Experimental Spectrum Sensing Measurements using USRP Software Radio Platform and GNU-Radio

(Invited Paper)

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***Abstract*—In cognitive radio, the secondary users are able to sense the spectral environment and use this information to opportunistically access the licensed spectrum in the absence of the primary users. In this paper, we present an experimental study that evaluates the performance of two different spectrum sensing techniques to detect primary user signals in real environment. The considered spectrum sensing techniques are: sequential energy and cyclosationary feature based detectors. An Universal Software Radio Peripheral platform with GNU-Radio is employed for implementation purpose. We analyzed the performances of both spectrum sensing methods by measuring the detection probabilities as a function of SNR for a given false alarm probability. As predicted theoretically, experimental measurements show that the cyclostationnary feature detector performs better than the sequential energy detector. However sequential energy detector can be used for reduction of sensing time in the presence of strong signals.**

I. INTRODUCTION

Cognitive Radio (CR) is an emerging concept to increase spectrum usage efficiency by allowing a secondary user (SU) to access some licensed spectrum bands temporarily unoccupied by the primary user (PU). Two basic approaches to spectrum sharing have been considered: spectrum *overlay* and spectrum *underlay*. According to the spectrum overlay approach, the secondary users sense and identify unused frequency bands and use them for communication purposes. Thus, the secondary users (SU) are responsible for detecting the unused bands and they should vacate the spectrum as soon as the primary user begins its [1] activities. The underlay approach imposes constraints on the secondary users' transmission power level so that it operates below the noise floor of primary users. Here, we focus on the implementation aspects of the overlay spectrum sharing. To determine the absence or presence of the primary user signals, several spectrum sensing techniques have been developed [2], [3], [4]. These techniques can be classified into three categories: (**i**) methods requiring both primary user signal and noise variance information, (**ii**) methods requiring only noise variance information (also called *semi-blind* methods), and (**ii**) methods not requiring any information on primary user signal or noise variance (also called *blind* methods). Examples of blind spectrum sensing methods would be wavelet based detection [5], eigenvalue based detection [6], second order statistical based detection [7], and symmetry property of cyclic autocorrelation function based detection [8].

In the practical implementation, the simplest spectrum sensing method capable of detecting the presence of a PU's signal is based on energy detection which is a semi-blind method. Several papers address experimental results of the energy detector and outline the impact of noise uncertainty on the performance of detection [9], [10]. Due to this shortcoming, there is an signal to noise ratio wall, SNR_{wall}, in which energy detector can not guarantee a detection performance.

The main aim of the conducted experiment is to sense the spectrum in a given frequency range and make as reliable decision as possible on the potential presence of the primary user (PU) signal in the observed spectrum fragment. In order to achieve this goal selected algorithms for spectrum sensing have been implemented in hardware.

This paper is organized as follows. In section II, the system model is presented as well as sequential energy and the cyclostationary feature-based detectors. In section III, we describe our experimental setup for both detectors using two different modulations for the PU. Section IV presents the experimental evaluation results of the considered algorithms. Finally, conclusions and future works are presented in section V.

II. SYSTEM MODEL AND SENSING TECHNIQUES

Thus, spectrum sensing and detecting the presence of a radio in the environment can be treated as a classical detection problem [11] [12]. Two binary hypotheses H_0 and H_1 can be defined to indicate the absence or the presence of the PU in the environment. The received signal at the SU, $r(t)$, can be expressed as:

$$r(t) = \begin{cases} n(t) + i(t) = \hat{n}(t) & \longrightarrow H_0 \\ h(t) \cdot s(t) + n(t) + i(t) = s(t) + \hat{n}(t) & \longrightarrow H_1 \end{cases} \quad (1)$$

where $s(t)$ and $h(t)$ stand for the PU signal and channel impulse response, respectively, $n(t)$ is an additive white Gaussian noise (AWGN), and $i(t)$ represents other sources of distortions such as ambient noise or interferences; the equivalent noise observed at the antenna input can be then represented as $\hat{n}(t)$. The objective of the spectrum sensing operation is to decide between H_0 et H_1 based on the observation of the received signal $r(t)$; one can find in the literature the papers dealing with interference mitigation or reduction during the spectrum sensing process. The detection performance is characterized by two probabilities: probability of detection, P_{d}, where the decision is H_1, while H_1 is true; and probability of false alarm, P_{fa}, which corresponds to the case where the decision is H_1 while H_0 is true.

