# Joint Switched Transmit Diversity and Adaptive Modulation in Spectrum Sharing Systems ${ }^{\star}$ 

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#### Abstract

Under the scenario of an underlay cognitive radio network, we propose in this paper an adaptive scheme using switched transmit diversity and adaptive modulation in order to minimize the average number of switched branches at the secondary transmitter while increasing the capacity of the secondary link. The proposed switching efficient scheme (SES) uses the scan and wait (SWC) combining technique where a transmission occurs only when a branch with an acceptable performance is found, otherwise data is buffered. In our scheme, the modulation constellation size and the used transmit branch are determined to achieve the highest spectral efficiency with a minimum processing power, given the fading channel conditions, the required error rate performance, and a peak interference constraint to the primary receiver. Selected numerical examples show that the SES scheme minimizes the average number of switched branches for the average and the high secondary signal-to-noise ratio range. This improvement comes at the expense of a small delay introduced by the SWC technique. For reference, we also compare the performance of the SES scheme to the selection diversity scheme (SDS) where the best branch verifying the modulation mode and the interference constraint is always selected.


Index Terms-Switched Diversity, Adaptive Modulation, Spectrum Sharing, and Performance Analysis.

## I. Introduction

The limited spectrum resource gave the concept of cognitive radio a higher importance. First introduced by Mitola and Maguire [1], the basic idea of cognitive radio is to allow a primary (licensed) and a secondary (unlicensed) users coexist in the same frequency spectrum. In underlay cognitive radio systems, called also spectrum sharing systems, the primary and the secondary users can transmit simultaneously as long as the interference of the secondary to the primary link stays below a predetermined threshold. In our paper, we use adaptive modulation and switched transmit diversity in an underlay cognitive radio scenario in order to efficiently use the spectrum.

Based on multiple thresholds, adaptive modulation [2], [3] can achieve high spectral efficiency over wireless channels. The key idea of adaptive modulation is to adapt the modulation parameters, such as constellation size, to fading

[^0]channel conditions while respecting the bit error rate (BER) requirements. Adaptive modulation increases the performance of the secondary link especially when the secondary channel gain is higher than that of the interference link. The second class of adaptive techniques that we use in our paper is transmit diversity. In particular, switched combining techniques improve the reliability of wireless fading channels by adapting the combiner structure to fading channel conditions and are very useful in mitigating the deleterious effect of fading [4]. In underlay cognitive radio, the performance of the secondary link can be badly affected because of the interference constraint to the primary user. In such scenarios transmit diversity techniques are very helpful in improving the performance of the secondary link while respecting the interference constraint to the primary user [5]. Dual-branch switch and stay (SSC) is one of these switching combining techniques that received a great deal of attention [6]-[9]. In this technique the current branch is used as long as the signal-to-noise ratio (SNR) is above the predetermined threshold, otherwise the transmitter switches and uses the second branch. Switch and examine (SEC) has been proposed as alternative of SSC to take advantage of additional diversity branches [10]. The scan and wait (SWC) technique was then proposed in [11] in order to improve the performance of SEC and other traditional combining techniques at the expense of some time delay.
In our paper, we take advantage of the performance of SWC in order to increase the capacity of the secondary link. We also choose the smallest switching threshold of the adaptive modulation technique as a combining threshold for SWC in order to minimize the average number of switched branches while satisfying the BER requirements. We analyze the SES scheme in terms of its spectral efficiency, the average number of switched branches, the BER, and the delay performance for several interference constraint levels and different number of available branches.
The remainder of the paper is organized as follows. Section II presents first the system and channel models then gives the details behind the adaptive transmission system and the mode of operation of the proposed scheme. While section III analyzes the performance of the SES scheme, section IV offers some selected numerical examples illustrating this performance. Finally, section V concludes the paper.

