicated to innovate activities.
$v(\xi)$	Stock market value of the assets issued by a firm, with $v(\xi) \in [0,1]$.
$\theta_\xi = \dfrac{\lambda^c}{\lambda^*} v(\xi) n_{sn}^{1-\phi}$	Arrival rate of the Poisson process z_ξ governing innovations.
$d\left(\dfrac{\xi}{\Gamma}\right) = \left(\Gamma - \dfrac{\xi}{\Gamma}\right) dz_\xi$	Increase in factor productivity caused by technological progress.
v	Value of the assets of the current innovator.
α_v	Increment in the value of the assets resulting from an innovation, with $\alpha_v > 1$.

We will restate these variables within the corresponding context as the model is built through the following sections.

The paper proceeds as follows. Section 2 illustrates the behavior of countries, as well as that of the consumers/investors and firms located within them. Section 3 studies the equilibrium optimality conditions and derives the main consequences of the model for convergence and structural stability. Section 4 describes the technological and financial implications of the model regarding the current European situation. Section 5 analyzes the policy implications derived from the current paper. Section 6 concludes and suggests potential extensions. Formal proofs are presented in the appendices.

2. THE BASIC MODEL

The *production side* of the system [real economic sector] coincides almost completely with the one introduced by López et al. (2011), except for the addition of the financial evolution equation (8) within the current environment and some other minor differences that will be highlighted through the presentation. However, despite the similarities with the setting of López et al. (2011), we will restate the main characteristics of the real economy in order to complement the presentation of the corresponding *financial side*, which is completely new to our model and was not considered by López et al. (2011). The basic assumptions defining the world economy and each country within it follow.

2.1. Countries

Without loss of generality, consider a world economy consisting of two countries. The infamous 'two-speed Europe' concept may provide a useful analogy. Consumers are identical in both countries, regarding both preferences and endowments. Thus, demand-pull effects or income differentials are excluded as possible divergence engines. Similarly, the structure of the labor base, i.e. human capital endowments, is assumed to be identical in both countries. Finally, the exchange rate remains fixed equal to one through the paper and identical unitary prices of the most technologically advanced goods being produced will be assumed in both countries.

Countries differ in their capacity to innovate, whose determinants have been empirically identified by Furman et al. (2002). Their general equilibrium model, defined for each country, consists of an innovation infrastructure, a cluster-specific innovation environment and the quality of the existing linkages between both. The empirical regularities described by Furman et al. (2002) illustrate how countries satisfy their [technological] equilibrium requirements only to a certain extent, with none of them fitting a perfect innovation model.[1] In this sense, it should be emphasized that the current model does not aim at analyzing the causes leading to initial imbalances in technological development levels between countries, but to study the ability of countries to converge in factor productivity and innovation capacity given the existing imbalances between their technological infrastructures.

2.2. Producers

There are always N workers per firm in each country. This assumption prevents us from considering brain drain based divergence processes and other migration related phenomena. The labor force is composed by skilled, n_s, and unskilled workers, n_u such that $n_s + n_u = 1$. Unskilled labor was not part of the labor force considered by López et al. (2011). We will not delve into the consequences derived from this assumption for the real economic sector, since we are more concerned with the financial side of the system. Note, however, that the technological obsolescence of human capital can be easily approximated by the youth unemployment rate, which in the second quarter of 2011 was equal to 42,9% for Greece and 45% for Spain, following an increasing trend since 2008, see Table 2: youth unemployment figures' in European Commission (2011a). Thus, a serious medium to long run human capital obsolescence problem, whose consequences should be accounted for in future studies, is being faced by these countries. For comparative purposes, Germany had a 8,9% rate in the second quarter of 2011 following a *decreasing* trend since 2009.

Skilled labor can be used to either innovate or manufacture, n_{sn} and n_{sm}, respectively, such that $n_{sn} + n_{sm} = n_s$. If unskilled labor is used to manufacture it generates output per period at a rate limited by the technological development level of the country, denoted by ξ. Skilled labor used to manufacture has a productivity δ (> 1) times higher than that of unskilled labor. On the other hand, skilled labor used in innovation activities increases the probability of achieving a higher productivity level the following period. The respective wages received by each type of labor are w_s and w_u, with $w_s > w_u$. Note that all skilled agents receive the same wage independently of whether they are employed to manufacture or to innovate.

Since the price of the [most technologically advanced] goods being produced in both countries is taken as the numeraire and the exchange rate is fixed equal to one, all wages are expressed in real terms and directly comparable between countries. It should be emphasized that we will be considering multiple industries within each country when modeling the financial side of the system. As a result, the uniform numeraire assumption just imposed eliminates any product specialization between countries, since all goods being produced have the same price. In other words, countries are assumed to be fully independent and production complementarities between them, which may take place within a cluster environment, are omitted. For expositional simplicity, a unique industry will be considered when modeling the real side of the system. A version with multiple industries can be easily inferred from the model of López et al. (2011) [see European Commission (2011d) for the data on the over-qualification [and subsequent underpayment] of the European labor force.

Despite the one third over-qualification rate obtained by Spain, based on data from 2008, we maintain our assumption on skilled labor wages. Once again, this will allow us to concentrate on the financial structure of the system, preventing migration flows and wage differentials from taking over the main focus of the paper.

Labor productivity is a function of the level of technological development of the country, ξ, an index that can be interpreted as a proxy variable for the amount and quality of the technological infrastructures existing in the country, i.e. industrial clusters allowing for scale economies, the level of both higher and general education required to generate innovations, incentive policies to R&D, and any other factors favoring the technological enhancement of the country. Thus, ξ could be interpreted as the relative development of the national innovation system of each country. It is important to note that ξ is not equivalent to the productivity of the technology employed by the country. However, in line with the empirical findings of Furman et al. (2002), it is assumed to affect the innovation probability of firms as well as the capacity utilization of the acquired innovations. In this way, we separate the productivity of the technology used by a country from its capacity to innovate. That is, the high costs of imitation illustrated in the literature, see Jovanovic (1997), translate into the (partial) ability of imitator countries to fully exploit the leading technology. As a result, owning a leading technology may help firms to develop a better one but only to the extent allowed by local constraints.

With these restrictions in mind, the production function available to *a firm that has not developed the current leading technological innovation*, if skilled labor is used at any point in time before an additional innovation takes place, is given by

$$Y_{sm} = \delta \xi A^{1-\alpha} K^{\alpha} n_{sm}^{1-\alpha} \quad (1)$$

where

$$\xi = \frac{\lambda^c}{\lambda^*} \tag{2}$$

The level of technological development reflects the existing technological distance between countries within a particular technological paradigm. That is, λ^* indicates the level of technological development necessary to generate the latest productivity improving innovation, i.e. it defines the technological frontier, while λ^c stands for the level achieved by the country. Similarly, the productivity of the technology used by the firm is given by A, which is assumed to be equal to the productivity frontier $\overline{A^*}$ at all points in time. The distinction between the technological and the productivity frontier should be intuitively clear from an economic viewpoint. As already noted, both countries have access to the same technology, since no trade barriers have been assumed. However, the knowledge implicit in each technological innovation is not freely available but directly related to the technological infrastructure of a country. The productivity frontier, to which countries have access by acquiring a given top of the line product, differs from the technological frontier, reachable only by the most technologically advanced country. The remaining part of equation (1) is standard, with K referring to the physical capital used in production and $\delta > 1$ defining the higher productivity achieved by skilled labor, as opposed to the unskilled one, whose $\delta = 1$.

Firms do not differ in the quality of their final product, as is the standard case in the quality ladders literature, but in their total factor productivity. Equations (1) and (2) imply that all firms in both countries are able to produce the most advanced technological good, but their factor productivity differs depending on the value of ξ.

Time is continuous and measured by (discrete) innovations, such that one unit of time lasts as much as it is required for the next innovation to appear. In case two innovations occur simultaneously, it will be assumed that continuity allows for them to be separated in two different units of time, see Aghion and Howitt (1992). Following López et al. (2011) we will assume that, at a given point in time, one of the firms within a country develops an innovation while the remaining ones behave as laggards. Contrary to the quality ladder theoretical literature, i.e. Grossman and Helpman (1991), it will be assumed that *all firms in both countries* gain immediate access to the most advanced production technology. As a consequence, all firms charge the same quality adjusted unitary price for the latest state of the art good being produced. Innovation incentives are provided by infinitely elastic demand functions in both countries that absorb all the production of the good per time period. Thus, innovations do not lead to a technological monopolies, but provide a factor productivity advantage over the rest of the firms. Profits will, therefore, be assumed increasing on the level of technological development, ξ, and total factor productivity, Γ.

The infinitely elastic demand assumption, together with the fixed unitary exchange rate and the same price being charged for the most technologically advanced goods produced in both countries, imply that no trade in goods takes place between countries. This autarchic environment is almost identical to the one defined by López et al. (2011). There exists, however, a considerable difference with respect to their model. That is, in the current paper, the behavior of consumers will be explicitly analyzed. Consumers must distribute their available income between consumption and investment activities. Their optimal distribution of income will depend, among other things, on the value of ξ achieved by a country. In order to keep things as simple as possible, we will assume that the demand generated within each country suffices to absorb all local production.

The skilled labor production function of *a firm after developing a leading technological innovation* is given by:

$$Y_{sn} = \Gamma^2 Y_s, \tag{3}$$

where $\Gamma > 1$, $\xi = 1$ and

$$Y_s = \delta A^{1-\alpha} K^{\alpha} n_{sm}^{1-\alpha}.$$

Thus, Equation (1) defines the production function of all the remaining firms that must manufacture the newly introduced technology. It has been implicitly assumed that a unique firm in only one of the countries develops one innovation per unit of time. We have also assumed that, as soon as an innovation is developed, all firms within both countries are endowed with a unit of the new production technology, the only difference among laggards being the corresponding values of ξ defined for each country. The industry innovator and the manufacturer within the country where the leading technology has been generated share a common infrastructure and an identical level of technological development, which translates into a value of $\xi = 1$ for both firms. Similarly, firms within the laggard country share a common $\xi < 1$ technological development value. It should be noted that, within the current framework, the catch-up process between innovator countries is immediate. That is, if a technologically underdeveloped country generates the next innovation, the value of ξ assigned to both its firms becomes immediately equal to one. As a result, developing a leading technological innovation allows the innovator firm to increment the production of its skilled labor by $\Delta Y = Y_{sn} - Y_{sm} = (\Gamma^2 - \xi) Y_s$, if the firm was previously a laggard. Clearly, the unskilled labor case is identical to the skilled one. Note that the skilled labor production function of the current industry leader is given by ΓY_s. Thus, an innovation must allow the corresponding innovator to increase its production over that of the current industry leader by a factor of Γ. At the same time, the innovator would update its relative technological development level as a result of the innovation. The increase in the output obtained from skilled labor, relative to that of the current industry leader, after an innovation takes place is given by

$$\Delta Y_s = \frac{\Delta Y}{\Gamma Y_s} = \frac{Y_{sn} - Y_{sm}}{\Gamma Y_s} = \left(\Gamma - \frac{\xi}{\Gamma}\right), \qquad (4)$$

if the firm was previously a laggard. Note that, if the firm was already an innovator, then $Y_{sm} = \Gamma Y_s$ and equation (4) translates into a productivity increase of $(\Gamma - 1)$. Thus, output grows at a constant rate of $(\Gamma - 1)$ per unit of time. This type of exponential progression in factor productivity is standard to endogenous growth models; see Aghion and Howitt (1992).

A similar effect, following from an identical type of reasoning, could be derived regarding the evolution of the value of the assets issued by a laggard firm that manages to innovate. In other words, the evolution of factor productivity and the value of financial assets reflect the Arrow effect, implying that no innovator would invest to improve its own leading technology, which defines the highest productivity level within each technological cycle.

2.3. Optimizing Countries

A unique representative industrial sector is considered, whose behavior reflects that of the remaining sectors in the country. We assume that there are two firms per industrial sector in each country. Alternatively, a countable number of firms per sector could be assumed, as is the case in López et al. (2011), but no generality would be gained and notation would become unnecessarily complicated. As emphasized in the previous section, there is only one innovator per sector, but the latest production technology is available to all the firms within the corresponding sectors in both countries; one may think of a system of licenses that become immediately available to all the firms within each industry as soon as a new technology is developed.

Consider the problem faced by a profit maximizing country that must decide how to distribute its labor force between unskilled and skilled workers, as well as what percentage of the latter ones dedicate to innovation and which to manufacturing activities. The Arrow effect dictates that the technological leaders of each industry do not have an incentive to undertake innovative activities and that, as a result, all innovation efforts are left to the manufacturers. As a result, the technological evolution of a given industry is based on the ability of the corresponding manufacturers to develop new innovations. Thus, each country maximizes the expected flow of profits obtained from its manufacturing firms limited by its level of technological development. Profits are determined by the output obtained from the skilled and unskilled labor used in manufacturing activities net of the respective wages received by the workers. At the same time, countries must account for the fact that unskilled workers may acquire the technological knowledge of the skilled ones via direct spillovers and learning by doing. This learning effect increases the productivity and resulting output obtained from the corresponding workers. The maximization problem faced by a country is therefore given by

$$\Pi(t) = E\int_{t}^{+\infty} e^{-\rho[\tau-t]}\pi(n_{sm}, n_{u})\,d\tau$$

where

$$\begin{aligned}\pi(n_{sm}, n_{u}) &= \\ &\delta\xi A^{1-\alpha}K^{\alpha}n_{sm}^{1-\alpha} + \\ &(\delta-1)\xi A^{1-\alpha}K^{\alpha}(\Psi_{sm} - n_{s})^{1-\alpha} \\ &+\xi A^{1-\alpha}K^{\alpha}n_{u}^{1-\alpha} - \\ &w_{s}(n_{sn} + n_{sm}) - w_{u}n_{u}\end{aligned},$$

$$\Psi_{sm} = \frac{1}{1+\left(\frac{1}{n_{s}}-1\right)e^{-\mu(\xi)N}}, \quad (5)$$

and ρ represents the rate of time preference for any given firm, assumed identical both among firms and between countries. The Ψ_{sm} expression corresponds to a logistic learning function whose derivation is presented in Appendix A. This learning function accounts for the fact that unskilled workers are able to acquire the tacit knowledge implicit in the most technologically advanced goods being produced through both knowledge spillovers from skilled workers and learning by doing. The absorptive capacity of unskilled workers, i.e. their ability to learn, has been denoted by $\mu(\xi)$, with $\mu'(\xi) > 0$. That is, the learning capacity of unskilled workers is constrained by the existing distance from the technology employed in the production of the most technologically advanced goods.