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In our experiments, two algorithms have been tested i.e. sequential version of energy detection, and a method for cyclostationarity-based detection called Symmetry Property of Cyclic Autocorrelation Function (SPCAF). Let us briefly summarize the theoretical basis of these solutions.

A. Traditional Energy-based spectrum sensing

One of the simplest way for primary user detection is to calculate the amount of received power in the considered frequency subband and compare this value with the noise variance. In the case that the received power is greater than the previously approximated power of noise the algorithm will make a decision on the spectrum occupancy by the primary user signal. In turn, the channel will be assumed to be vacant if the computed noise power will be close to the noise variance at the certain level of certainty. There are several parameters that influences the reliability of any spectrum sensing algorithm. In case of traditional energy detection, the crucial role is played by properly defined decision threshold, and in consequence, by the correctness of noise variance, and the duration of sensing time (expressed in seconds or - for discrete signals - in terms of number of gathered samples). Following [12], for the given values of probability of false alarm, P_{fa}, number of collected samples N, and the (equivalent-)noise variance $\hat{\sigma}_n^2$, the decision threshold can be defined as following:

$$\gamma_{\text{thr}} = \hat{\sigma}_n^2 \cdot Q^{-1}(P_{\text{fa}}) \cdot \left(\sqrt{2N} + N\right), \tag{2}$$

where $Q()$ represents the Q-function. Having in mind that the total power of N collected samples in the given frequency band can be represented as the random variable $P_N = \sum_{k=0}^{N-1} |r[n]|^2$, then based on (1) the generic decision rule D_N can be then modified to the considered case:

$$D_N = \begin{cases} P_N \leq \gamma_{\text{thr}} & \longrightarrow H_0 \\ P_N > \gamma_{\text{thr}} & \longrightarrow H_1 \end{cases} \tag{3}$$

It is worth noticing that the reliability of energy-detectors strongly depends on the received power and on the accuracy of approximated variance noise $\hat{\sigma}_n^2$. The latter can be improved by increasing the number of collected samples N. In practice, however, the sensing time will be fixed, thus it is feasible that for low signal-to-noise ratios the performance of the traditional energy detection algorithms will be relatively low (i.e. SNR_{wall}) [13] [14].

B. Sequential Energy detector

The behaviour of the traditional energy detector can be improved in various ways, e.g. by application of the adaptively modified threshold. In our experiment we have selected the double-threshold, sequential energy detector which possess the same reliability as the traditional one, but its application could reduce the sensing time. The main concept is based on the assumption that for very strong PU signal, or - contrarily - in the presence of noise only, the number of samples that should be collected for decision making can be reduced. If this is a case, the sensing time is minimized which increases the time devoted for data transmission and reduces the energy consumption of the observation phase. In order to achieve this goal two decision thresholds have to be applied, γ_{HI} and γ_{LO}, which will be used for decision making if the signal is or is not present in the observed frequency fragment. In a nutshell, the procedure can be realized in the iterative way. The energy detector collects the signal samples in the shorter period $N_s < N$ and tries to made the decision. If the amount of power is greater than γ_{HI}, the decision of the PU signal presence can be made; if the received power is lower than γ_{LO} one can state that the considered channel is vacant. If the calculated value falls between these thresholds, the sequential energy detector collects next block of samples and repeats the procedure. When the total number of gathered samples reaches the maximally allowed value (i.e. the maximum sensing time will be finished), the decision will be made as for traditional algorithms. The decision rule for i-th iteration can be written as follows:

$$D_N^i = \begin{cases} P_N^i \leq \gamma_{\text{LO}} & \longrightarrow H_0 \\ P_N^i \in (\gamma_{\text{LO}}, \gamma_{\text{HI}}) & continue \\ P_N^i \geq \gamma_{\text{HI}} & \longrightarrow H_1 \end{cases} \tag{4}$$

C. Cyclostationary feature based spectrum sensing

In wireless communications, the transmitted signals show very strong cyclostationary features [15]. Therefore, identifying a unique set a features of a particular radio signal can be used to detect its presence based on its cyclostationary features. In the context of spectrum sensing many works have been conducted in using the cyclostationary features to detect the presence of PU in the radio environement [4]. In general, this method can perform better than the energy based detector. However its main drawbacks are the complexity associated with the detection technique and needs of some a-priori knowledge of the PU signal.

