The Method of Discretization Signals to Minimize the Fallibility of Information Recovery

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***Abstract*:** The paper proposes a fundamentally new approach to the formulation of the problem of optimizing the discretization interval (frequency). The well-known traditional methods of restoring an analog signal from its discrete implementations consist in sequentially solving two problems: restoring the output signal from a discrete signal at the output of a digital block and restoring the input signal of an analog block from its output signal. However, this approach leads to a methodical fallibility caused by interpolation when solving the first problem and by regularizing the equation when solving the second problem. The aim of the work is to develop a method for signal discretization to minimize the fallibility of information recovery to determine the optimal discretization frequency.

The proposed method for determining the optimal discretization rate makes it possible to exclude both components of the methodological fallibility in recovering information about the input signal. This was achieved due to the fact that to solve the reconstruction problem, instead of the known equation, a relation is used that connects the input signal of the analog block with the output discrete signal of the digital block.

The proposed relation is devoid of instabilities inherent in the well-known equation. Therefore, when solving it, neither interpolation nor regularization is required, which means that there are no components of the methodological fallibility caused by the indicated operations. In addition, the proposed ratio provides a joint consideration of the properties of the interference in the output signal of the digital block and the frequency properties of the transforming operator, which allows minimizing the fallibility in restoring the input signal of the analog block and determining the optimal discretization frequency.

A widespread contradiction in the field of signal information recovery from its discrete values has been investigated. A decrease in the discretization frequency below the optimal one leads to an increase in the approximation fallibility and the loss of some information about the input signal of the analog-to-digital signal processing device. At the same time, unjustified overestimation of the discretization rate, complicating the technical implementation of the device, is not useful, since not only does it not increase the information about the input signal, but, if necessary, its restoration leads to its decrease due to the increase in the effect of noise in the output signal on the recovery accuracy. input signal. The proposed method for signal discretization based on the minimum information recovery fallibility to determine the optimal discretization rate allows us to solve this contradiction.

***Keywords*:** discretization rate, signal, information recovery, measuring channel, fallibility, analog-digital signal processing.

1. Introduction

The problem of the optimal choice of the discretization interval or frequency when carrying out analog-to-digital signal processing (ADSP) does not lose its relevance in measuring technology, including when recovering signals in information-measuring systems [1, 2].

In the classical setting, the problem of choosing the discretization frequency of an analog signal is well known and is solved by the Shannon-Kotelnikov theorem [3, 4]. However, in measuring practice, there is one fundamental feature that makes the direct application of the Shannon-Kotelnikov theorem and modern methods for optimizing the discretization rate of analog signals to minimize the recovery fallibility not quite adequate [5, 6]. Since this feature determines a fundamentally new approach to the formulation of the problem of optimizing the discretization interval (frequency), considered in this work, we will explain it in more detail. For this, we represent the generalized block diagram of the measuring channel as follows (Figure. 1).