Depending on the initial proportion of skilled workers and the absorptive capacity of the unskilled ones, at the end of a given time period there will be a Ψ_{sm} proportion of skilled agents, out of which $(\Psi_{sm} - n_{s})$ were unskilled workers that have been transformed into skilled *manufacturers*. Each transformed agent generates $(\delta - 1)$ extra units of output over her initial level as an unskilled worker.

Innovations are governed by a Poisson process whose arrival rate is given by

$$\theta_{\xi} = \frac{\lambda^{c}}{\lambda^{*}}v(\xi)n_{sn}^{1-\phi}, \quad (6)$$

where $(1-\phi)$ defines the elasticity of the skilled labor used in innovation activities, which is assumed to be higher than the manufacturing one, i.e. $\phi < \alpha$, see Aghion and Howitt (2005). The arrival rate of innovations depends both on the amount of skilled labor used in innovation-related activities and the level of technological development reached by the country. In this sense, the stock market value of the assets issued by a firm, denoted by $v(\xi) \in [0,1]$, depends positively on ξ. That is, firms located within more technologically developed countries, i.e. endowed with better national systems of innovation, are assumed to have better access to funding opportunities. The literature on national systems of innovation, see, among many others, Filippetti and Archibugi (2011), provides a justification for such an assumption.

Technological progress takes place through random discoveries that increase the factor productivity relative to that of the current innovator according to the following process

$$d\left(\frac{\xi}{\Gamma}\right)=\left(\Gamma-\frac{\xi}{\Gamma}\right)dz_{\xi} \tag{7}$$

The value of the assets issued by a firm that develops an innovation evolves in a similar way, increasing by a factor of $\alpha_v > 1$ over the value of the assets of the current innovator, denoted by v :

$$dv(\xi)=\left[\alpha_v v - v(\xi)\right]dz_{\xi}\,. \tag{8}$$

The dynamic structure of the model, in terms of factor productivity and asset value changes, depends on the technological development level of the country, which defines the stochastic arrival rate, θ_{ξ}, of the corresponding Poisson process z_{ξ}. It should be noted that, despite the continuous form of the profit and learning functions, the model is in nature discrete. The continuous flow of technology has a direct effect on the profit function of each country, which changes every period. At the same time, countries are subject to the same decision problem after each innovation takes place, that is, how to allocate their resources among the different labor types, given their relative level of technological development and the probability that any of their firms develops the next innovation.

The current theoretical setting is designed to highlight the lack of intertemporal effects derived from manufacturing activities, since only innovation increments the productivity and asset value of firms. Thus, even though manufacturing requires an investment in human capital to exploit the leading technology, its effect on the production function of the country is temporal. As a result, a country is able to generate the technological base that allows it to grow through time only by innovating continuously.

The Bellman equation defining the intertemporal optimization problem of each country is given by (see Appendix B for its derivation)

$$\rho V\left(\frac{\xi}{\Gamma},v(\xi)\right)= \max_{n_u,n_{sm},n_{sn}}\left[\begin{matrix}\pi(n_{sm},n_u)+\\ \theta_{\xi}\left[\begin{matrix}V(\Gamma,\alpha_v v)-\\ V\left(\dfrac{\xi}{\Gamma},v(\xi)\right)\end{matrix}\right]\end{matrix}\right], \tag{9}$$

which allows for a direct comparison between the immediate benefits obtained from manufacturing, $\pi(n_{sm},n_u)$, and the expected ones derived from innovation related activities,

$$\theta_{\xi}\left[V(\Gamma,\alpha_v v)-V\left(\frac{\xi}{\Gamma},v(\xi)\right)\right].$$

The first order conditions defining the optimal behavior of countries are:

$$(1-\alpha)\xi A^{1-\alpha}K^{\alpha}n_u^{-\alpha}=w_u\,;$$

$$\begin{matrix}(1-\alpha)\delta\xi A^{1-\alpha}K^{\alpha}n_{sm}^{-\alpha}+\\ (1-\alpha)(\delta-1)\xi \\ A^{1-\alpha}K^{\alpha}(\Psi_{sm}-n_s)^{-\alpha}[\Psi'_{sm}-1]=w_s\end{matrix}\;;$$

$$\begin{matrix}(1-\alpha)(\delta-1)\xi A^{1-\alpha}K^{\alpha}\\ (\Psi_{sm}-n_s)^{-\alpha}[\Psi'_{sm}-1]+\\ (1-\phi)\xi\, v(\xi)n_{sn}^{-\phi}\left[\begin{matrix}V(\Gamma,\alpha_v v)-\\ V\left(\dfrac{\xi}{\Gamma},v(\xi)\right)\end{matrix}\right]=\\ w_s\end{matrix} \tag{10}$$

Note that the first condition states that the wage of unskilled workers increases, as does their marginal productivity, in the level of technological development of the country. As a result, unskilled workers have a clear incentive to migrate to technologically developed countries, providing them with a relatively large amount of potentially skilled workers. Nevertheless, we will not be considering migration related issues in the current paper, though their importance as mechanisms allowing for an efficient allocation of resources between countries is acknowledged.

The last two conditions can be simplified to obtain a break up rule defining the optimal allocation of skilled labor based on the value of ξ

$$(1-\alpha)\delta\xi A^{1-\alpha}K^{\alpha}n_{sm}^{-\alpha} =$$

$$(1-\phi)\xi\, v(\xi) n_{sn}^{-\phi}\begin{bmatrix} V(\Gamma,\alpha_v v) - \\ V\left(\frac{\xi}{\Gamma}, v(\xi)\right)\end{bmatrix}. \tag{11}$$

The left hand side of equation (11) corresponds to the instantaneous gained obtained from manufacturing, given by the marginal productivity derived from using an additional unit of skilled labor in manufacturing activities. The right hand side defines the *expected* gain (through the marginal increase in the arrival rate of the Poisson innovation process) from using an additional unit of skilled labor in innovative activities. Equation (11) can be written as follows:

$$\frac{n_{sn}^{\phi}}{n_{sm}^{\alpha}} =$$

$$H\left(\frac{1-\phi}{1-\alpha}\right)\begin{bmatrix} V(\Gamma,\alpha_v v) - \\ V\left(\frac{\xi}{\Gamma}, v(\xi)\right)\end{bmatrix}$$

$$\text{with } H = \frac{v(\xi)}{\delta A^{1-\alpha}K^{\alpha}}. \tag{12}$$

Equation (12) defines the optimal distribution of skilled labor between innovative and manufacturing activities as a function of the relative productivity and asset value gains derived from a successful innovation. That is, large increments in factor productivity and the value of assets relative to the current levels defined by ξ for a given country would promote the use of skilled labor in innovative activities over manufacturing ones. In other words, countries with a relatively low level of technological development should use a larger proportion of their human capital resources to innovate rather than to manufacture, which relates the current model to Gerschenkron's (1962) advantage of backwardness.

2.4. Consumers

Consumers are modeled following the quality ladder theoretical assumptions introduced by Grossman and Helpman (1991). Since consumers have been assumed to be identical in both countries, related notational distinctions will not be considered. Through this section, an identical continuum of industrial sectors, indexed by $\omega \in [0,1]$, will be defined within each country. Without loss of generality, each sector will be assumed to consist of four firms, two per country. Products within each sector can be supplied in a countable-infinite number of qualities. Quality j of a product in sector ω is defined by $q_j(\omega) = \gamma^j$, where $\gamma > 1$ is assumed to be identical in all sectors. That is, a j quality value implies that a product j has been improved times with respect to its initial quality level, which is assumed equal to one and identical for all products in all sectors.

The common intertemporal utility function defined for all consumers in both countries is given by

$$U = \int_{t}^{+\infty} e^{-\beta[\tau - t]} u(c(t))\, d\tau\ , \tag{13}$$

where β is the subjective discount rate applied by consumers, $u(c(t))$ represents the flow of utility at time t,

$$u(c(t)) = \int_{0}^{1} \log\left[\sum_{j=0}^{+\infty} q_j(\omega) c_{jt}(\omega)\right] d\omega\ , \tag{14}$$

and $c_{jt}(\omega)$ denotes the consumption of a quality j product from sector ω at time t. At the same time, the flow of consumption expenditure at time t is defined as follows

$$e(t) = \int_{0}^{1} \left[\sum_{j=0}^{+\infty} p_{jt}(\omega) c_{jt}(\omega)\right] d\omega \tag{15}$$

where $p_{jt}(\omega)$ denotes the price of a quality j product from sector ω at time t.

The solution to the static optimization problem faced by a consumer is standard, with $e(t)$ being allocated to maximize $u(c(t))$, given prices at time t. The corresponding static demand function is defined by:

$$c_{jt}(\omega) = \begin{cases} \dfrac{e(t)}{p_{jt}(\omega)} & if\ j = h_t(\omega), \\ 0 & otherwise, \end{cases} \tag{16}$$

where $h_t(\omega)$ denotes the unique quality level that carries the lowest quality-adjusted price, $p_{jt}(\omega) \,/\, q_j(\omega)$. The uniform numeraire assumption imposed on all sectors in both countries constitutes an oversimplification that, at the cost of eliminating the ability of countries to specialize in different industrial sectors, concentrates all the attention on their financial capacities (and constraints). This assumption, together with the concavity of the utility function, imply that consumers allocate identical expenditure shares to all markets. Besides, they all consume the most technologically advanced [lowest quality-adjusted price] product from each market, supplied at different productivity rates depending on the technological development level of the corresponding firms [and countries]. Relaxing this latter constraint would introduce interesting frictions, such as production-planning, capacity and capability management constraints, see Wu et al. (2005) and Bessant et al. (2011), that firms should deal with based on the expected outcomes from their innovation activities.

Note that, if we were to relax the uniform numeraire assumption, then quality adjusted prices would differ among sectors, with each sector's relative productivity varying based on the number of innovations introduced within it and the technological development level of the country. This heterogeneity in productivity levels would lead to the type of Bertrand competition in [quality adjusted] prices highlighted by the quality ladders literature and the subsequent sectoral specialization of countries. In this case, technologically developed countries could exclude laggards from competing in a progressively increasing amount of sectors, while specializing in a subset of them. The example of Cuba's biotech sector described in López et al. (2011) represents this idea. Clearly, developing significantly a given industrial sector does not guarantee either convergence or structural development via internal technology spillovers.

Consumers face also a dynamic optimization problem, where they maximize their intertemporal utility subject to a stochastic budget constraint that is based on the expected state of the world economy after an innovation within any industry takes place. Besides expenditure, $e(t)$ consumers must decide how to distribute their budgets to finance innovation activities among all available manufacturers within both countries.

Consider a consumer who owns assets from the current innovator firm within industry ω. The amount of assets owned will be defined by either a_n^n or $a_{m|n}^m$, depending on whether the current innovator was previously located within an innovator or a laggard country, respectively. In both cases, the value of each asset will be given by v . The evolution of a consumer's financial wealth, in asset value terms, if the next innovation is introduced by the manufacturing firm within the country where the current innovation has been developed is given by:

$$d(v_n a_n^m) = \begin{bmatrix} w + va_n^n + \dfrac{va_{m|n}^m}{2} - \\ v_n a_n^m - v_m a_m^m - e \end{bmatrix} dt + , \left[\alpha_v va_n^m - v_n a_n^m\right] dz_n \tag{17}$$

while the stochastic differential equation defining the evolution of a consumer's financial wealth if the next innovation is introduced by one of the manufacturing firms within the laggard country reads

$$d\left(\frac{v_m a_m^m}{2}\right) = \begin{bmatrix} w + va_n^n + \dfrac{va_{m|n}^m}{2} - \\ v_n a_n^m - v_m a_m^m - e \end{bmatrix} dt + . \left[\frac{\alpha_v va_m^m}{2} - \frac{v_m a_m^m}{2}\right] dz_m \tag{18}$$

In both equations, w denotes the wage received by consumers, e corresponds to their consumption expenditure, $v_n a_n^m$ is spent to purchase assets from the manufacturer within the innovator country at a price of v_n , while $v_m a_m^m$ represents the expenditure in assets from both manufacturing firms located within the laggard country at a price of v_m , which is assumed lower than v_n due to the differences in technological development between countries. Investments made in a laggard country are assumed to be equally divided between both its (identical) manufacturing firms. Therefore, the financial wealth derived from a current innovator that was previously located within a laggard country must be defined over half the amount invested initially in the country.