The cyclostationary detector can be realized by analyzing the Cyclic Autocorrelation Function (CAF) of a received signal $r(k)$. The CAF of a received signal $r(k)$ at the SU can be expressed as illustrated in (5).

$$R_r(k,\tau) = \sum_{\alpha} R_r^{\alpha}(\tau) \mathrm{e}^{2\pi j \alpha k} \tag{5}$$

where τ is lag associated to the autocorrelation function, α the cyclic frequency and $R_r^{\alpha}(\tau)$ is given by (6).

$$R_r^{\alpha}(\tau) = \lim_{N\to\infty} \frac{1}{N} \sum_{k=0}^{N-1} R_r(k,\tau) \mathrm{e}^{-2\pi j \alpha k} \tag{6}$$

1) Classical Cyclostationary feature based detector: The classical approach to realize the cyclostationary detector is based on the Cyclic Spectrum Density (CSD) or the spectral correlation function of the received signal $r(k)$.

$$S_r^{\alpha}(f) = \frac{1}{N} \sum_{k=0}^{N-1} R_r^{\alpha}(\tau) \mathrm{e}^{-2j\pi f \tau} \tag{7}$$

The CSD function presented in (7), exhibits peaks when the cyclic frequency α equals the the fundamental frequencies of $s(k)$ the transmitted signal. Under the H_0 hypothesis, the CSD function does not have peaks since the noise is generally non-cyclostationary.

Using this technique, it is possible to distinguish even weak PU signals from the noise at very low SNR, where the energy detector is not applicable.

2) SPCAF detector: The discrete-time consistent and unbiased estimation of the CAF of a random process is given as:

$$\tilde{R}^{\alpha}_{rr^*}(\tau) = \frac{1}{M}\sum_{k=0}^{M-1} r(k)r^*(k+\tau)\mathrm{e}^{-2j\pi\alpha k} \quad (8)$$

For a given lag parameter $\tau \in \{1, 2, \ldots, L\}$, the cyclic autocorrelation function (CAF) can be seen as Fourier transform of $V = [r(0)r^*(0+\tau), r(1)r^*(1+\tau), \ldots, r(M-1)r^*(M-1+\tau)]$, where M is FFT size. As shown in the work of Khalaf *et al.* [8], the CAF is an M-dimensional sparse vector in cyclic frequency domain for a fixed lag parameter τ. Moreover, it presents a symmetry property as illustrated in (9).

$$||\tilde{R}^{\alpha}_{rr^*}(\tau)||_2 = ||\tilde{R}^{-\alpha}_{rr^*}(\tau)||_2 \quad (9)$$

Using a compressed sensing (CS) recovery technique like the Orthogonal Matching Pursuit (OMP) algorithm [16], we can accurately estimate the CAF using a limited and small number of received samples $N << M$. If the obtained CAF verifies the property (9) then H_1 is true otherwise H_0 is true. Its important to note that even under H_0 the obtained CAF verifies the symmetry property. However, when using a small number of samples, the probability to obtain a symmetrical CAF under H_0 is very small [8]. This SPCAF technique, can perform with a limited number of samples and consequently with lower complexity and shorter observation time compared to the classical cyclostationary feature detector.

III. Spectrum Sensing Experimental Setup

A. Hardware/Software overview

The performance of the selected spectrum sensing algorithms has been verified in conducted experiments realized by means of Universal Software Radio Peripheral (USRP) boards by Ettus Research. USRP platforms, as the low-cost and high-quality realization of the software-defined-radio (SDR) concept, delivers to the users various functionalities allowing for efficient, real-time realization of even very complicated wireless systems that operate in the radio-frequency (RF) band. The main role of the URSP platform is to convert the digital base-band signal delivered from the computer to analogue signal in the RF band. This process is realized in two-steps. In the first step the digital signal is converted to the digital intermediate-frequency (IF) domain; this phase is realized in the so-called mother-board, being the basis of the USRP platform. After that the signal is processed in the dedicated daughter-board, which is responsible for transforming of the digital IF signal to its analogue form in RF band. Finally, the signal is radiated by means of the mounted RF aerial. The variety of available daughter-boards creates big opportunities to the user, since these are designed to convert the IF signal to different part of the RF spectrum. Being the realization of the SDR concept, USRP are steered from the software level, i.e. the whole data processing in the base-band is realized on the computer side. Various software platforms can be applied for that purposes, including commercial and open-source solutions.