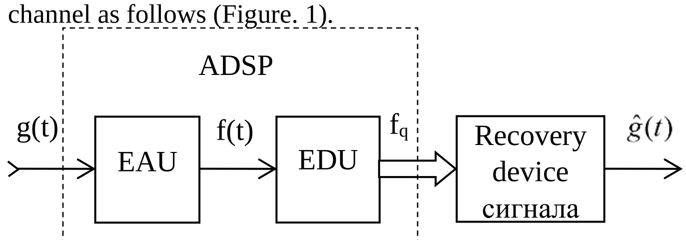

Figure 1. Block diagram of the measuring channel

In this scheme, in the part of the measuring channel (or in the automatic digital signal processing device- device - ADSP), analog-to-digital processing of the input signal, including analog-to-digital conversion, is carried out. The ADSP device contains a series-connected equivalent analog unit (EAU) and an equivalent digital unit (EDU). The term “equivalent” emphasizes that these blocks are distinguished not by their structural or functional features, but by the type of operations (analog or digital) that are performed on the input signal in the ADSP. Therefore, the EAU includes not only separate analog functional blocks, but also the analog part of the analog-to-digital converter (ADC). The term “equivalent” emphasizes that these blocks are distinguished not by their structural or functional features, but by the type of operations (analog or digital) that are performed on the input signal $g(t)$ in the ADSP. Therefore, the EAU includes not only separate analog functional blocks, but also the analog part of the analog-to-digital converter (ADC). Therefore, the EAU includes not only separate analog functional blocks, but also the analog part of the analog-to-digital converter (ADC). In the EDU, the operations of discretization and quantization of the output signal of the EAU $f(t)$ are necessarily performed, usually with a constant strictly specified discretization interval Δt, but digital signal processing can also be performed. The codes of discrete values f_q of the signal from the EDU output are sent to the signal recovery device, where they are converted into an analog signal $\hat{g}(t)$, which is an image of the input signal with a certain recovery fallibility.

2. Literature Analysis and Problem Statement

The articles provide many examples of the use of methods for recovering an analog signal from a discrete one.

Thus, in articles [1-6] the realization of frequency characteristics, in particular, EAU is given, at the frequency tending to infinity, they decrease, approaching zero. However, the frequency response, which is strictly zero at frequencies above the cutoff frequency, cannot be realized by the Paley-Wiener test. Therefore, according to the Shannon-Kotelnikov theorem, at any finite sampling rate, accurate reconstruction of the signal is impossible, because the high-frequency components of the output signal cannot be restored.

In articles [7, 8] the known traditional methods of recovery of an analog signal from discrete have resulted. These methods consist of the sequential solution of two problems: recovery of the output signal of the $f(t)$ EAU from the discrete signal f_q at the output of the EDU and recovery of the input signal of the $g(t)$ EAU from its output signal $f(t)$. However, this approach leads to a methodological error caused, firstly, by interpolation in solving the first problem, and secondly, by regularization of the equation in solving the second problem. This is due to the fact that in solving the second problem the initial equation is equal to

$$\int_{-\infty}^{t} h(t-\tau)\, g(\tau) d\tau = f(t) \tag{1}$$

where $h(t-\tau)$ the impulse transient response of the EAU.

The limits of integration in (1) are determined by the area of existence of the input signal and time. The equation has an unstable solution and is used to identify stable approximate solutions.

Articles [9 -15] use methods for solving incorrect problems, in particular the Fredholm equation of the first kind, most of which are based on the replacement of the operator, the exact approximate $g(\tau)$ (adjustable) operator.

But all methods of recovery of an analog signal from discrete do not exclude a methodological error of recovery. Therefore, the development of a method that deprives the obtained results of the recovery of the signal of methodological error is relevant.

The proposed method for determining the optimal discretization rate makes it possible to exclude both components of the methodological recovery fallibility. This was achieved due to the fact that to solve the recovery problem instead of equation (1), an equation is used that connects the input signal of the $g(\tau)$ EAU with the output discrete signal of the EDU:

$$\int_{-\infty}^{t_q} h\left(t_q-\tau\right) g(\tau)\, d\tau = f_q \tag{2}$$

where $f_q \equiv f(t_q)$ are the discrete values of the EAU output signal $f(t)$ obtained using the ADS; t_q - moments of signal discretization $f(t)$.

Equation (2) is devoid of instabilities inherent in equation (1). Therefore, when solving it, neither interpolation nor regularization is required, which means that there are no components of the methodological fallibility caused by the indicated operations. In addition, Eq. (2) provides a joint consideration of the properties of the interference in the output signal f_q of the EDU and the frequency properties of the converting operator, which makes it possible to minimize the fallibility in reconstructing the input signal $g(\tau)$ of the EAU and to determine the optimal discretization frequency.

As will be shown below, equation (2), in contrast to (1), has a stable solution even if $\hat{h}$ is specified exactly, and not an approximate (regularized) operator. Moreover, the fallibility in restoring the original signal turns out to be uniquely related to the discretization frequency of the input signal of the EAU $f(t)$, since with its increase (or with a decrease in the discretization interval Δt), equation (2) approaches (1). In this case, the stability with respect to interference in the signal $f(t)$ decreases, and, consequently, the fallibility in signal reconstruction $g(\tau)$ increases. In other words, the signal discretization frequency (or interval) plays the role of a regularization parameter, and its value directly determines the component of the reconstruction fallibility caused by noise in the sampled signal f_q. Let's call it the interference component of the recovery fallibility.