Note that we should have considered a particular $\xi(\omega)$ value per industrial sector. In this case, the resulting integral could have either been defined over the entire set of industrial sectors or a unique ξ index, based on the number of innovator sectors located within each country, could have been built. In both settings, the innovator country should be defined as the one with a higher percentage of innovator sectors. If we were to actually follow the latter approach, equation (17) would be given by:

$$d\left(\int_0^1 v_n(\xi) a_n^m(\omega)\, d\omega\right) = \begin{bmatrix} w + v\displaystyle\int_0^1 a_n^n(\omega)\, d\omega + \\ \dfrac{v}{2}\displaystyle\int_0^1 a_{m|n}^m(\omega)\, d\omega - \\ \displaystyle\int_0^1 v_n(\xi) a_n^m(\omega)\, d\omega - \\ \displaystyle\int_0^1 v_m(\xi) a_m^m(\omega)\, d\omega - e \end{bmatrix} dt + \begin{bmatrix} \alpha_v v\displaystyle\int_0^1 a_n^m(\omega)\, d\omega - \\ \displaystyle\int_0^1 v_n(\xi) a_n^m(\omega)\, d\omega \end{bmatrix} dz_n . \tag{19}$$

A similar extension in ω terms should be applied to the remaining stochastic differential equations described in this section when following either the previous or the $\xi(\omega)$ approach. It should be emphasized that the main results obtained through the paper hold within both these settings. However, the additional notational requirements do not add any economic intuition to the results while complicating the presentation considerably. Thus, the simplest possible notational environment has been chosen to perform the analysis.

The Poisson processes defining the innovation arrival rates within the current innovator and laggard countries are respectively given by z_n and z_m. In both cases, the value of the assets acquired from the corresponding manufacturing firms within any of the countries increases by α_v (>1) times the value of the assets of the current innovator. Note that z_n and z_m describe the arrival rate of innovations for the whole country. If we were to consider innovations on an industrial sector basis, a continuum of equations, one per sector, should be defined. Besides tractability problems, even if we were to consider a countable number of sectors, the resulting micro level analysis would not modify the main results obtained in aggregate financial flows terms. Moreover, consumers have been assumed to constitute an homogeneous group in both preference and information terms. However, heterogeneity seems a more plausible assumption. If we were to impose it, the deterministic part of Equations (17) and (18) should be expanded to account for failure rates in the financial investments made by consumers. In other words, the expression

$$va_n^n + \frac{va_{m|n}^m}{2}\text{, i.e.}$$

$$v\int_0^1 a_n^n(\omega)\,d\omega + \frac{v}{2}\int_0^1 a_{m|n}^m(\omega)\,d\omega\,,$$

refers to the income available during a given time period due to successful financial investment decisions made in the previous one. It should be intuitively clear that implicit in this expression is a measure of the [relative] success of the financial choices made by consumers, where, for expositional simplicity, the assets of the firms failing to innovate during the previous period have been assigned a value of zero. If accounted for, the implicit financial success rate could be assumed to depend on the value of ξ, the quantity and quality of financial information, and the education level of consumers, among many other variables.

The (subjective) expected financial wealth of a consumer (per unit of time) is defined as follows:

$$E[va] = \mu_n(\theta_n)v_n a_n^m + \mu_m(\theta_m)\frac{v_m a_m^m}{2}\,, \tag{20}$$

where $\mu_i(\theta_i)$, with $i = m, n$, stands for the subjective probability assigned to country i becoming the next innovator, which is assumed to be an increasing function of θ_i, and such that $\mu_m(\theta_m) + \mu_n(\theta_n) = 1$. When calculating *the evolution* of their expected budget constraints, consumers take as given the arrival rates corresponding to the Poisson processes that define the innovation intensity of each country, $\theta_i = \xi\, v_i(\xi)\, n_{sn}^{1-\phi}$, and generate their own subjective innovation probabilities, $\mu_i(\theta_i)$. Differences in information acquisition and processing capacities among consumers could be assumed to justify this assumption. Note that the expected financial wealth of consumers is a stochastic variable whose very own definition accounts implicitly for the dynamic process determining the evolution of the expected budget constraint of a consumer presented in equation (21). Thus, given equations (17) and (18), the stochastic evolution of the expected budget constraint of a consumer becomes (its derivation is presented in Appendix B)

$$d\left(\mu_n(\theta_n)v_n a_n^m + \mu_m(\theta_m)\frac{v_m a_m^m}{2}\right) =$$

$$\left[\begin{array}{l} w + va_n^n + \dfrac{va_{m|n}^m}{2} - \\ v_n a_n^m - v_m a_m^m - e \end{array}\right] dt +$$

$$\left[\mu_n(\theta_n)\left[\alpha_v va_n^m - v_n a_n^m\right]\right]dz_n +$$

$$\left[\mu_m(\theta_m)\left(\frac{\alpha_v va_m^m}{2} - \frac{v_m a_m^m}{2}\right)\right]dz_m. \tag{21}$$

Finally, we use equation (21) to obtain the Bellman equation defining the optimization problem of a consumer, which is based on the stochastic evolution of the innovation processes that determine the (subjectively defined) expected budget constraint of each consumer (additional details are provided in Appendix B)

$$\rho V(E(va)) = \max_{e, a_n^m, a_m^m} \begin{cases} u(c) + \\ V_{E(va)}\left[E(va)\right]\begin{bmatrix} w + va_n^n + \\ \frac{va_{m|n}^m}{2} - v_n a_n^m - \\ v_m a_m^m - e \end{bmatrix} + \\ \theta_n \begin{bmatrix} V\left[\mu_n(\theta_n)(\alpha_v va_n^m) + \mu_m(\theta_m)\frac{v_m a_m^m}{2}\right] - \\ V\left[\mu_n(\theta_n)v_n a_n^m + \mu_m(\theta_m)\frac{v_m a_m^m}{2}\right] \end{bmatrix} + \\ \theta_m \begin{bmatrix} V\begin{bmatrix} \mu_n(\theta_n)v_n a_n^m + \\ \mu_m(\theta_m)\left(\frac{\alpha_v va_m^m}{2}\right) \end{bmatrix} - \\ V\left[\mu_n(\theta_n)v_n a_n^m + \mu_m(\theta_m)\frac{v_m a_m^m}{2}\right] \end{bmatrix} \end{cases}. \tag{22}$$

The first order conditions for the above optimization problem read as follows:

$$u'(c) = V'\left[E(va)\right]$$

$$v_n V'\left[E(va)\right] = \mu_n(\theta_n)\begin{bmatrix} \theta_n \alpha_v v V'\left[\Delta E(va) \mid n\right] + \\ \theta_m v_n V'\left[\Delta E(va) \mid m\right] - \\ v_n V'\left[E(va)\right] \end{bmatrix}$$

$$v_m V'\left[E(va)\right] = \mu_m(\theta_m)\begin{bmatrix} \theta_n \frac{v_m}{2} V'\left[\Delta E(va) \mid n\right] + \\ \theta_m \frac{\alpha_v v}{2} V'\left[\Delta E(va) \mid m\right] - \\ \frac{v_m}{2} V'\left[E(va)\right] \end{bmatrix} \tag{23}$$

where $V'\left[X\right]$ denotes the first order derivative of $V\left[X\right]$ with respect to the X variable. $V'\left[\Delta E(va) \mid n\right]$ (respectively $V'\left[\Delta E(va) \mid m\right]$) stands for the marginal value derived from an increase in the expected financial wealth of consumers that follows from an innovation taking place within a manufacturing firm located within the innovator (resp. laggard) country.

Substituting for $V'\left[E(va)\right]$ in the last two equations and operating we obtain:

$$\theta_n \left(\frac{\mu_n(\theta_n)\alpha_v v}{(1+\mu_n(\theta_n))v_n} - \frac{\mu_m(\theta_m)}{2+\mu_m(\theta_m)}\right) V'\left[\Delta E(va) \mid n\right] =$$

$$\theta_m \left(\frac{\mu_m(\theta_m)\alpha_v v}{v_m(2+\mu_m(\theta_m))} - \frac{\mu_n(\theta_n)}{1+\mu_n(\theta_n)}\right) V'\left[\Delta E(va) \mid m\right]. \tag{24}$$

Note that the model has been constructed so that consumption relies only implicitly on the expected evolution of the innovation processes of countries, while financial investment decisions are directly determined by it. Consider the first optimality condition. Clearly, if $V\left[E(va)\right]$ is concave, decrements in expected financial wealth lead to a relative decrease in the optimal consumption level of decision makers. Similarly, if $V\left[E(va)\right]$ is convex, then a lower $E(va)$ value leads to a relatively higher consumption level. This optimal redistribution of expenditure does not affect the main results obtained and has therefore been kept to the simplest possible form.

Equation (24) determines the optimal financial investment distribution of consumers when deciding how to allocate their funds among the manufacturing firms located within both the innovator and the laggard country. The expected marginal value gain obtained from an increase in the subjective expected financial wealth of consumers that follows from an innovation taking place within the innovator country must equal the expected marginal value gain generated by any additional investment made in the laggard country. In both cases, the respective expected marginal value gains are weighted by the relative change in the value of the corresponding firm's assets after an innovation takes place. Equation (24) simplifies to

$$\frac{V'\left[\Delta E(va) \mid n\right]}{V'\left[\Delta E(va) \mid m\right]} =$$

$$\frac{\theta_m v_n \begin{bmatrix} \alpha_v v(1+\mu_n(\theta_n))\mu_m(\theta_m) - \\ v_m(2+\mu_m(\theta_m))\mu_n(\theta_n) \end{bmatrix}}{\theta_n v_m \begin{bmatrix} \alpha_v v(2+\mu_m(\theta_m))\mu_n(\theta_n) - \\ v_n(1+\mu_n(\theta_n))\mu_m(\theta_m) \end{bmatrix}}. \tag{25}$$

It should be emphasized that the main results obtained would not be [qualitatively] modified if we were to consider N firms per industrial sector instead of two. It should also be noted that we have defined the innovation probability of a firm in terms of the value of its assets, and not the total amount of funds received. While this distinction is relevant, the results obtained regarding the evolving growth [and financial] patterns of countries and poverty traps hold independently of it. Clearly, given

$$\theta_i = \xi\, v_i(\xi)\, n_{sn}^{1-\phi}, \; \frac{\theta_m}{v_m} = \xi\, n_{sn}^{1-\phi}$$

converges to zero as the technological gap widens between countries. In this sense, several main results hold based on a relatively simple analysis. However, if we were to consider the total amount of funds received by a firm when defining θ_i, we should account for the sum of all individual contributions, which would lead θ_i to depend on a_i^m, with $i = m, n$. The additional notation required, as well as the effect of $\theta_i(a_i^m)$ on the corresponding equilibrium expressions, would complicate the analysis considerably without modifying the main results obtained. We have therefore assumed implicitly that the contribution of each consumer to the innovation probability of a firm is negligible and that this fact is acknowledged by all consumers, who act accordingly. Nevertheless, we acknowledge the fact that the total amount of funds received by a firm, as well as the value of its assets, are not only defined according to economic considerations, but also political and/or speculative factors play an important role, particularly in technologically underdeveloped countries. Thus, we will not apply the simplification described above when analyzing the behavior of θ_m and v_m, but consider them as different variables that, while related, follow different converging patterns in ξ.

3. CONVERGENCE AND STRUCTURAL STABILITY

The stochastic evolution of the equilibrium defined for the economic system presented in the previous section is determined by equations (12) and (25)

$$\frac{n_{sn}^{\phi}}{n_{sm}^{\alpha}} = H\left(\frac{1-\phi}{1-\alpha}\right)\left[\begin{matrix} V(\Gamma, \alpha_v v) - \\ V\left(\frac{\xi}{\Gamma}, v(\xi)\right) \end{matrix}\right] \text{(PP)}$$

$$\frac{V'\left[\Delta E(va) \mid n\right]}{V'\left[\Delta E(va) \mid m\right]} = \frac{\theta_m v_n \left[\begin{matrix} \alpha_v v(1+\mu_n(\theta_n))\mu_m(\theta_m) - \\ v_m(2+\mu_m(\theta_m))\mu_n(\theta_n) \end{matrix}\right]}{\theta_n v_m \left[\begin{matrix} \alpha_v v(2+\mu_m(\theta_m))\mu_n(\theta_n) - \\ v_n(1+\mu_n(\theta_n))\mu_m(\theta_m) \end{matrix}\right]}. \text{(FF)}$$

Equation (PP) defines the equilibrium in the production side of the economy, such that labor markets clear and human capital is distributed according to the productivity and asset value gains expected to be obtained from a successful innovation. Note that we have omitted the unskilled labor equilibrium condition when defining (PP). This has been done to avoid considering the human capital frictions suffered by the [technological] laggard countries and concentrate the analysis on the financial incentives that follow from modifying the value functional differentials within this equation. Accounting for differences in human capital indicators would worsen the divergent results obtained both formally and numerically. At the same time, equation (FF) corresponds to the equilibrium in the financial side of the economy, where investment is optimally allocated depending on the marginal value differentials between countries caused by innovation-based changes in the subjective expected financial wealth of consumers. It follows directly from equation (PP).

3.1. Lemma

There exists a (technological) poverty trap generated by the production side of the economic system.

Proof. As the technological development gap between countries increases, so does the value gain obtained from a successful innovation,

$$\left[V(\Gamma, \alpha_v v) - V\left(\frac{\xi}{\Gamma}, v(\xi)\right)\right],$$

leading countries with a relatively low level of technological development to dedicate an increasingly larger proportion of their skilled labor force to innovation related activities. However, the probability of a firm generating the next innovation is also an increasing function of ξ, i.e.

$$\theta_\xi = \xi\, v(\xi)\, n_{sn}^{1-\phi}.$$

Therefore, the innovation probability of these countries does not necessarily increase and would eventually converge to zero in the limit as the technological development gap increases through time. ■

Equation (FF) defines the optimal investment distribution policy applied by consumers when financing the innovation activities of countries through the international asset market. It leads to the following result.