In our experiments two USRP boards have been utilized: the PU signal has been generated by means of the first board,

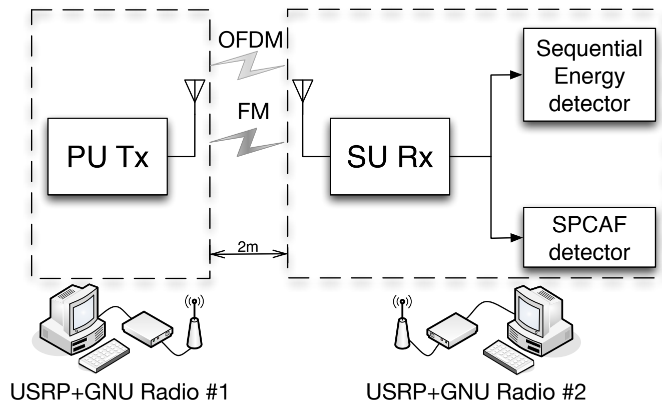

Fig. 1. Schematic system diagram

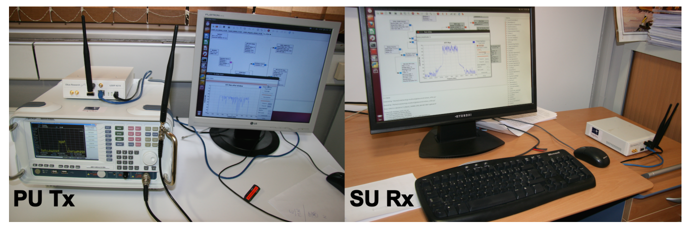

Fig. 2. PU transmitter and SU receivers realized by means of USRP board and personal computers

whereas the second one has been used for spectrum sensing purposes and acted as the secondary user. The whole software processing has been realized in the open-source GNU-Radio environment [17]. This set of libraries together with the appropriate drivers for manipulating the USRP boards and graphical programming environment allowed for efficient and accurate implementation of the selected spectrum sensing algorithms. In our experiment two sensing scenarios have been considered: first, where the narrow-band frequency-modulated radio signal, and second, where the multicarrier signal should be detected. The schematic diagram and the dedicated photographs of the experimentation setup are shown in Fig. 1 and 2, respectively. Finally, the whole system is presented in Fig. 3.

B. Transmitter side

At the transmitter side two types of signals were generated, the narrowband FM signal, and wideband mutlicarrier signal based on orthogonal frequency division multiplexing. In the former case the composite radio signal has been created and frequency modulated before sending to the USRP board via

Fig. 3. The whole experimentation setup

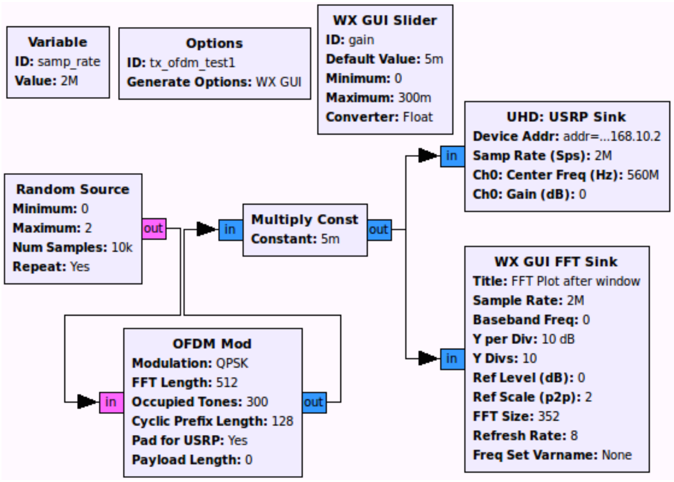

Fig. 4. Diagram of the PU OFDM transmitter realized in the GNU radio (Screenshot from GRC)