There is one more component of the input signal reconstruction fallibility, which is also related to its discretization frequency $g(\tau)$. It does not depend on interference and is caused by the fact that with an increase in the discretization interval Δt of a signal $f(t)$, the number of degrees of freedom in a discrete signal f_q decreases, and this, when the signal is restored $g(\tau)$, leads to the loss of information about its small details. This component of the fallibility depends on the signal discretization fallibility. In the known methods of approximation (stepwise, linear, etc.) of an analog signal by its discrete readings, the form of the approximating function can be different and is set a priori [16, 17]. In the proposed method, the approximating function, as will be shown below, is related to the impulse response of the EAU so that the considered component of the fallibility cannot be reduced without additional, a priori information about the signal $g(\tau)$. Thus, the second component of the reconstruction fallibility is completely determined by the type of the input signal $g(\tau)$, the discretization frequency and the impulse response of the EAU. Let's call it the approximation fallibility.

To develop a method for signal discretization to minimize the fallibility of information recovery to determine the optimal discretization frequency. The proposed method makes it possible to exclude the components of the methodological fallibility of information recovery.

3. The Proposed Mechanism

When synthesizing the ADSP, it should be taken into account that the signal discretization frequency $f(t)$ affects both the approximation fallibility and the interference component of the signal $g(\tau)$ reconstruction fallibility, and with an increase in the discretization frequency, the approximation fallibility decreases, and the interference component of the fallibility increases. Therefore, for each class of input signals, with the known transfer function of the EAU and the statistical characteristics of the noise in the output signal f_q of the ADC, the optimal discretization frequency can be determined. To do this, one can use, for example, either the criterion for the minimum of the total recovery fallibility, which includes both specified fallibility components, or the criterion for the minimum of one component of the recovery fallibility at a given level of the other component of the fallibility, or the information criterion (maximum information in the signal $g(\tau)$ that can be obtained from a discrete signal f_q). The existence and determination of the optimal discretization rate, the overestimation of which, as well as the underestimation, increases the fallibility in recovering the input signal $g(\tau)$, is the essence of the proposed method. Regardless of the criterion used to determine the optimal discretization rate, it is necessary to find estimates of both components of the reconstruction fallibility as a function of the discretization rate. For this, it is necessary to obtain a solution to equation (2), i.e. find the input signal f_q using a known discrete signal $g(\tau)$. Equation (2) has many solutions, which will be shown below. The solution that has the smallest norm and does not contain a priori information about the input signal $g(\tau)$ will be called an approximating (skeletal) signal.

We emphasize that even in the case when the restoration of the input signal $g(\tau)$ from a discrete signal f_q is not carried out, the approximating signal $g(\tau)$ determines the information about the signal potentially contained in the signal depending on the discretization frequency and, therefore, makes it possible to reasonably determine it. Let us find a regularized solution to equation (2), representing the approximating signal.

In equation (2), we denote $h(t_q-\tau)=h_q(\tau)$ and consider the system of functions $\{h_q(\tau)\}$ as a basis (in the general case, nonorthogonal) in the space of functions. Then the quantities f_q, which can be seen from (2), can be considered as projections of the input signal $g(\tau)$ onto the subspace L, "spanned" by the system of functions

$$\{h_q(\tau)\}:\ (g,h_q)=f_q, \tag{3}$$

where $(g,h_q)=\int_{-\infty}^{t_q}h_q(\tau)g(\tau)\,d\tau$ — is the value representing the dot product of the signal $g(\tau)$ and the function $h_q(\tau)$.