3.2. Lemma

There exists a (technological) poverty trap generated by the financial side of the economic system if,

1. $V\left[\cdot\right]$ is concave and v_m converges to zero at a lower rate than θ_m,
2. $V\left[\cdot\right]$ is convex and v_m converges to zero at a higher rate than θ_m.

Proof. Assume that the right hand side (RHS) of the (FF) equation equals one. At this point, consumers would equally finance innovation activities in both the innovator and laggard countries, as the increase in the expected financial wealth derived from an innovation taking place within any of the countries would generate the exact same marginal value. It can be easily observed that the term in brackets within the denominator of equation (FF) is always positive for any $\mu_n(\theta_n) > \mu_m(\theta_m)$. Consequently, so must be the term in brackets defined in the numerator for any increasing $V\left[\cdot\right]$. Indeed, both these terms simply weight, either up or down, the effect of the,

$$\frac{\theta_m v_n}{\theta_n v_m}$$

expression within equation (FF). For example, it can be easily shown after some basic algebra that the expression in brackets within equation (FF) is smaller than one if,

$$\frac{v_n}{v_m} < \frac{2\mu_n(\theta_n) + \mu_n(\theta_n)\mu_m(\theta_m)}{\mu_m(\theta_m) + \mu_n(\theta_n)\mu_m(\theta_m)}. \quad (26)$$

Thus, consumers' optimal financial investment distribution among the firms located within both countries is determined by,

$$\frac{\theta_m v_n}{\theta_n v_m}.$$

Consider now the (1.) case, while keeping in mind that (2.) follows an identical logic. Clearly, θ_n, $v_n > 0$ at all times, which simplifies the RHS of (FF) to be fully determined by the evolution of θ_m and v_m. Therefore, if v_m converges to zero at a lower rate than θ_m, the optimal allocation of funds must be such that the marginal value generated by an increase in the expected financial wealth of consumers conditional on an innovation taking place within the innovator country converges to zero as the gap in the level of technological development between countries widens through time. This would be the case for any positive but bounded above value of $V'\left[\Delta E(va) \mid m\right]$. As a result, all financial funds would be progressively driven towards the manufacturing firm located in the innovator country and away from the firms within the laggard one. If, in this case, $V\left[\cdot\right]$ were a convex function, the opposite effect in the distribution of financial funds would be induced, and the poverty trap just described could be averted by laggard countries. ■

Both lemmas lead to the main result in the paper.

3.3. Theorem

A (technological) poverty trap is generated as the technological development gap between countries widens through time if

1. $V\left[\cdot\right]$ is concave and v_m converges to zero at a lower rate than θ_m,
2. $V\left[\cdot\right]$ is convex and v_m converges to zero at a higher rate than θ_m.

3.4. Corollary

The convergence process of laggard countries depends on the phase of the technological cycle on which both countries are located, i.e. on the shape of the value function $V\left[\cdot\right]$, in the following possible ways

1. If $\lim_{\xi \to 0}\left(\frac{\theta_m}{v_m}\right) = 0$ and $V\left[\cdot\right]$ is convex,

laggard countries will keep on receiving funds to invest in innovation related activities and may converge depending on their innovation probability, which is, at the same time, based on the width of the technological development gap between countries.

2. If $\lim_{\xi \to 0}\left(\frac{\theta_m}{v_m}\right) = 0$ and $V\left[\cdot\right]$ is concave,

laggard countries will receive proportionally less funds to invest in innovation related activities as the technological gap widens and would eventually, i.e. in the limit, stop receiving funds, which prevents any innovation and, therefore, convergence process from taking place.

3. If $\lim_{\xi \to 0} \left(\frac{\theta_m}{v_m} \right) > 0$,the RHS of (FF) is higher (lower) than one and $V\left[\cdot\right]$is convex (concave), laggard countries will receive a positive but smaller amount of funds than the innovator country.

4. If $\lim_{\xi \to 0} \left(\frac{\theta_m}{v_m} \right) > 0$,the RHS of (FF) is lower (higher) than one and $V\left[\cdot\right]$ is convex (concave), laggard countries will receive a larger amount of funds than the innovator country.■

Case (1.) defines a purely speculative scenario. That is, even though the probability of a laggard country developing an innovation approaches zero at a relatively fast rate, the value of the assets of its manufacturing firms converges at a slower rate, with the expected value gains derived from an innovation becoming increasingly larger due to the convexity of $V\left[\cdot\right]$. However, if the [financial] value gains derived from an innovation are not sufficiently large, as is the case in (2.), the very same speculative setting leads to a poverty trap. Clearly, for identical θ_i and v_i values, with $i = n, m$, the relative [financial value] gains derived from a laggard's innovation are much smaller in (2.) than in (1.), and funds flow towards safer, i.e. more probable, expected returns. In other words, relatively lower financial value gains weaken the speculative funding motive.

Cases (3.) and (4.) represent intermediate scenarios. In both cases there is room for financial speculation, but the innovation probability of the laggard country does not necessarily decrease at a higher rate than the value of its firms' assets. The main difference between both settings relies on the proportion of funds received by the laggard country: partial divergence occurs if the value of its firms' assets decreases relatively fast during expansionary periods – based on a convex $V\left[\cdot\right]$ – or increases at a relatively fast rate during contractionary ones, i.e. when $V\left[\cdot\right]$ is concave. Thus, even though the speculative funding motive exists, it is not sufficiently strong for funds to be shifted towards the laggard country.

Note, however, that the speculative motive is stronger in (4.), due either to large productivity gains being met by relatively high asset prices in the convex $V\left[\cdot\right]$ framework or to relatively low asset prices being defined when catching up within a concave $V\left[\cdot\right]$ setting. This final scenario could be described as countercyclical, since the economic system tries to equate the asset and productivity values between countries instead of promoting the generation of a new innovation-based cycle. On the other hand, (3.) is clearly more procyclical, with the financial compensation schema not being sufficiently strong to prevent funds from flowing towards the innovator country. In the same way, counter and procyclical financial effects can be identified in cases (1.) and (2.), respectively.

Finally, it should be highlighted that the current analysis could be extended to an environment where one the countries evolves through the concave region of a given technological cycle while the other remains within the convex one, i.e based on a sigmoid $V\left[\cdot\right]$ function with either one or several sequential cycles being considered. In both these cases multiple *financial* equilibria would exist and the possibility of discontinuous jumps taking place within a cycle or among them should be accounted for.

4. EUROPEAN INSTABILITY REDUX

Consider two groups of countries that differ in their corresponding ξ values. Countries within the low technological development group tend to converge due partly to their advantage of backwardness, which leads to additional investments in innovative activities (even though their innovation probabilities do not necessarily increase), and partly because of relative increments in the amount of international funds received to finance innovation. Thus, if the technological gap does not widen excessively – and θ_m and v_m are such that countries belong to either case (1.) or (4.) within Corollary 3.4 –, convergence may eventually occur, even though it does so with an increasingly lower probability as the technological gap widens through time.

The achievements realized by countries in innovation capacity and technological development terms are often analyzed by the literature on national systems of innovation. For example, Filippetti and Archibugi (2011) analyze the main structural characteristics of the highly heterogeneous national systems of innovation of the countries composing the Euro-27 group using data from the European Innovation Scoreboard of 2008. We use data from the Innovation Union Scoreboard of 2011, see European Commission (2011b), to simulate the main implications derived from the current model for the development and innovation processes taking place among various reference countries within the Euro-27 group, with particular emphasis being placed on the Southern European states. The data available from the latest Innovation Union Scoreboard, see European Commission (2014), will be used to verify the capacity of the model to account for the potential financial constraints faced by the countries under analysis. The variables retrieved from the Innovation Union Scoreboard are presented in the first two columns of Tables 2 and 3.

The summary innovation index (SII) variable – calculated by the Innovation Union Scoreboard – will be used as a proxy for the quality and strength of the national innovation system of each country. Our variable ξ constitutes a proxy for such a complex structural concept. This simplification is justified to some extent by the generality of the purely macroeconomic approach under consideration. The

Table 2. Structural parameters and optimal values when $\Gamma = 1.3$ and $\alpha_v v = 1.5$

Country	Summary Innovation Index (SII)	Finance and Support	n_{sn} $V(x) = x^2$	n_{sn} $V(x) = x^{1/2}$	V'(n)/V'(m) $V(x) = x^2$	V'(n)/V'(m) $V(x) = x^{1/2}$
Sweden	0.755	0.895	0.9815	0.0719	-	-
Germany	0.700	0.584	0.9391	0.0297	0.1370	-0.0831
Finland	0.691	0.833	0.9786	0.0775	0.1809	0.0700
EU27	0.539	0.584	0.9417	0.0447	0.1069	-0.0712
Italy	0.441	0.349	0.7376	0.0082	0.0120	-0.0285
Portugal	0.438	0.522	0.9191	0.0392	0.0717	-0.0709
Spain	0.406	0.466	0.8860	0.0278	0.0501	-0.0666
Greece	0.343	0.188	0.1862	0.0001*	-0.0184	-5.4162e-004

**Irrational negative solution that has been changed to its current value for comparability purposes.*

Table 3. Structural parameters and optimal values when $\Gamma = 1.5$ *and* $\alpha_v v = 2$

Country	Summary Innovation Index (SII)	Finance and Support	n_{sn} $V(x) = x^2$	n_{sn} $V(x) = x^{1/2}$	V'(n)/V'(m) $V(x) = x^2$	V'(n)/V'(m) $V(x) = x^{1/2}$
Sweden	0.755	0.895	0.9991	0.4036	-	-
Germany	0.700	0.584	0.9967	0.1626	0.1604	-0.0676
Finland	0.691	0.833	0.9989	0.3839	0.1850	0.0696
EU27	0.539	0.584	0.9967	0.2005	0.1268	-0.0657
Italy	0.441	0.349	0.9822	0.0352	0.0684	-0.0233
Portugal	0.438	0.522	0.9953	0.1622	0.0969	-0.0663
Spain	0.406	0.466	0.9931	0.1147	0.0825	-0.0575
Greece	0.343	0.188	0.8714	0.0025	0.0166	-0.0018

Definition of the indicators: *The calculation of and additional intuition regarding the summary innovation index variable can be found at* http://www.proinno-europe.eu/inno-metrics/page/7-technical-annex-0. *The finance and support variable includes R&D expenditure in the public sector and venture capital. Both definitions and additional information on this variable can be found at* http://www.proinno-europe.eu/sites/default/files/page/12/02/annex_C_1.pdf.

finance and support variable accounts for the financial capacity with which a country is endowed when designing its innovation policy. In the current paper, this concept has been represented through the function $v(\xi)$ and it's respective v_n and v_m variables.

It should be noted that the Innovation Union Scoreboard considers other structural characteristics, such as human capital and further refinements on the composition of national innovation systems, as main codeterminants of the innovation capacity of countries. Including additional sources of friction among countries in the current model does not modify the main results obtained, while shifting attention from the main focus of the paper on the links between the financial capacity and the technological development processes of several countries within the Euro-27 group.

The following numerical values have been exogenously assigned through the simulations to the parameters that we will not be controlling for

$$\begin{aligned} &\delta = 1.2; \\ &A = 1; \\ &K = 1; \\ &\alpha = 0.3; \\ &\phi = 0.2. \end{aligned} \tag{27}$$

It should be emphasized that these numerical values do not affect the main results obtained, and that we are aware of the fact that, for example, different countries exhibit different labor elasticities ϕ and α. In this regard, several numerical modifications could be applied to an extended version of the current model, allowing for some of these differences to be included and explicitly analyzed.

Moreover, the following simplifications have been imposed in the [general] simulations of equations (PP) and (FF) presented in Figures 1 to 4.

(PP) When solving equation (PP) for n_{sn}, given different n_{sm} and ξ values ranging within $[0,1]$, we have assumed that the relation between the variables contained in

$$V\left(\frac{\xi}{\Gamma}, v(\xi)\right)$$

simplifies to $\xi \in [0,1]$. A similar comment applies to the numerator term $v(\xi)$ included within H, which has been assumed equal to ξ. This has been done to provide the most general visual account of the possible equilibrium scenarios. Different sets of values as well as relations between the variables contained in $V(\cdot)$ could have been assumed. However, the resulting effects are equivalent to those following from increments in the value function differentials and illustrated in Figures 1 and 2, where Γ has been shifted from 1.3 to 1.5 and $\alpha_v v$ from 1.5 to 2 within both concave $V(x) = x^{1/2}$ and convex $V(x) = x^2$ value settings [i.e. sections of the technological cycle].

In this regard, note that the optimization results obtained in Sections 2 and 3 are sensitive to the type of value function under consideration, that is, to the section of the technological cycle through which countries evolve. Consequently, we provide two different sets of simulations based on a concave and a convex value function, respectively, within both Figures 1 and 2. Clearly, the incentives to use skilled labor for innovative activities are much higher when moving through the convex section of the cycle. This effect is ameliorated to some extent when the potential gains from innovation, Γ and $\alpha_v v$, increase. In this case, the incentives of those countries going through the concave section of the cycle to dedicate a larger proportion of skilled labor to innovative activities increase, as is clear when Figures 1 and 2 are compared.