Ethernet cable. It means that assumed frequency deviation ($\pm$ 75 kHz deviation from the assisted center frequency) the bandwidth of the spectrum occupied by the FM signal is narrow (144 kHz). On the contrary, the spectrum of the multicarrier signal is assumed to be wider - the OFDM symbol with $N_{\text{OFDM}} = 512$ subcarriers of the width 1.2 MHz has been used. As it has already been mentioned, the whole baseband processing has been realized on the PC computer in the GNU Radio environment, and in particular in the graphical tool called GNU Radio Companion (GRC), where the whole system is built from blocks. The screen-shot from the GRC illustrating the OFDM transmitter side is shown in Fig. 4. One can observe the presence of the signal source block (*Random Source*) that generates repeatedly random data, which are mapped to QPSK symbols and then are subject to OFDM modulation (realized in *OFDM Mod* block). Only 300 subcarriers from 512 available has been occupied, and the cyclic prefix of the size equal to one-quarter of that of IFFT was used. Finally, after proper power adjustment, the signal was sent to the local spectrum analyzer (*FFT plot*) and to the USRP block (*USRP Sink*), responsible for sending data to the USRP platform. It can be noticed that the complex sampling frequency has been set to 2 MHz, and the center frequency was set to 560 MHz. This frequency band has been chosen intentionally - it is within the TV band and is not occupied in the physical location where the experiment was conducted (i.e. no interference from the distance digital-television station could be observed).

C. Receiver side

As indicated in Fig. 1, two spectrum sensing algorithms have been implemented: the one based on energy detection, and the second that analyzes the cyclostationarity features of the received data. Analogously to the transmitter side, the whole base-band processing - that will be performed by the SU wireless terminal - has been realized in the computer side using the GNU Radio environment. The schematic diagram of the receiver is shown in Fig. 5. One can observe the presence of the *USRP Source* block responsible for delivering data from RF spectrum to the computer; it operates at the center frequency equal to 560 MHz and covers the band of 1MHz (what corresponds to complex sampling frequency equal to 1 Msps, as well). In order to evaluate the influence of noise on the performance of selected spectrum sensing algorithms, additional block for noise generation has been used and the noise-signal of appropriate power has been added to the signal produced by the *USRP Source* block. After, the signal is split into parallel chains: one dedicated for energy detection, and one for cyclostationariy feature-based algorithm. In such a configuration both algorithms operates on the same received samples making the comparison fair. One can also observe the presence of the *FFT plot* block used for displaying the received signal on the computer screen. Let us focus on the sequential energy detection algorithm (lower processing chain in the analyzed figure). The signal disturbed by the additive white Gaussian noise is then transformed to the frequency domain by means of *FFT* block. The algorithm for sequential energy detection, implemented in C language and assigned to the *DTED* block, make the decision on the occupancy of each frequency bin separately. In other words for the presented case 256 decisions will be made. The decisions are then transferred to the graphical sink. In the upper processing chain, devoted for cyclostationary feature spectrum sensing algorithm, the signal is converted from complex to real type and such modified signals are subject to processing in the *SPCAF_v1* block, realizing the functionality of the SPCAF algorithm described in the previous sections. All of the decisions are stored into the files.

IV. Experimental results

In order to compare the performance of the selected spectrum sensing algorithms let us analyze the results obtained during the conducted experiments.

A. FM signal as PU

Here, we compare performance of the SPCAF based blind detector with the sequential energy detector. A frequency modulated signal is used as primary user's signal. In the experiments, the central carrier frequency is set to 560 MHz. We compute the good detection probability (P_d) for the two detectors at different values of estimated SNR. In order to estimate the SNR, the noise power σ^2 is estimated at the receiver with no transmitted signal. Then, the transmitter is switched on and its transmission power is varied to obtain different signal-to-noise ratios (SNRs) at the receiver. Fig. 6 shows the detection probability of the two detectors obtained through experiments as function of SNRs. As concluded from the measurements, the probability of false alarm (P_{fa}) is approximately equal to 0.08 for both detectors. Furthermore, for the SPCAF detector, the maximum value of the lag parameter is $L = 5$ and the FFT size is $M = 2048$. It is clear from Fig. 6 that the performance of the SPCAF is better than the sequential ED. Another important point to note is that the number of received samples used by SPCAF is $N = 256$. However, the sequential ED requires at least N samples for detection.