## II. Models and Mode of Operation

## A. System and Channel Models

We consider the underlay cognitive system model shown in Fig. 1. A secondary user is allowed to share the spectrum


Fig. 1. System model.
with a primary link as long as an interference constraint to the primary receiver (PR) with a peak value $Q$ is respected. Assuming that the secondary link is not affected by the primary transmission, we aim to increase the spectral efficiency of the secondary link by applying adaptive modulation. Equivalently to the down link of a cellular system, the secondary transmitted (ST) is equipped with $L$ antennas and the secondary receiver (SR) has a single antenna. Using SWC, the ST cyclically switches between the $L$ antennas in order to find the antenna that both verifies the interference constraint and reaches the required modulation threshold. Whenever the interference constraint is satisfied, the PR sends a binary ACK to the ST through a reliable feedback channel, otherwise it sends a NACK. We also assume that there is a reliable feedback channel between the SR and the ST. This channel is implemented in a discrete-time fashion, more specifically, short guard periods are periodically inserted into the transmitted signal. During these guard periods, the SR selects a suitable antenna and the signal constellation to be used for transmission throughout the subsequent data burst. Under the assumption of frequency flat fading, we use a block-fading model assuming that different antennas experience roughly the same fading conditions (or equivalently the same SNR) during the data burst and its preceding guard period.

For our study, we assume that the received signal from each diversity branch experiences independent identically distributed (i.i.d.) Rayleigh fading. As such, the received SNRs from the $i$ th antenna at the SR and the PR, denoted by $\gamma_{s_{i}}$ and $\gamma_{p_{i}}$ respectively, $(i=1,2, \ldots, L)$, follow an exponential distribution, with common probability density function (PDF) and cumulative distribution function (CDF) given by

$$
\begin{equation*}
f_{\gamma_{s}}(x)=\frac{1}{\overline{\gamma_{s}}} \exp \left(-\frac{x}{\overline{\gamma_{s}}}\right), \quad x \geq 0 \tag{1}
\end{equation*}
$$

and

$$
\begin{equation*}
F_{\gamma_{s}}(x)=1-\exp \left(-\frac{x}{\overline{\gamma_{s}}}\right), \quad x \geq 0 \tag{2}
\end{equation*}
$$

respectively, where $\overline{\gamma_{s}}$ is the common average faded SNR for the secondary link (the common average faded SNR for the interference channel will be denoted $\overline{\gamma_{p}}$ ).

## B. Adaptive Transmission System

We consider the constant-power variable-rate uncoded $M$ ary QAM [3] as an adaptive modulation system for our proposed adaptive transceiver. With this adaptive modulator, the SNR range is divided into $N+1$ fading regions and the constellation size $M=2^{n}$ (where $n$ is the number of bits per symbol) is assigned to the $n$th region ( $n=0,1, \ldots N$ ). The selection of a constellation size is based on the fading channel state. Specifically, we partition the range of the SNR after diversity combining into $N+1$ regions, which are defined by the switching thresholds $\left\{\gamma_{T n}\right\}_{n=1}^{N}$. The constellation size $n$ is used for transmission whenever the combined SNR is greater or equal to $\gamma_{T_{n}}$ but strictly less than $\gamma_{T_{n+1}}$. The BER of $2^{n}$-QAM constellations with SNR of $\gamma$ is given in [3] by

$$
\begin{equation*}
P_{b_{n}}(\gamma)=\frac{1}{5} \exp \left(\frac{-3 \gamma}{2\left(2^{n}-1\right)}\right) \tag{3}
\end{equation*}
$$

Given a target instantaneous BER equal to $P_{b_{0}}$, the region boundaries (or adaptive modulator switching thresholds) $\gamma_{T_{n}}$ for $n=0,1, \ldots N$ are given in this case by

$$
\begin{equation*}
\gamma_{T_{n}}=-\frac{2}{3} \ln \left(P_{b_{0}}\right)\left(2^{n}-1\right) ; n=0,1, \ldots N . \tag{4}
\end{equation*}
$$

## C. Mode of Operation

The aim of the SES scheme is to reduce the processing power consumption, by minimizing the average number of branch switching, while respecting the BER error rate requirement and the peak interference constraint to the primary receiver.