Figure 1. (PP)-based n_{sn} values when $\Gamma = 1.3$ and $\alpha_v v = 1.5$

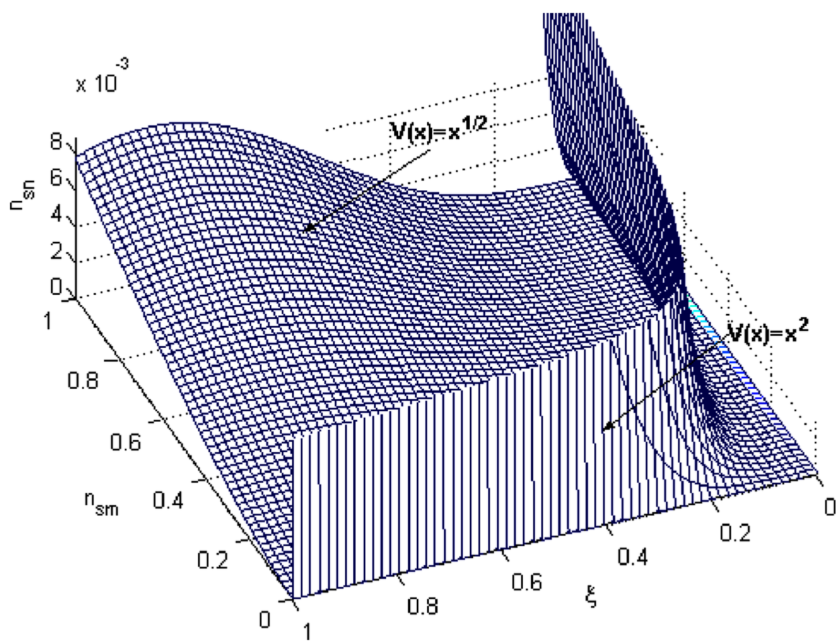
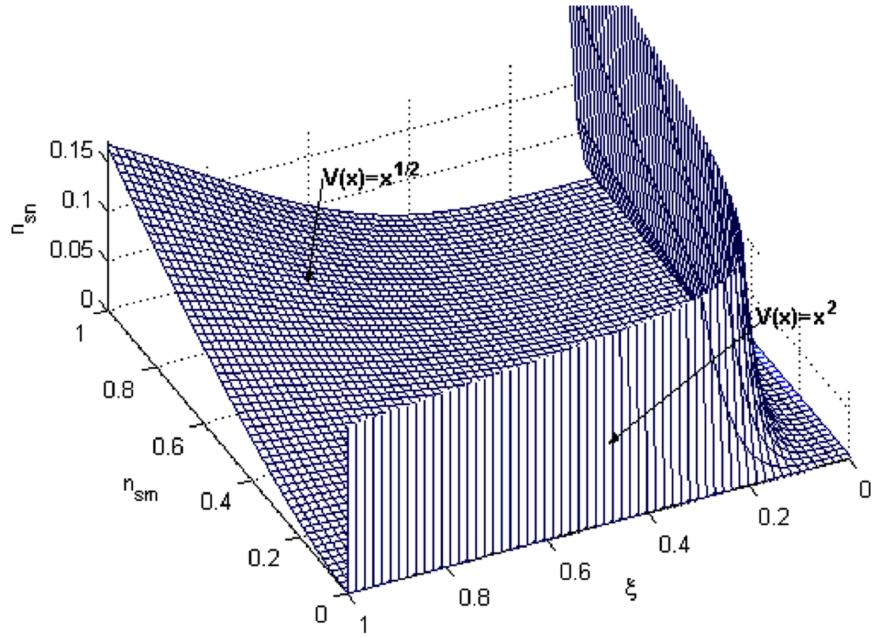


Figure 2. (PP)-based n_{sn} values when $\Gamma = 1.5$ and $\alpha_v v = 2$



Note also that the evolution of the technological development level of a country is not exclusively determined by the variable n_{sn}, but that several other factors affect the behavior of ξ. We do not want to concentrate our analysis on approximating the technological effort of countries through their choice of n_{sn}, but on the financial consequences derived from the existence of a technological gap and their effect on the stability of the Euro zone, given the absence of a strong unified technological policy at the European level.

(FF) When simulating equation (FF) in order to provide a graphical representation of the financial incentives of consumers based on n_{sn} and ξ, see Figures 3 and 4, we have assumed:

$$\theta_m = \xi v_m n_{sn}^{(1-\phi)};\ \theta_n = \xi v_n \overline{n}_{sn}^{(1-\phi)};$$

$$v_m = 0.5;\ v_n = 1;\ \overline{n}_{sn} = 0.5. \tag{28}$$

The v_m and $\overline{n}_{sn}$ numerical values have been chosen as average reference points and such imposition will be relaxed when considering the actual European data. Clearly, the n_{sn} variable defined within θ_m differs from that of the innovator country, with both of them resulting from the simulation that solves the (PP) equation for each respective country. We return to this point below.

The direct and indirect [via (PP)] effects that Γ and $\alpha_v v$ have on (FF) are also visible in Figures 3 and 4, where

$\dfrac{V'\left[\Delta E(va) \mid n\right]}{V'\left[\Delta E(va) \mid m\right]}$, denoted by

$V'(n)/V'(m)$, has been calculated assuming that the subjective expectations of consumers [i.e. the $\mu_i(\theta_i)$, $i = m, n$, variables] coincide with the corresponding θ_i's, with

$i = m, n$. Adding subjectivity (and, consequently, additional speculative incentives) to the valuation of assets is formally allowed for in the model. In this regard, the subjective valuation of R&D projects [through either real options or discounted cash flow methods] and their effect on the value of firms' assets, see Hemantha et al. (1999), could be considered in future extensions of the current paper.

Thus, assuming no subjective asset valuations, we observe how an increase in the productivity and financial gains from innovating increments the LHS of equation (FF) when moving through the convex section of the cycle, i.e. when $V(x) = x^2$. Note that the increment in $V'(n) / V'(m)$ implies that the gain derived from the assets of the innovator country relative to the laggard ones increases with respect to the one obtained in a previous equilibrium. At the same time, improvements in expected innovation payoffs trigger an advantage of backwardness effect in financial terms for the laggards. The fact that the resulting $V'(n) / V'(m)$ values are relatively low and below one illustrates the superiority exhibited by the innovator in financial terms with respect to the laggards.

On the other hand, the concave value setting exposes the inability of the laggards [actually, most countries] to raise sufficient funds to invest in the innovative activities of their respective firms. This result is illustrated numerically in the last column of Tables 1 and 2, where the $V'(n) / V'(m)$ values of the reference innovator country relative to itself have been omitted. The negative sign of the corresponding laggard $V'(n) / V'(m)$ values implies that the innovator must face financial loses [negative gains] in order for consumers to be indifferent between investing in the innovator country or any of the laggards (except Finland). Clearly, differences in the specification of the model and the existence of speculative financial incen-

Figure 3. (FF)-based $\dfrac{V'[\Delta E(va) \mid n]}{V'[\Delta E(va) \mid m]}$ *values when* $\Gamma = 1.3$ *and* $\alpha_v v = 1.5$

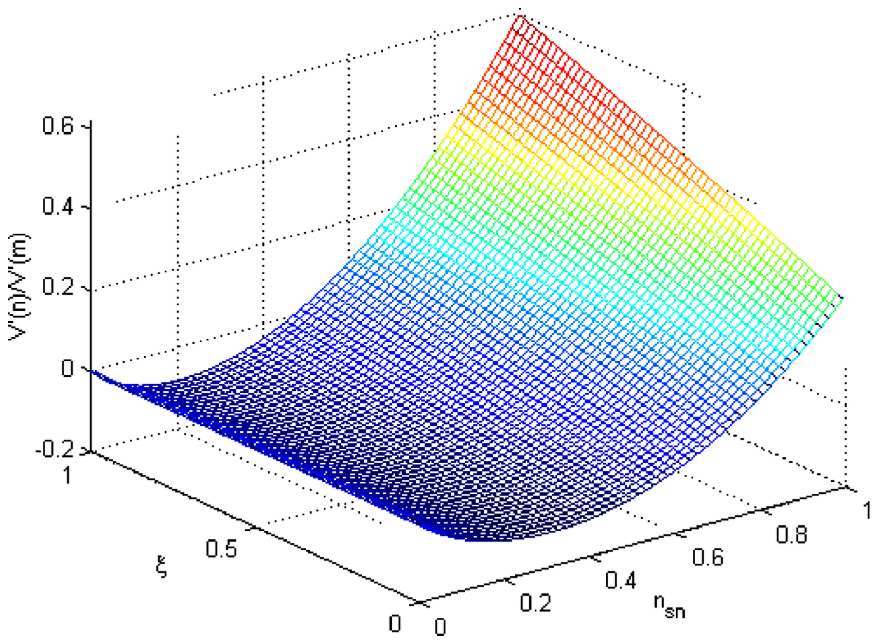
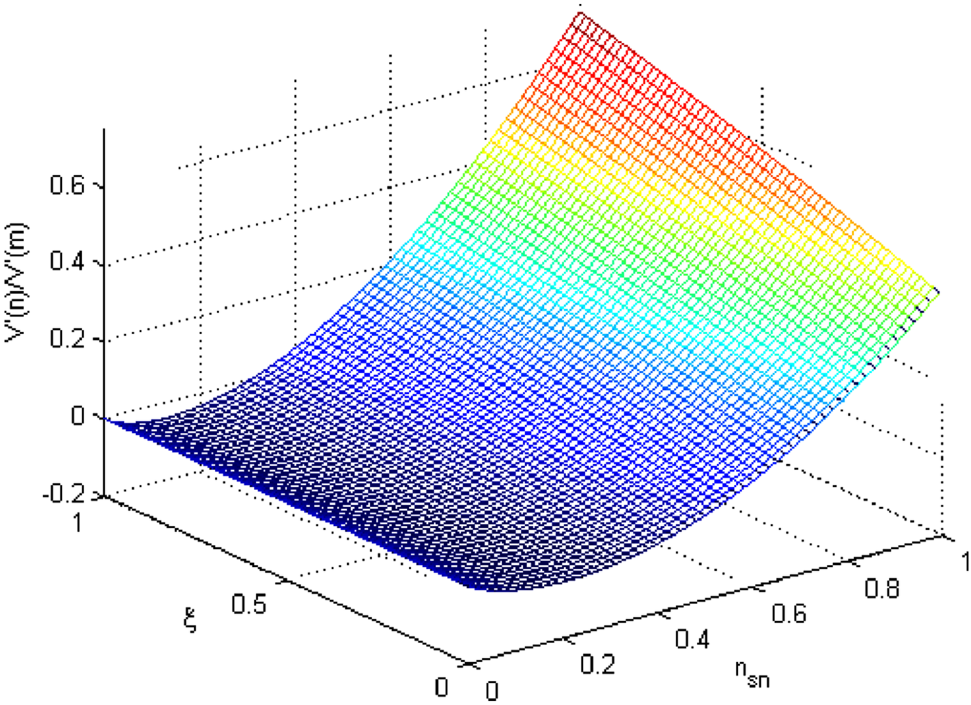


Figure 4. (FF)-based $\frac{V'[\Delta E(va)\mid n]}{V'[\Delta E(va)\mid m]}$ *values when* $\Gamma = 1.5$ *and* $\alpha_v v = 2$



tives could be used to smooth the severity of this effect.

In summary, the expected returns obtained by investors depend on the relative technological development level of the corresponding countries. Figures 3 and 4 have illustrated how an increment in the returns obtained from an innovation both in productivity and asset value terms incentivize the flow of financial capital towards the laggards. The higher numerical values obtained in the convex case result from the higher incentives of both the laggards and the technological leader to innovate. Note, however, that the higher expected returns obtained from an innovation affect also the incentives of the innovator country, leading to a relatively lower investment in the laggards as Γ and $\alpha_v v$ increase. This result is reflected in the higher values obtained for $V'(n) / V'(m)$ within the convex case as Γ and $\alpha_v v$ increase. At the same time, the negative values obtained in the concave case indicate an overinvestment in the technological leader, with Finland, Germany, the EU27 average and Portugal, in this order, receiving a correspondingly lower amount of funds. We will provide additional intuition on this result when analyzing Figure 8.

The simplifications described above have been omitted and the original model followed when applying the data retrieved from the Innovation Union Scoreboard to equations (PP) and (FF). Two remarks must however be considered. First, we have assumed the relation between the variables contained in the value functions within (PP), i.e. $V\left(\cdot,\cdot\right)$, to be multiplicative, emphasizing the existing complementarity between the financial and technological sides of an economic system. Second $v(\xi) = fc / fs$ within both (PP) and (FF), where fc is the value of the finance and support parameter in the laggard country under consideration and fs corresponds to the innovator one, i.e. Sweden. Thus, v_m is determined by fc and fs, while v_n equals one by definition. Trivially, $v(\xi) = v_m$ when referring to the laggard country and v_n in the innovator one.

Given the value of the structural parameters and the equilibrium results obtained from equations (PP) and (FF), we have represented the n_{sn} and $V'(n)/V'(m)$ values achieved by the European countries under consideration in Figures 5 and 6. It should be noted that, in the data-based simulations presented in these figures we have

$$\theta_m = \xi\, v_m\, n_{sn}^{(1-\phi)}\,;\ \theta_n = \xi\, v_n\, n_{sn}^{(1-\phi)}\,. \qquad (29)$$

As already stated, the n_{sn} of the laggard differs from that of the innovator. We have not added the notation required to differentiate between both percentages since it should be intuitively clear that the n_{sn} obtained for Sweden constitutes the reference value against which comparisons are made in Figures 5 and 6, while 0.5 was [exogenously] taken as the corresponding reference value in Figures 3 and 4.

Consequently, the convergence curves observed in Figures 5 and 6 illustrate the equilibrium combinations of n_{sn} and $V'(n)/V'(m)$ for all the countries except Sweden, which constitutes the reference indicator. Note that the labor and financial incentives to invest in innovation activities are higher when Γ and $\alpha_v v$ increase and, obviously, when moving through the convex section of the cycle. The concave one presents a much less appealing scenario, whose economic policy consequences for convergence and stability are described in the policy section of the paper.