B. OFDM signal as PU

In this part, the primary user signal is an orthogonal frequency division multiplexing (OFDM) signal. Fig. 7 shows

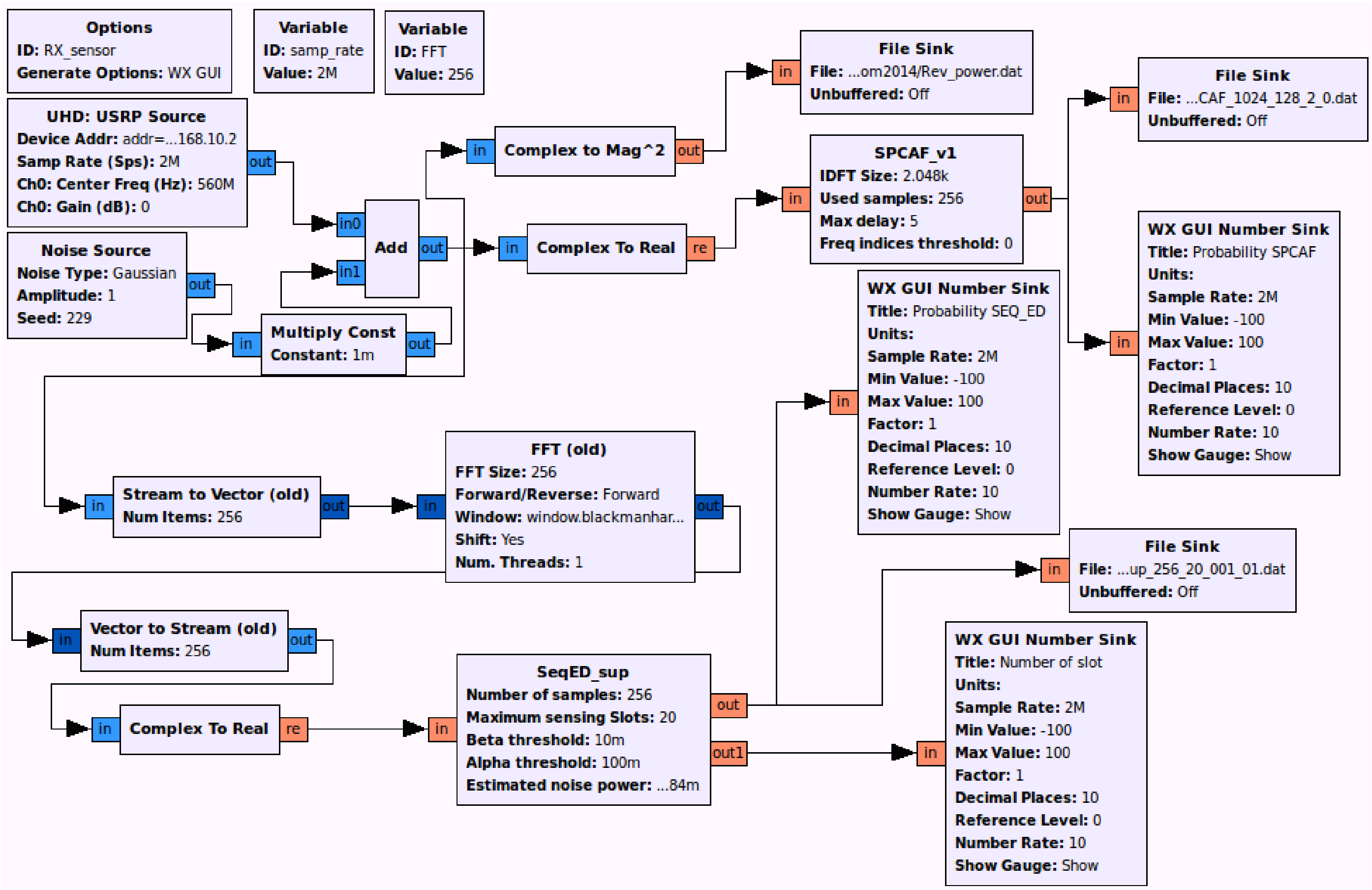

Fig. 5. Diagram of the SU receiver realized in the GNU radio (Screenshot from GRC)