Fig. 2. Switching efficient scheme.
The mode of operation of the SES scheme is summarized in a flowchart given in Fig. 2. In the beginning of each data burst, the base station transmits a training sequence using
an optimal power level $P_{\mathrm{opt}}{ }^{1}$. The combiner in the mobile's side tries to increase the output SNR above the threshold for the lowest constellation size by performing SWC with $\gamma_{T_{1}}$ as output threshold. More specifically, the combiner looks for the branch that (i) has a received SNR at the secondary above $\gamma_{T_{1}}$ and (ii) satisfies the interference constraint to the primary link (i.e. receives an ACK from the PR). Whenever the received SNR from the $i$ th branch verifies these conditions, the mobile determines the highest feasible constellation index $n$ for the given $\gamma_{s_{i}}$ by comparing this SNR to different switching thresholds $\left\{\gamma_{T_{n}}\right\}_{n=1}^{N}$. If $\gamma_{s_{i}}$ is greater than $\gamma_{T_{n}}$ but smaller than $\gamma_{T_{n+1}}$, the mobile selects the constellation size $n\left(2^{n}\right.$-QAM $)$ and the $i$ th branch is used for the secondary communication during the upcoming data burst. In the next guard period, the $i$ th branch is the first to be tested for conditions (i) and (ii).

If none of the branches meets the $\gamma_{T_{1}}$ threshold with an ACK from the primary receiver, the base station buffers the data and wait for a channel coherence time before going again through a scanning of available branches. The process of scanning and waiting are repeated indefinitely until an acceptable branch is found.

## III. Performance of the SES Scheme

In this section, the performance of SES is analyzed in terms of the average spectral efficiency and the average number of branch switching. We first start by characterizing the statistics of the output SNR with the SES scheme.

## A. Statistics of the Output SNR

Let $P_{i}=\left(1-\operatorname{Pr}\left[\gamma_{s_{i}} \geq \gamma_{T_{1}} \& \gamma_{p_{i}} \leq Q\right]\right)=P$ be the probability that the $i$ th branch fails to be selected and let $P_{\text {no }}$ be the probability of no transmission. The transmission is delayed if none of the $L$ available branches is able to meet the constellation size for the lowest modulation and to verify the interference constraint to the primary receiver. The probability of no transmission is then given by

$$
\begin{equation*}
P_{\mathrm{no}}=\prod_{i=1}^{L} P_{i}=\left[1-\left(1-F_{\gamma_{s}}\left(\gamma_{T_{1}}\right)\right) F_{\gamma_{p}}(Q)\right]^{L}=P^{L} \tag{5}
\end{equation*}
$$

Let $\gamma_{S}$ denote the received SNR at the secondary receiver. Applying the mode of operation of the SES scheme as summarized in Fig. 2, the CDF of the SNR at the secondary receiver can be shown to be given by

$$
\begin{align*}
& F_{\gamma_{S}}^{\mathrm{SES}}(x)=\sum_{n=0}^{\infty} P_{\mathrm{no}}^{n}\left(\operatorname{Pr}\left[\gamma_{s_{1}} \geq \gamma_{T_{1}} ; \gamma_{p_{1}} \leq Q ; \gamma_{s_{1}} \leq x\right]\right) \\
& \times\left(1+\sum_{i=2}^{L} \prod_{j=2}^{i} P_{j}\right) \\
& =\frac{\sum_{i=1}^{L} P^{i-1}}{1-P^{L}}\left(F_{\gamma_{s_{1}}}(x)-F_{\gamma_{s_{1}}}\left(\gamma_{T_{1}}\right)\right) F_{\gamma_{p_{1}}}(Q) \mathcal{U}\left(x-\gamma_{T_{1}}\right) \\
& =\frac{1}{1-P}\left(F_{\gamma_{s_{1}}}(x)-F_{\gamma_{s_{1}}}\left(\gamma_{T_{1}}\right)\right) F_{\gamma_{p_{1}}}(Q) \mathcal{U}\left(x-\gamma_{T_{1}}\right) \tag{6}
\end{align*}
$$

[^1]where $\mathcal{U}(\cdot)$ represents the unit step function.
Differentiating (7) with respect to $x$, the PDF of the received SNR at the secondary receiver for the SES scheme is obtained as
\[

$$
\begin{equation*}
f_{\gamma_{S}}^{\mathrm{SES}}(x)=\frac{1}{1-P} f_{\gamma_{s_{1}}}(x) F_{\gamma_{p_{1}}}(Q) \mathcal{U}\left(x-\gamma_{T_{1}}\right) . \tag{7}
\end{equation*}
$$

\]