The system of functions $\{h_q(\tau)\}$ is not complete in the general case and forms a subspace in the space of input signals $g(\tau)$, which can be divided into a subspace L and its orthogonal complement $\bar{L}$, so that the signal $g(\tau)$ can be represented in the form $g(\tau)=g_L(\tau)+\bar{g}_L(\tau)$,

where $g_L(\tau)$ the functions belong to the subspace L, and the functions $\bar{g}_L(\tau)$ - belong to the subspace $\bar{L}$ and are orthogonal to the functions $g_L(\tau)$, i.e. all functions $h_q(\tau)$:

$(g_L,\bar{g}_L)=0$; $(\bar{g}_L,h_q)=0$.

Equations (2) and (3) do not allow the functions $g(\tau)$ to be determined unambiguously. Adding any function $g_L(\tau)$ from the orthogonal complement $\bar{L}$ to the function $\bar{g}_L(\tau)$ does not change the equations, since

$$(h_q,g_L+\bar{g}_L)=(h_q,g_L)=f_q$$

This is similar to when in ordinary three-dimensional space, for example, two projections of a vector on the axis in the XY plane are known, and the component of the vector along the Z axis remains arbitrary.

Thus, equation (2) only defines the input signal component $g_L(\tau)$, and the signal component $\bar{g}_L(\tau)$ cannot be found from (2) or (3). To determine it (if required), it is necessary to attract additional (a priori) information that is not contained in the main equation (2), which can to some extent reconstruct the signal component $\bar{g}_L$ lost in the process of analog-to-digital conversion. But this will lead to an increase in the signal rate (its energy or power). Indeed, of all possible solutions to equations (2) or (3), the solution $g_L(\tau)$ has the smallest norm. This follows from the fact that for the square of the norm $g(\tau)$, taking into account the orthogonality of the functions $g_L(\tau)$ and $\bar{g}_L(\tau)$, the equality is true:

$$\|g(\tau)\|^2=\|g_L+\bar{g}_L\|^2=\|g_L\|^2+\|\bar{g}_L\|^2$$

This shows that the signal has the smallest norm $g(\tau)=g_L(\tau)$ at $\bar{g}_L(\tau)=0$. In the case when the signal energy $g_L(\tau)$ is unlimited, for example, for a periodic signal, the norm is understood as the average signal power $g_L(\tau)$. So, the signal $g_L(\tau)$ does not contain a priori information about the input signal $g(\tau)$ and has a minimum norm, and therefore, according to the definition introduced above, $g_L(\tau)$ it is an approximating (skeletal) signal. Assuming the functions $h_q(\tau)$ to be linearly independent, we write

$$g_L(\tau)=\sum_{n=-\infty}^{\infty}g_nh_n(\tau) \tag{4}$$

where g_n are coefficients that are not equal to zero at the same time. The function $g_L(\tau)$ approximates the input signal $g(\tau)$ and, as can be seen from (4), impulse functions are the basis functions $h_n(\tau)=h(t_n-\tau)$. Consequently, when constructing an approximating signal $g_L(\tau)$, the basis functions, in contrast to the known approximation methods, are not set a priori, but are directly related to the properties of the ADC, expressed by its impulse response. Substituting (4) into (3), we obtain a system of equations for determining the coefficients g_n

$$\sum_{n=-\infty}^{\infty} k_{qn} g_n = f_q \,. \tag{5}$$

where the matrix k_{qn} is

$$k_{qn} = \int_{-\infty}^{\infty} h_q(\tau) h_n(\tau) d\tau = (h_q, h_n) \tag{6}$$

The determinant of a matrix $Det\|k_{qn}\|$ is the Gram determinant of a system of functions $h_q(\tau)$ and, if they are independent, does not have zero eigenvalues. Therefore, the solution of the system of equations (5) for the quantities is unique [18, 19]. We get it for an unlimited time interval and uniform discretization with an interval Δt. The matrix k_{qn} described by expression (6) is infinite-dimensional, depending on the difference between the indices:

$$k_{qn} = \int_{-\infty}^{\infty} h(q\Delta t - \tau) h(n\Delta t - \tau) d\tau \text{, or}$$

$$k_{qn} = \int_{-\infty}^{\infty} h[(q-n)\Delta t + x]\, h(x) dx = k(q-n) \,. \tag{7}$$