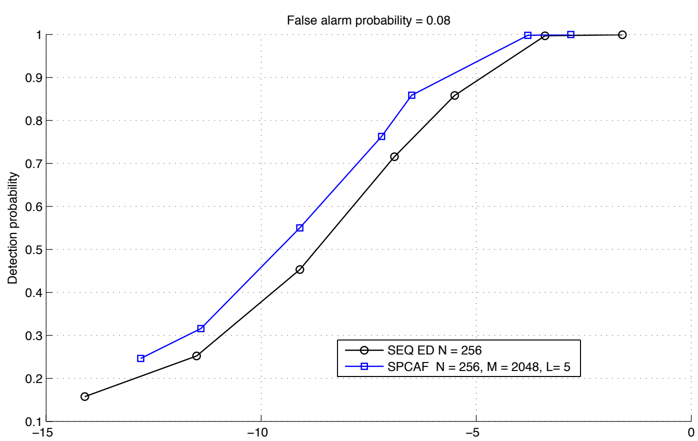

Fig. 6. Probability of detection for both algorithms when using FM signal as PU ($P_{fa} = 0.08$)

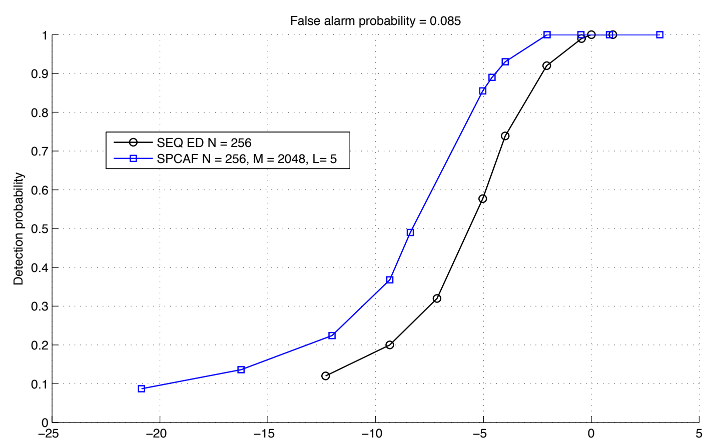

Fig. 7. Probability of detection for both algorithms when using OFDM signal as PU ($P_{fa} = 0.085$)

the detection probability achieved by the secondary user using SPCAF and sequential ED, while maintaining the false alarm probability below 0.08. Based on the Fig. 7, it can be concluded that the SPCAF gives better results compared to the sequential Energy detection method at low signal-to-noise ratios (SNRs).

V. Conclusions and Future Work

One can observe that for high SNRs both algorithms behave similar achieving high detection efficiency. However, it is not so challenging to detect strong signal, since the influence of noise in such a case will be minimal. Thus it is more important to focus on the low-SNR regions for both: narrowband and wideband signals. As it could be expected, the energy detector achieves much poorer results comparing to the cyclostationary feature-based detector. It is due to the fact that the latter are not so sensitive to the signal imperfections and channel influence. However, let us remind that the reason for application of sequential energy-detection algorithm was to reduce the sensing time in the case when the reliable decision could be made before collecting the maximum allowed number of samples for spectrum sensing. The obtained results confirm that for high SNRs values the performance of sequential algorithms is as good as the performance of more advanced ones but the price paid for it - understood as the computational complexity - is much less. This bring us to the concept of the hybrid structure of spectrum sensing algorithm. In such a case, the low-complex double-threshold algorithm should be applied in the first phase, followed by the cyclostationarity-based one. When the signal of the PU will be strong enough or the observed signal variance will be close to the noise variance, the sequential energy detection algorithm will made reliable decision in the very short time, and the application of the more complicated algorithms will be not necessary. On the other hand, if the energy-detection procedure will not finish after collecting of N signal samples, the cyclostationary based algorithm shall be applied for final decision. Such a scenario will be investigated in the future. However, beside measurements of the sensing-time reduction obtained in the hybrid approach as well as of its overall performance, the whole system will be implemented on the FPGA chips. It will allow for detailed analysis of the energy consumed in each phase of the hybrid algorithm. Thus, the conclusions on the real energy-efficiency of the proposed hybrid approach could be drawn.

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