Consider now the 2011 data presented in Tables 2 and 3. Clearly, there is a substantial

Figure 5. $\dfrac{V'[\Delta E(va)\,|\,n]}{V'[\Delta E(va)\,|\,m]}$ *and* n_{sn} *when* $V(x) = x^2$, $\Gamma = 1.3$ *and* $\alpha_v v = 1.5$

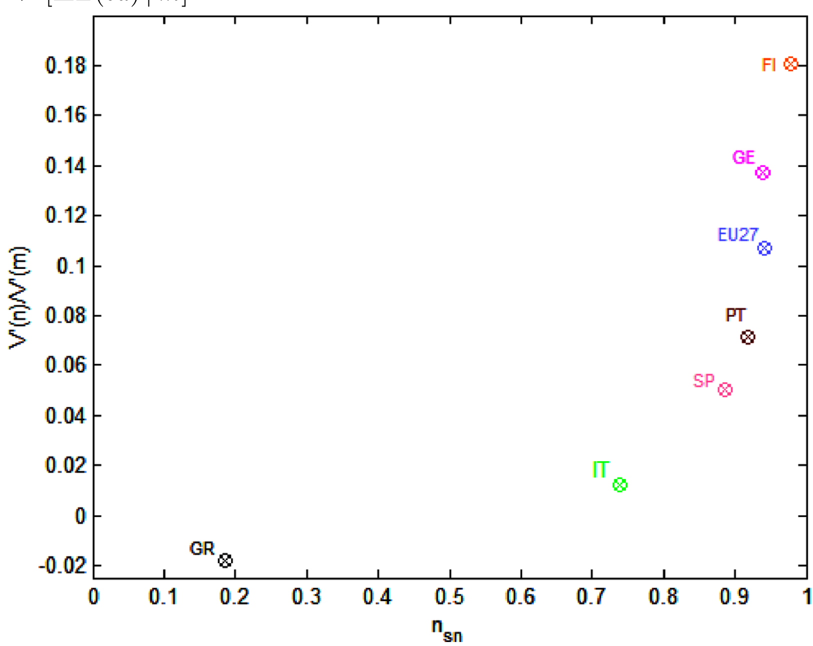
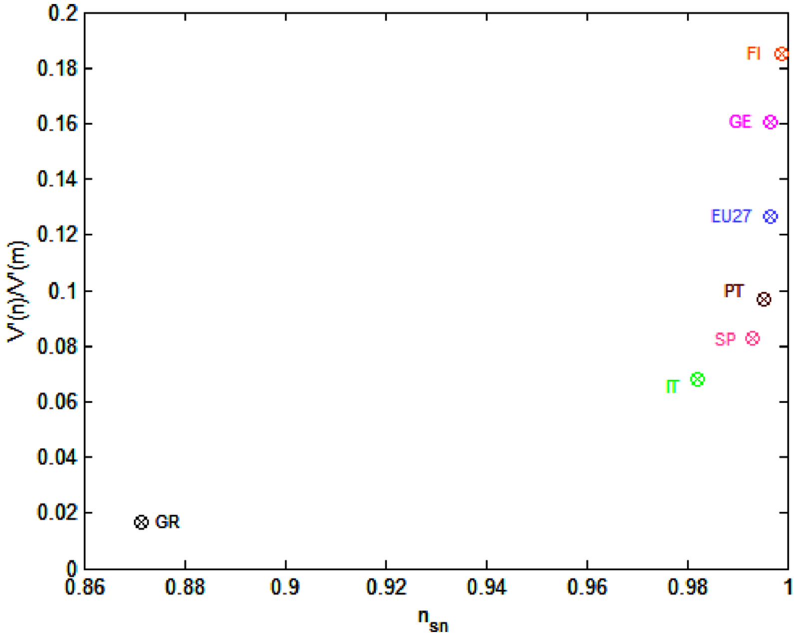


Figure 6. $\frac{V'[\Delta E(va) \mid n]}{V'[\Delta E(va) \mid m]}$ *and* n_{sn} *when* $V(x) = x^2$, $\Gamma = 1.5$ *and* $\alpha_v v = 2$



gap at the structural level between the technologically developed countries and the southern laggards that widens when accounting for the finance and support indicator. Given these values, case (2.) within Corollary 3.4 seems a plausible scenario within which the European Monetary Union could be classified. Moreover, the economic recession faced during this period indicates that the EMU countries should have been moving through the contractionary section of a given technological cycle, with severely damaged financial systems and considerably low innovation probabilities among its weaker members, in particular, Greece and Spain. In this case, the flow of international funds received to finance innovation related activities will tend to decrease progressively through time and convergence will eventually cease as the innovation probability approaches zero. Similarly, case (3.) also constitutes a reasonable scenario, where laggards are endowed with some basic industrial structure but facing serious structural and financial disadvantages with respect to the main innovators of the area, such as, Finland, Germany and Sweden. In this case, the divergent trend is not necessarily reversed after the innovator countries start growing, even if the funds received by the laggards to invest in their industrial structures were to increase their innovation probabilities.

The above intuition is confirmed when the behavior of both the Summary Innovation Index and the 'Finance and support' variables is observed through time. The corresponding data, acquired from the most recent Innovation Union Scoreboard, see European Commission (2014), is presented in Tables 4 and 5 and represented in Figures 7 and 8. In particular, Figure 7 illustrates the evolution of the technological gap (in terms of ξ) between the technologically developed countries and the laggard ones within the EU area. Note how the gap increases between the European Union average (EU27) and the least developed laggards, i.e. Greece and Spain, while remaining almost identical in the case of Italy; only Portugal manages to reduce its gap with respect to the European average, but its technological development still remains at a considerably low level as of 2013.

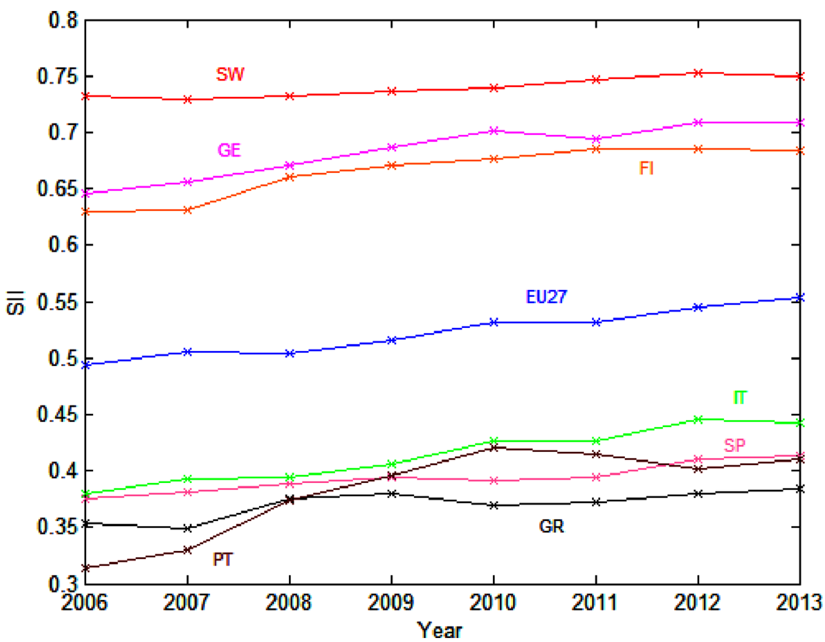
Clearly, this divergent trend observed among the laggards is not only the result of the recent financial crises, since a considerable technological gap existed beforehand. As stated in the introduction, the maintenance and upgrading process of the technological infrastructures of a country requires substantial investments, with technological knowledge accumulating at a lower rate as a given cycle is exhausted. This situation constitutes the

Table 4. Evolution of the Summary Innovation Index (SII)

Country	2006	2007	2008	2009	2010	2011	2012	2013
Sweden	0.732	0.729	0.732	0.737	0.739	0.746	0.752	0.750
Germany	0.646	0.656	0.671	0.687	0.701	0.694	0.708	0.709
Finland	0.630	0.631	0.660	0.670	0.676	0.685	0.685	0.684
EU27	0.493	0.506	0.504	0.516	0.531	0.532	0.545	0.554
Italy	0.380	0.393	0.394	0.406	0.427	0.427	0.446	0.443
Portugal	0.314	0.330	0.374	0.396	0.420	0.415	0.402	0.410
Spain	0.375	0.381	0.389	0.395	0.391	0.395	0.411	0.414
Greece	0.353	0.349	0.375	0.379	0.370	0.372	0.380	0.384

The Summary Innovation Index, together with other variables composing the Innovation Union Scoreboard, can be retrieved from http://ec.europa.eu/enterprise/policies/innovation/policy/innovation-scoreboard/index_en.htm

Figure 7. Evolution of the Summary Innovation Index (SII) among the selected countries



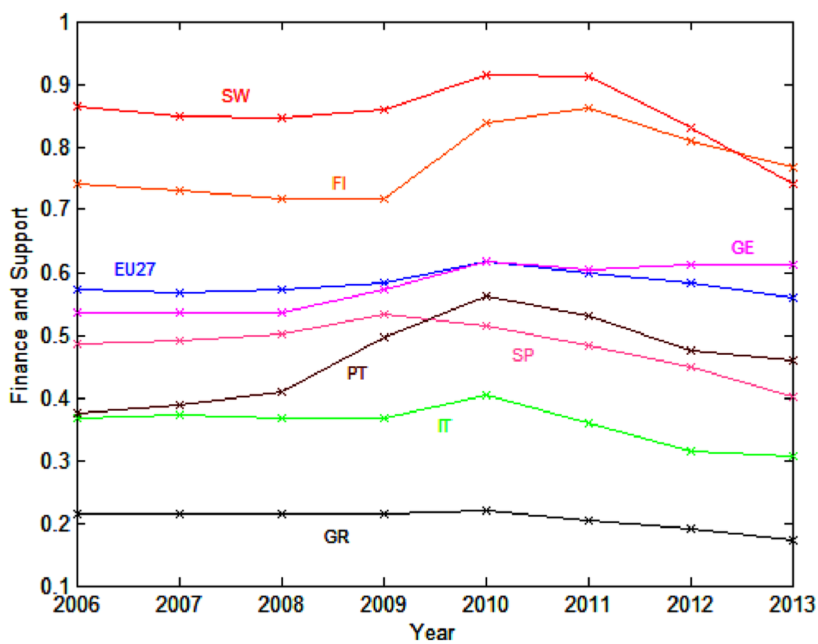
concave case described in our theoretical setting. In this regard, the concave behavior exhibited by the variable ξ is clearly illustrated in Figure 7. At the same time, knowledge accumulates at a faster rate when a new technology is developed and introduced in the market, i.e. through the convex section of a technological cycle. The advantage of backwardness argument builds on these cycle differentials but clashes with the incapacity of laggards to use this knowledge to improve their technological capabilities, a fact that has been illustrated repeatedly in the literature, as indicated in the introduction.

Table 5. Evolution of the finance and support variable

Country	2006	2007	2008	2009	2010	2011	2012	2013
Sweden	0.865	0.850	0.845	0.860	0.914	0.911	0.831	0.741
Germany	0.536	0.536	0.536	0.572	0.618	0.605	0.613	0.613
Finland	0.742	0.731	0.716	0.716	0.838	0.861	0.809	0.767
EU27	0.573	0.568	0.573	0.583	0.617	0.599	0.584	0.558
Italy	0.368	0.373	0.368	0.368	0.405	0.360	0.314	0.306
Portugal	0.374	0.389	0.410	0.497	0.562	0.529	0.476	0.458
Spain	0.486	0.491	0.501	0.532	0.515	0.482	0.448	0.402
Greece	0.215	0.215	0.215	0.215	0.219	0.205	0.191	0.172

This variable, together with those composing the Innovation Union Scoreboard, can be retrieved from http://ec.europa.eu/enterprise/policies/innovation/policy/innovation-scoreboard/index_en.htm

Figure 8. Evolution of the Finance and support variable among the selected countries



The numerical results obtained within the concave scenario for both Γ and $\alpha_v v$ environments indicate that Portugal should receive relatively more funds than the other laggards, due to the higher expected returns realized if developing an innovation, followed by Spain, Italy and Greece. This pattern is confirmed in Figure 8. Note that this financial ranking does not necessarily coincide with the order provided by the corresponding value of the variable ξ in Figure 7. On the side of the technological leaders, Finland should be receiving the highest amount of funds. We can see in Figure 8 how it even overtakes Sweden in terms of financial support. Once again, the financial support received by the countries does not necessarily follow the ranking defined by the variable ξ in Figure 7.

Before proceeding with the policy implications derived from these results, it should be emphasized that we have considered only one period of time, i.e. the year 2011, to generate the numerical results. The dynamic behavior of the main structural variables could be further captured by averaging their values for each country over a given period of time. However, we have preferred to concentrate on the capacity of the model to account for the potential financial constraints that may be faced by different countries when the availability of information is limited.

5. POLICY IMPLICATIONS: ON A TWO-SPEED EUROPE

We are not going to review here the structural requirements guaranteeing the stability of an optimum currency area when absorbing asymmetric shocks to a group of its members, see Mundell (1961). However, contractionary fiscal policies and decrements in the R&D expenditure of the weakest countries do not seem the best options to apply, particularly when considering the implications derived from the literature on national innovation systems when applied to the EMU countries, see Filippetti and Archibugi (2011). In this sense, the current paper provides a word of caution against the contractionary policies being undertaken at the lower end of the European development spectrum. The cumulative and highly persistent nature of technological development implies that R&D austerity programs, which may lead θ_m to converge to zero at a relatively fast rate, could not only worsen the divergence and imbalances among the members of the currency area, but also weaken its stability and long term sustainability.