## B. Average Number of Branch Switching

The power consumption and the processing complexity of a given switched diversity system can be quantified in terms of the average number of branch switching. Based on the mode of operation of SES scheme, a further branch needs to be checked if the previous one fails to meet the modulation and the interference requirements. Using the same steps as in [11], the average number of switched branches $\bar{N}_{s}$ can be expressed as

$$
\begin{align*}
\overline{N_{s}} & =\sum_{n=0}^{\infty} \sum_{l=1}^{L}(n L+l) \operatorname{Pr}\left[N_{s}=n L+l\right] \\
& =\frac{1}{1-P} \tag{8}
\end{align*}
$$

Since we are considering an i.i.d fading scenario, we notice from (8) that the average number of switched branches is not a function of $L . N_{s}$ is only a function of the switching threshold for the minimum modulation mode and of the peak interference constraint.

## C. Average Spectral Efficiency

A general expression of the average spectral efficiency of an adaptive modulation system is given in [3, Eq. (33)] by

$$
\begin{equation*}
\eta=\sum_{i=1}^{N} n p_{n} \tag{9}
\end{equation*}
$$

where $p_{n}$ denotes the probability that the $n$th constellation is used. The expression of this probability is given for the SES scheme by

$$
\begin{equation*}
p_{n}=F_{\gamma_{S}}^{\mathrm{SES}}\left(\gamma_{T_{n+1}}\right)-F_{\gamma_{S}}^{\mathrm{SES}}\left(\gamma_{T_{n}}\right) \tag{10}
\end{equation*}
$$

Using the above expression of $p_{n}$ and (9) we obtain the following expression of the average spectral efficiency of the SES scheme

$$
\begin{equation*}
\eta=N-\sum_{n=1}^{N} F_{\gamma_{S}}^{\mathrm{SES}}\left(\gamma_{T_{n}}\right) \tag{11}
\end{equation*}
$$

## D. Average Error Rate

The general expression of the average BER for an adaptive modulation system is given in [3, Eq. (35)] as

$$
\begin{equation*}
\overline{P_{b}}=\frac{1}{\eta} \sum_{n=1}^{N} n \overline{P_{b_{n}}} \tag{12}
\end{equation*}
$$

where $\overline{P_{b_{n}}}$ is the average BER for constellation size $n$, and is given, using (3), by

$$
\begin{equation*}
\overline{P_{b_{n}}}=\int_{\gamma_{T_{n}}}^{\gamma_{T_{n+1}}} P_{b_{n}}(\gamma) f_{\gamma_{S}}^{\mathrm{SES}}(\gamma) d \gamma \tag{13}
\end{equation*}
$$

## E. Delay Statistics

If none of the branches is able to both satisfy the minimum modulation size and the interference constraint, the system has to wait until an acceptable path is found. Let $N_{w}$ be the number of slot times that the system waits until a transmission occurs. Thus $N_{w}$ is a standard geometric random variable with probability mass function (PMF) given by

$$
\operatorname{Pr}\left[N_{w}=n\right]=P_{\mathrm{no}}^{n}\left(1-P_{\mathrm{no}}\right)
$$

The mean and the variance of $N_{w}$ are then given by

$$
\overline{N_{w}}=\frac{P_{\mathrm{no}}}{1-P_{\mathrm{no}}}
$$

and,

$$
\operatorname{Var}\left[N_{w}\right]=\frac{P_{\mathrm{no}}}{\left(1-P_{\mathrm{no}}\right)^{2}}
$$

respectively.


Fig. 3. Average number of switched branches versus $\overline{\gamma_{s}}$ (in dB ) for different levels of $Q$.

## IV. NumERICAL Examples

The performance of the proposed scheme is illustrated in this section with some selected numerical results. These numerical examples are obtained by Monte-Carlo simulations and confirmed by analytical results. For these examples we set the number of available transmit diversity branches $L=6$, the number of signal constellations $N=4$, and the BER constraint as $P_{b_{0}}=10^{-3}$. For reference, we compare the performance of our scheme to the selection diversity scheme (SDS) where the best branch verifying the modulation mode and the interference constraint is always selected. The SDS scheme will also buffer data if none of the branches verifies the two required conditions.