Therefore, the solution of the system of equations (5), i.e. the coefficients g_q and, therefore, the approximating signal $g_L(\tau)$, according to (4), can be found explicitly using the Fourier transform. Let us introduce the Fourier transforms of a discrete signal f_q and a system of coefficients g_q:

$$\begin{cases} F(\omega) = \sum\limits_{q=-\infty}^{\infty} f_q e^{-jq\omega\Delta t}; \\ G(\omega) = \sum\limits_{q=-\infty}^{\infty} g_q e^{-jq\omega\Delta t}. \end{cases} \tag{8}$$

Since the functions $F(\omega)$ and $G(\omega)$ are periodic with a period $2\pi/\Delta t$, the values of the frequency ω are limited by the interval $-\pi/\Delta t \le \omega \le \pi/\Delta t$.

The inverse Fourier transforms for the functions $F(\omega)$ and $G(\omega)$ in (8) have the form

$$\begin{cases} f_q = \dfrac{\Delta t}{2\pi} \int\limits_{-\pi/\Delta t}^{\pi/\Delta t} F(\omega) e^{jq\omega\Delta t} d\omega; \\ g_q = \dfrac{\Delta t}{2\pi} \int\limits_{-\pi/\Delta t}^{\pi/\Delta t} G(\omega) e^{jq\omega\Delta t} d\omega. \end{cases} \tag{9}$$

Performing the Fourier transform of equations (5), we obtain

$$G(\omega) = F(\omega)/\lambda(\omega) \,.$$

where

$$\lambda(\omega) = \sum_{q=-\infty}^{\infty} k(q) e^{-jq\omega\Delta t} = k(0) + 2\sum_{q=1}^{\infty} k(q) \cos q\omega\Delta t \tag{10}$$

- eigenvalues (spectrum) of the operator with matrix elements $k_{qn} = k(q-n)$.

Substituting the equality for g_q from (9) into (4), we find the solution to equations (5):

$$g_L(\tau) = \frac{\Delta t}{2\pi} \int_{-\pi/\Delta t}^{\pi/\Delta t} \frac{F(\omega)\psi(\omega,\tau)}{\lambda(\omega)} d\omega \,, \tag{11}$$

where

$$\psi(\omega,\tau) = \sum_{q=-\infty}^{\infty} h(q\Delta t - \tau) e^{jq\omega\Delta t} \,. \tag{12}$$

The system of functions $\psi(\omega,\tau)$ forms an orthogonal (but not normalized) basis in the space L. Indeed, using expressions (7), (10) and the equality

$$\sum_{q=-\infty}^{\infty} e^{jq(\omega'-\omega)\Delta t} = \frac{2\pi}{\Delta t}\delta(\omega'-\omega) \text{ at } -\frac{\pi}{\Delta t} \le \omega; \omega' \le \frac{\pi}{\Delta t}, \tag{13}$$

get

$$\int_{-\infty}^{\infty} \psi(\omega,\tau)\psi(\omega',\tau) d\tau = \frac{2\pi}{\Delta t}\lambda(\omega)\delta(\omega'-\omega) \text{ at}$$

$$-\frac{\pi}{\Delta t} \le \omega; \;\; \omega' \le \frac{\pi}{\Delta t}, \tag{14}$$

where $\delta(\omega'-\omega)$ - delta function.

Let us associate the functions $\psi(\omega,\tau)$ and eigenvalues $\lambda(\omega)$ with the Fourier transform of the transfer function of the EAU. We denote through $q = [\tau/\Delta t]$ the integer part, and through the $\gamma = \{\tau/\Delta t\}$ fractional part of the value $\tau/\Delta t$, then $\tau = (q+\gamma)\Delta t$. Shifting the origin in (12), we find

$$\psi(\omega,\gamma) = e^{jq\omega\Delta t} H^*(\omega,\gamma) \tag{15}$$

where

$$H(\omega,\gamma) = \sum_{i=1}^{\infty} h[(i-\gamma)\Delta t] e^{-ji\omega\Delta t} \tag{16}$$

- Fourier transform of the transfer function or the frequency response of the EAU; $*$ - complex conjugation sign.