In particular, the technological and financial implications following from the information assimilation interactions described through the paper are reflected in Figures 7 and 8 for the European countries under consideration. Differences in the development of the information assimilation structures of countries determine the limited capacity of firms to generate further knowledge and finance the required research. Clearly, direct transfers of capital among European countries may be used to prevent purely speculative flows and ameliorate the divergent trend caused by productivity differentials, BBC News (2012), but do not necessarily suffice to close the technological gap between them despite containing its expansion, see OECD (2012). The model highlights the importance of managing technological information flows at both the industry and national levels while considering their potential interconnections among European countries. Otherwise, standard redistributive policies would not suffice per se to counter the expansion of the technological differences among countries while possibly triggering a countercyclical response from the (European) economic system.

We have assumed that the availability of identical technology for all countries integrates them within the same phase of the technological cycle though at different distances. As a result, the financial and production sides of the economy complement each other through the cycle. This assumption may also be interpreted in terms of the intensity of the flow of remanent information received by laggard countries, with lower flows provided through (concave) almost fully developed technological cycles while higher flows originate from (convex) developing cycles. As a result, a countercyclical or a purely speculative motive is required for the system not to destabilize, with countries receiving funds to counter the spread within a technological cycle instead of financing the generation of a new one. Thus, the management of technological information and financial resources must take place at a European level instead of at a national one, which, as our model and Figures 7 and 8 illustrate, may not suffice to stabilize the system.

Otherwise, as our numerical simulations illustrate, laggards will keep on facing stricter constraints when gathering financial resources to develop their national innovation systems. This will be the case even when the expected returns from innovation are relatively high and countries move through an expansionary section of the technological cycle. However, if the technologically developed countries start growing while the laggards remain stuck within a concave section of the cycle, then the latter ones would only be able to grow through direct transfers of capital from the innovator(s), absent any expected compensation in return. At the same time, this type of compensation policy may constitute a severe shock to the incentives of the innovator industries to perform additional innovation activities. The severity of these structural imbalances will be exacerbated when wider development gaps and larger funding requirements arise as the technological differentials among countries increase, a phenomenon recognized – to some extent – by the European Commission (2011c) though with a different expected equilibrium resulting from it.

This result brings back to the policy discussion the main ideas from the information management literature described by Loh et al. (1998) regarding whether technologies must be effectively adapted based on the development level of the countries and whether the incapacity to assimilate and operate with superior technologies constraints the economic development of developing countries (Checchi et al. 2012). Clearly, such a problem persists when the transmission and appropriateness of knowledge takes place among developed nations located at different (technological) stages, with laggards suffering the increasing obsolescence of their capital, both human and technological.

6. CONCLUSION AND EXTENSIONS

The theoretical environment described in the paper has been designed to create a convergence-prone structure that ameliorates the increments in the technological and financial gaps generated between the innovator country and the laggard ones. In order to do so, we have imposed several simplifying assumptions. The main ones were defined to control the extension of the gap and generate incentives supporting technological investments within the laggard countries. These assumptions were given by the immediate convergence of the technological development levels and asset values of firms after an innovation takes place within a country, independently of its previous level of technological development. These and other complementary assumptions could be relaxed in future versions of the current model aimed at extending it and providing a larger set of potential results and evaluations. Some of these extensions are presented below.

1. Several frictions that would worsen the divergence results obtained have been assumed not to affect the dynamic behaviour of the model. In this case, extension of the current setting should consider
    a. Relaxing the immediate convergence in technological development levels and asset values of firms, with the increments in the values of ξ and $\alpha_v v$ being determined by the level of (technological) development of the country where the innovation is generated. Note that this modification would decrease the incentives to promote innovative activities among laggard countries.

b. Including the pecuniary and strategic costs in the form of patents and path dependence that must be incurred when acquiring a more developed technology that must be adapted to the local production (and innovation) system. Moreover, imported technology is not generally available to be immediately implemented among laggard countries or their firms and adaptation frictions following from the corresponding learning curves should also be considered.

2. The (partial) equalizing mechanism provided by migration should also be studied. As stated through the paper, migration constitutes an important mechanism allowing for an efficient reallocation of human capital resources among countries. In addition, it also decreases unemployment-related pressures over the technological laggards both from a social and a suboptimal income allocation perspective.
3. A considerable simplification imposed in order to highlight the macro and financial perspectives of the model is the existence of a unique industrial sector per country. The model should be extended so as to allow for multiple industrial sectors and the resulting potential specialization among countries based on both their technological development levels and those of the respective industries. The corresponding innovation probabilities should be modified accordingly and considered both on a country and industry basis.
4. Building on the previous point, the strategic management of technological knowledge in different industrial sectors treated as a compensating device for the stagnation of others should also be analyzed. This opens the way to consider full versus partial catching up processes in technological infrastructures between differently developed countries and the resulting specialization and knowledge spillovers taking place among complementary industrial sectors.
5. Heterogeneity among the consumer bases of different countries should also be accounted for. In this case, it may be assumed that consumers from underdeveloped countries have a lower income endowment than those from technologically developed ones. Analyzing the existence of (local) substitute product markets and their effect on the economic system of the less developed countries should follow.
6. Finally, the financial investors operating in the international economy should be differentiated from the consumers located within a given country. Moreover, noisy subjective valuations should substitute the correct expectations assumed when defining the expected financial wealth of investors. That is, subjectively distorted beliefs should be allowed for and the resulting speculative movements following from the set of international financial investors studied.

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ENDNOTES

1 Their sample of countries consists of Australia, Austria, Canada, Denmark, Finland, France, Germany, Italy, Japan, New Zealand, Netherlands, Norway, Spain, Sweden, Switzerland, the United Kingdom and the United States, and covers the 1973 to 1995 period.

Debora Di Caprio completed her Masters and PhD in Pure Mathematics at York University, Toronto, Canada. Her Undergraduate studies were in Mathematics at the Seconda Università degli studi di Napoli, Italy, where she has also been a Researcher and was awarded the title of Subject Expert in Geometry at the Dipartimento di Matematica. She holds a tenure teaching position at the Polo Tecnologico I.I.S.S. G. Galilei, Bolzano, Italy. She is a Visiting Researcher at the Department of Mathematics and Statistics of York University, Canada, and a Research Member in the ICEI and Grinei Groups in Economics and Innovation Policy at the Universidad Complutense de Madrid, Spain. Her research interests focus on set-theoretical topology, hyperspace topologies, utility and choice theory, information and uncertainty. She has published more than thirty research papers in academic journals such as Information Sciences, Applied Mathematics and Computation, Benchmarking: An International Journal, Business & Information Systems Engineering, Applied Mathematical Modelling, Technological and Economic Development of Economy, Journal of Mathematical Economics, International Journal of Operational Research, International Journal of Strategic Decision Sciences, Canadian Applied Mathematics Quarterly, International Journal of Applied Decision Sciences, Topology and its Applications, Acta Mathematica Hungarica.

Francisco J. Santos-Arteaga is a researcher in the ICEI and Grinei Groups in Economics and Innovation Policy at the Universidad Complutense de Madrid, Spain. He completed a PhD in Mathematical Economics at York University, Toronto, Canada, from where he also graduated with a Masters degree in Economics. He also completed a PhD in Economics of Innovation at the Universidad Complutense de Madrid. Prior to this, he completed his Undergraduate studies in Economics at the Universidad de Salamanca, Spain. He was awarded grants from the Spanish Education and Culture Ministry and the European Union and also the Dean's Academic Excellence Award from York University. His research interests include systemic risk, choice and information theory. He has published more than thirty research papers in academic journals such as Information Sciences, Applied Mathematics and Computation, Benchmarking: An International Journal, Business & Information Systems Engineering, Applied Mathematical Modelling, Technological and Economic Development of Economy, Journal of Mathematical Economics, International Journal of Operational Research, International Journal of Strategic Decision Sciences, Canadian Applied Mathematics Quarterly, International Journal of Applied Decision Sciences, The Manchester School, Structural Change and Economic Dynamics.

Madjid Tavana is a professor of Business Systems and Analytics and the Lindback Distinguished Chair of Information Systems and Decision Sciences at La Salle University, where he served as Chairman of the Management Department and Director of the Center for Technology and Management. He is a Distinguished Research Fellow at Kennedy Space Center, Johnson Space Center, Naval Research Laboratory at Stennis Space Center, and Air Force Research Laboratory. He was recently honored with the prestigious Space Act Award by NASA. He holds a MBA, PMIS, and PhD in Management Information Systems and received his Post-Doctoral Diploma in Strategic Information Systems from the Wharton School at the University of Pennsylvania. He is the Editor-in-Chief of Decision Analytics, International Journal of Applied Decision Sciences, International Journal of Management and Decision Making, International Journal of Strategic Decision Sciences, and International Journal of Enterprise Information Systems. He has published several books and over one hundred and fifty research papers in academic journals such as Information Sciences, Decision Sciences, Information Systems, Interfaces, Annals of Operations Research, Advances in Space Research, Omega, Information and Management, Knowledge-Based Systems, International Journal of Production Research, Expert Systems with Applications, European Journal of Operational Research, Journal of the Operational Research Society, Computers and Operations Research, Energy Economics, Applied Soft Computing, and Energy Policy.

APPENDIX A

Deriving Ψ_{sm}

The derivation of Ψ_{sm} follows from the imitation model of Iwai (2000), and the interested reader is referred to section 2.2 in Iwai's paper or any text in ordinary differential equations, i.e. Braun (1983), for additional details on logistic growth functions.

Consider the proportion of unskilled workers in a given economy, n_d. The probability that one of these workers is able to acquire the required knowledge to become skilled per unit of time is $\mu\left(\xi\right)Nn_s dt$. The proportion of skilled workers in the economy increases whenever any of the unskilled ones, whose proportion is $\left(1-n_s\right)$, acquires the required knowledge. Given the previous probability, the expected increase in the proportion of skilled worker per unit of time is $\mu\left(\xi\right)Nn_s dt\left(1-n_s\right)$. Assuming a sufficiently large number of workers allows for the application of the law of large numbers so as to obtain the following approximation to the rate of change in n_s:

$$\dot{n}_s = \mu(\xi)Nn_s(1-n_s) \tag{30}$$

which is a logistic differential equation of the form:

$$\dot{n}_s = an_s(1-n_s)$$

whose solution is equal to [see section 1.5 in Braun (1983)]:

$$\Psi_{sm} = \frac{an_s}{an_s + (a - an_s)e^{-a(t-T)}}, \qquad t \geq T \tag{31}$$

Letting $a = \mu(\xi)N$ and dividing the entire fraction by n_s we obtain the required result

$$\Psi_{sm} = \left(1 + \left(\frac{1}{n_s} - 1\right)e^{-\mu(\xi)N(t-T)}\right)^{-1}, \qquad t \geq T \tag{32}$$

It should be noted that the first order conditions resulting from the optimization problem of a given country are greatly simplified by the assumption imposed when defining Ψ_{sm}, such that both types of skilled workers may increase the proportion of unskilled ones who become skilled. Differences in the optimal allocation of skilled labor would arise if the evolution of Ψ_{sm} would only be determined by either n_{sm} or n_{sn}.

APPENDIX B

Bellman Equations

B.1. Basic Theory

We build on the paper of Wälde (1999) in order to introduce a set of Poisson processes within the stochastic framework of the model. Wälde employs the following version of Ito's lemma. Let $\mathbf{z} \equiv (z_1, z_2)^T$ be a vector-valued Poisson process consisting of two independent Poisson processes, z_1 and z_2. Let $f(\mathbf{x}) \equiv (f_1(\mathbf{x}), f_2(\mathbf{x}))^T$, $g(\mathbf{x})$ and $\sigma(\mathbf{x}) \equiv (\sigma_1(\mathbf{x}), \sigma_2(\mathbf{x}))^T$ be continuous real functions of $\mathbf{x} \equiv (x_1, x_2)$. Note that $f_i, g, \sigma_i : \Re^2 \to \Re$.

Let $\mathbf{x}$ follow $d\mathbf{x} = f(\mathbf{x})dt + \sigma(\mathbf{x})d\mathbf{z}$, then the differential $dg(\mathbf{x})$ equals:

$$dg(\mathbf{x}) = [g_{x_1}(\mathbf{x})f_1(\mathbf{x}) + g_{x_2}(\mathbf{x})f_2(\mathbf{x})]dt + \left[g\left(x_1 + \sigma_1\left(\mathbf{x}\right), x_2\right) - g\left(\mathbf{x}\right)\right]dz_1 + \left[g\left(x_1, x_2 + \sigma_2\left(\mathbf{x}\right)\right) - g\left(\mathbf{x}\right)\right]dz_2 \tag{33}$$

If a unique Poisson process defines the stochastic evolution of $g(\mathbf{x})$, i.e. $dz_1 = dz_2 = dz$, the differential $dg(\mathbf{x})$ becomes

$$dg(\mathbf{x}) =$$
$$[g_{x_1}(\mathbf{x})f_1(\mathbf{x}) + g_{x_2}(\mathbf{x})f_2(\mathbf{x})]\,dt +$$
$$[g(x_1 + \sigma_1(\mathbf{x}), x_2 + \sigma_2(\mathbf{x})) - g(\mathbf{x})]\,dz \tag{34}$$

At the same time, the differential generator $Diff$ can be applied to $dg(\mathbf{x})$.

$$Diff\; g(\mathbf{x}) =$$
$$g_{x_1}(\mathbf{x})f_1(\mathbf{x}) + g_{x_2}(\mathbf{x})f_2(\mathbf{x}) +$$
$$[g(x_1 + \sigma_1(\mathbf{x}), x_2) - g(\mathbf{x})]\,a_1 +$$
$$[g(x_1, x_2 + \sigma_2(\mathbf{x})) - g(\mathbf{x})]\,a_2$$

where $a_i\,dt$, $i = 1,2$, denotes the probability per unit of time that x_i jumps with an amplitude of $\sigma_i(\mathbf{x})$, while $Diff\; g(\mathbf{x})$ represents the expected change of $g(\mathbf{x})$ per unit of time.