In Fig. 3 we depict the average number of switched branches with the SES scheme in function of the average secondary SNR per branch $\overline{\gamma_{s}}$. In this figure we fixed the common average faded SNR for the interference channel $\overline{\gamma_{p}}$ to 0 dB and compared the average number of switched branches for different levels of the peak interference constraint $Q_{\mathrm{dB}}$. For small values of $Q_{\mathrm{dB}}$ the transmitter needs to wait for more


Fig. 4. Average number of switched branches versus $\overline{\gamma_{p}}$ (in dB ) for different levels of $Q$.
time before finding a path that satisfies the interference constraint and the minimum modulation requirements especially in the low SNR range. This explains the higher number of estimated branched for $Q_{\mathrm{dB}}=-3 \mathrm{~dB}$. When $Q_{\mathrm{dB}}$ gets bigger, the transmission of the cognitive user will be in independent from the interference link and the average number of switched branches will only depend on the ability to reach the lowest constellation size. From this figure we also see that our scheme outperforms the SDS scheme in terms of the average number of switched branches. While the SDS scheme always estimates all the branches before finding an acceptable branch, our scheme tests the branches one by one and does not need to estimate all branches especially in the high SNR range.


Fig. 5. Average spectral efficiency versus $\overline{\gamma_{s}}$ (in dB ).

In Fig. 4, we also depict the average number of switched branches with the SES scheme but this time in function of the average interference SNR per branch $\overline{\gamma_{p}}$ for a fixed $\overline{\gamma_{s}}$. When $\overline{\gamma_{p}}$ increases a lower number of branches will be able to reach the interference constraint. In such case, data is buffered until a good branch is available for transmission, which explains
the high number of switched branches in the high interference SNR region.

The spectral efficiency of the proposed scheme is presented in Fig. 5 in function of $\overline{\gamma_{s}}$ for $\bar{\gamma}_{p B}=0 \mathrm{~dB}$ and $Q_{\mathrm{dB}}=10 \mathrm{~dB}$ and compared to the spectral efficiency of the SDS scheme. It is very important to note that the spectral efficiency is not a function of the interference constraint if $\overline{\gamma_{p}}$ is fixed. In fact, in case the interference constraint is not met, no transmission takes place which will be reflected in terms of delay while $\eta$ only depends on $\overline{\gamma_{s}}$. Fig. 5 also shows that the processing power that we gained using the SES scheme compared to the SDS scheme comes at the expense of a reduced spectral efficiency. This is explained by the fact that SDS compares all available branches and then selects the best reaching the interference constraint and the BER requirement, on the other hand, the proposed SES scheme selects the first branch with acceptable quality.


Fig. 6. Average BER versus $\overline{\gamma_{s}}$ (in dB ).

In Fig. 6 we show the bit error performance of the proposed scheme $\overline{\gamma_{s}}$. For reference we also included the BER performance of another version of our scheme that we call always transmit scheme (ATS) where the secondary will transmit its data independently from the channels condition. Our proposed scheme performs better in terms of the BER performance especially in the low SNR range. This performance improvement gained by the use of SWC comes at the expense of a time delay that we depict in Fig. 7 versus the peak interference level $Q_{\mathrm{dB}}$. When the number of available branches increases it becomes more likely to find an acceptable branch which decrease the average delay. Also when $Q$ increases more branches will be able to verify the interference constraint and thus be accepted if they verify the minimum modulation requirement.

## V. Conclusion

We have proposed in this paper, a switching efficient scheme using adaptive modulation and the scan and wait transmit diversity technique. The aim of this scheme is to reduce the average number of branch switching in the secondary transmitter and thus minimize the processing power consumption of the


Fig. 7. Average number of slot times required before finding an acceptable branch versus the peak interference level $Q_{\mathrm{dB}}$ for ${\overline{\gamma_{s}}}_{\mathrm{dB}}={\overline{\gamma_{p}}}_{\mathrm{dB}}=10 \mathrm{~dB}$ and for different values of $L$.
system. This scheme improves the switching efficiency of the SDS scheme at the expense of a reduced spectral efficiency and a slightly increased complexity. As a future work, we can extend this switching efficient scheme to a bandwidth efficient scheme (BES) that aims to reach a higher constellation size but at the expense of an increased number of branch switching. We also aim to reduce the average delay to fit delay sensitive applications and multiuser scenarios.

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[^1]:    ${ }^{1}$ The transmitted power $P_{\mathrm{opt}}$ is assumed to correspond to a constant level of output power that is optimized and set to minimize the average interference to the primary user.