If a unique Poisson process is assumed and the differential generator $Diff$ is applied to $g(\mathbf{x})$ we obtain

$$Diff\; g(\mathbf{x}) =$$
$$g_{x_1}(\mathbf{x})f_1(\mathbf{x}) + g_{x_2}(\mathbf{x})f_2(\mathbf{x}) +$$
$$[g(x_1 + \sigma_1(\mathbf{x}), x_2 + \sigma_2(\mathbf{x})) - g(\mathbf{x})]\,a_1. \tag{35}$$

B.2. Countries

Consider the unique Poisson process (per country) triggering the increments in the productivity and asset value of the firms located within the country where the next innovation is developed:

$$d\left(\frac{\xi}{\Gamma}\right) = \left(\Gamma - \frac{\xi}{\Gamma}\right) dz_\xi \tag{36}$$

$$d\,v\left(\xi\right) = [\alpha_v v - v(\xi)]\,dz_\xi \tag{37}$$

Note that we should have differentiated between firms (and, indeed, between countries) when defining the corresponding intertemporal optimization problem of each country, in a similar way as we did in López et al. (2011). However, the additional notational requirements would not modify or add any intuition to the main results obtained and the simplest possible approach has therefore been followed.

Replace the corresponding variables in equation (B.4) according to

$$\mathbf{x} = \left(\frac{\xi}{\Gamma}, v(\xi)\right),\; f_1(\mathbf{x}) = f_2(\mathbf{x}) = 0$$

$$g(\mathbf{x}) = V(\mathbf{x})$$

$$\sigma_1(\mathbf{x}) = \left(\Gamma - \frac{\xi}{\Gamma}\right)$$

$\sigma_2(\mathbf{x}) = \alpha_v v - v(\xi)$ and $a_1 = \theta_\xi$ in order to obtain:

$$E\left(\frac{dV\left(\frac{\xi}{\Gamma}, v(\xi)\right)}{dt}\right) =$$

$$\theta_\xi \left[V\left(\frac{\xi}{\Gamma} + \left(\Gamma - \frac{\xi}{\Gamma}\right),\ v(\xi) + \alpha_v v - v(\xi) \right) - V\left(\frac{\xi}{\Gamma}, v(\xi)\right) \right] \tag{38}$$

which simplifies to

$$E\left(\frac{dV\left(\frac{\xi}{\Gamma}, v(\xi)\right)}{dt}\right) =$$

$$\theta_\xi \left[V\left(\Gamma, \alpha_v v\right) - V\left(\frac{\xi}{\Gamma}, v(\xi)\right) \right] \tag{39}$$

Finally, include the expected temporal evolution of the value function in the Bellman equation corresponding to this type of stochastic optimization problems (refer to any standard optimization text, i.e. Kamien and Schwartz (1981), Section 21), which is given by

$$\rho V\left(\frac{\xi}{\Gamma}, v(\xi)\right) =$$

$$\max_{n_{sn},n_{sm},n_u}\left[\begin{array}{l}\pi(n_{sm},n_u)+\\ E\left(\dfrac{dV\left(\dfrac{\xi}{\Gamma},v(\xi)\right)}{dt}\right)\end{array}\right] \tag{40}$$

to obtain the final Bellman equation optimized by each country based on their respective values of ξ :

$$\rho V\left(\frac{\xi}{\Gamma},v(\xi)\right)=$$

$$\max_{n_{sn},n_{sm},n_u}\left[\begin{array}{l}\pi(n_{sm},n_u)+\\ \theta_{\xi}\left[V\left(\Gamma,\alpha_v v\right)-V\left(\dfrac{\xi}{\Gamma},v(\xi)\right)\right]\end{array}\right] \tag{41}$$

B.3. Consumers

Consider the stochastic evolution of the asset value of firms, which defines the budget constraint of consumers. The value of the assets purchased from a manufacturing firm located within the country hosting the current innovation, $v_n a_n^m$, evolves through time according to:

$$\begin{array}{l}d(v_n a_n^m)=\\ \left[w+va_n^n+\dfrac{va_{m|n}^m}{2}-v_n a_n^m-v_m a_m^m-e\right]dt+\\ \left[\alpha_v va_n^m-v_n a_n^m\right]dz_n\end{array} \tag{42}$$

while the assets purchased from a manufacturer located within the laggard country, $\dfrac{v_m a_m^m}{2}$, follow:

$$\begin{array}{l}d\left(\dfrac{v_m a_m^m}{2}\right)=\\ \left[w+va_n^n+\dfrac{va_{m|n}^m}{2}-v_n a_n^m-v_m a_m^m-e\right]dt+\\ \left[\dfrac{\alpha_v va_m^m}{2}-\dfrac{v_m a_m^m}{2}\right]dz_m\end{array} \tag{43}$$

We are interested in the evolution of the expected budget constraint subjectively defined by consumers per unit of time:

$$E[va] = \mu_n(\theta_n)v_n a_n^m + \mu_m(\theta_m)\frac{v_m a_m^m}{2} \tag{44}$$

The differential $dg(\mathbf{x})$ given in equation (33) determines the stochastic evolution of $E[va]$ once the corresponding variables are replaced according to:

$$\mathbf{x} = \left(v_n a_n^m, \frac{v_m a_m^m}{2} \right)$$

$$f_1(\mathbf{x}) = f_2(\mathbf{x}) =$$

$$\left[w + va_n^n + \frac{va_{m|n}^m}{2} - v_n a_n^m - v_m a_m^m - e \right]$$

$$g(\mathbf{x}) =$$

$$\mu_n(\theta_n)v_n a_n^m + \mu_m(\theta_m)\frac{v_m a_m^m}{2} \tag{45}$$

$$\sigma_1(\mathbf{x}) = \left[\alpha_v va_n^m - v_n a_n^m \right]$$

$$\sigma_2(\mathbf{x}) = \left[\frac{\alpha_v va_m^m}{2} - \frac{v_m a_m^m}{2} \right]$$

$$dz_1 = dz_n$$

$$dz_2 = dz_m$$

The evolution of the expected budget constraint subjectively defined by consumers per unit of time reads:

$$d\left(\mu_n(\theta_n)v_n a_n^m + \mu_m(\theta_m)\frac{v_m a_m^m}{2} \right) =$$

$$\left[\begin{array}{l} w + va_n^n + \dfrac{va_{m|n}^m}{2} - \\ v_n a_n^m - v_m a_m^m - e \end{array} \right] dt +$$

$$\left[\begin{pmatrix}\mu_n(\theta_n)\left(v_n a_n^m + \left[\alpha_v v a_n^m - v_n a_n^m\right]\right) + \\ \mu_m(\theta_m)\dfrac{v_m a_m^m}{2}\end{pmatrix} - \right. \tag{46}$$

$$\left.\left(\mu_n(\theta_n) v_n a_n^m + \mu_m(\theta_m)\frac{v_m a_m^m}{2}\right)\right] dz_n +$$

$$\left[\begin{pmatrix}\mu_n(\theta_n) v_n a_n^m + \\ \mu_m(\theta_m)\left(\dfrac{v_m a_m^m}{2} + \left[\dfrac{\alpha_v v a_m^m}{2} - \dfrac{v_m a_m^m}{2}\right]\right)\end{pmatrix} - \right.$$

$$\left.\left(\mu_n(\theta_n) v_n a_n^m + \mu_m(\theta_m)\frac{v_m a_m^m}{2}\right)\right] dz_m$$

which simplifies to

$$d\left(\mu_n(\theta_n) v_n a_n^m + \mu_m(\theta_m)\frac{v_m a_m^m}{2}\right) =$$

$$\begin{bmatrix} w + v a_n^n + \dfrac{v a_{m|n}^m}{2} - \\ v_n a_n^m - v_m a_m^m - e \end{bmatrix} dt + \tag{47}$$

$$\left[\mu_n(\theta_n)\left[\alpha_v v a_n^m - v_n a_n^m\right]\right] dz_n +$$

$$\left[\mu_m(\theta_m)\left(\frac{\alpha_v v a_m^m}{2} - \frac{v_m a_m^m}{2}\right)\right] dz_m,$$

We use equation (47) and the differential generator given in equation (35) to obtain the Bellman equation defining the optimization problem of a consumer, which is based on the stochastic evolution of the innovation processes that determine the (subjectively defined) expected budget constraint of each consumer. Substituting the corresponding variables according to:

$$\mathbf{x} = E(va)$$

$$f_1(\mathbf{x}) = \begin{bmatrix} w + v a_n^n + \dfrac{v a_{m|n}^m}{2} - \\ v_n a_n^m - v_m a_m^m - e \end{bmatrix}$$

$$f_2(\mathbf{x}) = 0$$
$$g(\mathbf{x}) = V(E(va))$$

$$\sigma_1(\mathbf{x}) = \left[\mu_n(\theta_n)\left[\alpha_v va_n^m - v_n a_n^m\right]\right] \tag{48}$$

$$\sigma_2(\mathbf{x}) =$$
$$\left[\mu_m(\theta_m)\left(\frac{\alpha_v va_m^m}{2} - \frac{v_m a_m^m}{2}\right)\right]$$
$$a_1 = \theta_n$$
$$a_2 = \theta_m$$

leads to:

$$E\left(\frac{dV\left(E(va)\right)}{dt}\right) =$$
$$V_{E(va)}[E(va)]\begin{bmatrix} w + va_n^n + \\ \dfrac{va_{m|n}^m}{2} - v_n a_n^m - \\ v_m a_m^m - e \end{bmatrix} +$$

$$\theta_n\begin{bmatrix} V\begin{bmatrix} E(va) + \\ \mu_n(\theta_n)\left[\alpha_v va_n^m - v_n a_n^m\right] \end{bmatrix} - \\ V[E(va)] \end{bmatrix} + \tag{50}$$

$$\theta_m\begin{bmatrix} V\begin{bmatrix} E(va) + \\ \mu_m(\theta_m)\left[\dfrac{\alpha_v va_m^m}{2} - \dfrac{v_m a_m^m}{2}\right] \end{bmatrix} - \\ V[E(va)] \end{bmatrix}$$

which, at the same time, can be written as:

$$E\left(\frac{dV\left(E(va)\right)}{dt}\right) =$$

$$V_{E(va)}[E(va)]\begin{bmatrix} w + va_{n}^{n} + \dfrac{va_{m|n}^{m}}{2} - \\ v_{n}a_{n}^{m} - v_{m}a_{m}^{m} - e \end{bmatrix} +$$

$$\theta_{n}\begin{bmatrix} V\begin{bmatrix} \mu_{n}(\theta_{n})\begin{bmatrix} v_{n}a_{n}^{m} + \alpha_{v}va_{n}^{m} - \\ v_{n}a_{n}^{m} \end{bmatrix} + \\ \mu_{m}(\theta_{m})\dfrac{v_{m}a_{m}^{m}}{2} \end{bmatrix} - \\ V\left[\mu_{n}(\theta_{n})v_{n}a_{n}^{m} + \mu_{m}(\theta_{m})\dfrac{v_{m}a_{m}^{m}}{2}\right] \end{bmatrix} + \tag{51}$$

$$\theta_{m}\begin{bmatrix} V\begin{bmatrix} \mu_{n}(\theta_{n})v_{n}a_{n}^{m} + \\ \mu_{m}(\theta_{m})\begin{bmatrix} \dfrac{v_{m}a_{m}^{m}}{2} + \dfrac{\alpha_{v}va_{m}^{m}}{2} - \\ \dfrac{v_{m}a_{m}^{m}}{2} \end{bmatrix} \end{bmatrix} - \\ V\left[\mu_{n}(\theta_{n})v_{n}a_{n}^{m} + \mu_{m}(\theta_{m})\dfrac{v_{m}a_{m}^{m}}{2}\right] \end{bmatrix}$$

Finally, we substitute the previous expression in the Bellman equation optimized by consumers based on their (subjectively defined) *expected* state of the economic system:

$$\rho V\left(E(va)\right) = \max_{e, a_n^m, a_m^m}\left[u(c) + E\left(\frac{dV\left(E(va)\right)}{dt}\right)\right]$$

where:

$$E\left(\frac{dV\left[E(va)\right]}{dt}\right) = V_{E(va)}[E(va)]\begin{bmatrix} w + va_n^n + \dfrac{va_{m|n}^m}{2} - \\ v_n a_n^m - v_m a_m^m - e \end{bmatrix} +$$

$$\theta_n\begin{bmatrix} V\begin{bmatrix} \mu_n(\theta_n)\left(\alpha_v va_n^m\right) + \\ \mu_m(\theta_m)\dfrac{v_m a_m^m}{2} \end{bmatrix} - \\ V\left[\mu_n(\theta_n) v_n a_n^m + \mu_m(\theta_m)\dfrac{v_m a_m^m}{2}\right] \end{bmatrix} + \tag{52}$$

$$\theta_m\begin{bmatrix} V\begin{bmatrix} \mu_n(\theta_n) v_n a_n^m + \\ \mu_m(\theta_m)\left(\dfrac{\alpha_v va_m^m}{2}\right) \end{bmatrix} - \\ V\left[\mu_n(\theta_n) v_n a_n^m + \mu_m(\theta_m)\dfrac{v_m a_m^m}{2}\right] \end{bmatrix}$$