Since the function $\gamma = \{\tau/\Delta t\}$ is periodic in τ terms of a period Δt, the function $H(\omega,\gamma)$ is also periodic in τ terms of a period Δt. Its inverse Fourier transform

$$h[(i-\gamma)\Delta t] = \frac{\Delta t}{2\pi} \int_{-\pi/\Delta t}^{\pi/\Delta t} H(\omega,\gamma) e^{ji\omega\Delta t} d\omega \tag{17}$$

Let us express the eigenvalues $\lambda(\omega)$ in terms of the frequency response $H(\omega,\gamma)$. To do this, we use relation (7), replacing it x with $(-x)$ and dividing the integration interval into sections Δt:

$$k(q) = \int_{-\infty}^{\infty} h(q\Delta t - x) h(-x) dx = \sum_{i=-\infty}^{\infty} \int_{i\Delta t}^{(i+1)\Delta t} h(q\Delta t - x) h(-x) dx.$$

In each integral, we make the change of variables $x = (i+\gamma)\Delta t$, than

$$k(q)=\Delta t\sum_{i=-\infty}^{\infty}\int_0^1 h[(q-i-\gamma)\Delta t]\,h[-(i+\gamma)\Delta t]\,d\gamma .$$

Let us substitute expression (17) into this equality, replacing for convenience of calculation $h[-(i+\gamma)\Delta t]$ by the complex conjugate (equal to it due to reality) term. Using (13), we obtain

$$k(q)=\frac{(\Delta t)^2}{2\pi}\int_0^1 d\gamma\int_{-\pi/\Delta t}^{\pi/\Delta t} d\omega\left|H(\omega,\gamma)\right|^2 e^{jq\omega\Delta t} .$$

Taking this expression into account, we transform (10) to the form

$$\lambda(\omega)=\Delta t\int_0^1\left|H(\omega,\gamma)\right|^2 d\gamma . \quad (18)$$

The physical meaning of formula (18) is quite obvious: the eigenvalues or spectrum of the operator $\hat{k}$ (matrix k_{qn}) is obtained by averaging the square of the modulus of the frequency response of the $H(\omega,\gamma)$ EAU over the initial moment (initial phase) of the discrete signal f_q. One of the consequences of such averaging is that even if $H(\omega,\gamma)$ it has isolated zeros (for example, for the frequency response of the current average), then these zeros are eliminated in the spectrum $\lambda(\omega)$, i.e. skeletal solution $g_L(\tau)$ remains regular in this case.

After substituting relations (15) and (18) into (11), we obtain the final formula for the regularized solution of equation (2):

$$g_L(\tau)=\frac{1}{2\pi}\int_{-\pi/\Delta t}^{\pi/\Delta t} F(\omega)e^{jq\omega\Delta t}H^*(\omega,\gamma)\,d\omega\times \left[\int_0^1\left|H(\omega,\gamma)\right|^2 d\gamma\right]^{-1} . \quad (19)$$

It is interesting to compare the resulting skeletal solution $g_L(\tau)$ with the known regularized ones. Thus, in the method proposed in [20], the regularized solution is obtained in the form

$$g_L(\tau)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{F(\omega)e^{j\omega\tau}H^*(\omega)}{\left|H(\omega)\right|^2+\alpha^2}\,d\omega ,$$

where $H(\omega)$ - Fourier transform of the transfer function; α Is a regularization parameter that dampens small values of the frequency response in the high frequency region.

The regularizing parameter in (19) is the discretization interval Δt. Damping of small values of the frequency response $H(\omega,\gamma)$ of the EAU in the high-frequency region is achieved by folding the spectrum in the periodicity interval $-\pi/\Delta t<\omega<\pi/\Delta t$, and the elimination of isolated function zeros $H(\omega)$ is achieved by averaging the frequency $H(\omega,\gamma)$ response over the discretization interval Δt.

Let us compare the skeletal solution $g_L(\tau)$, defined by formula (19) with the one obtained by replacing the integral in (2) by the sum, i.e. provided that not only the output signal of the EAU is sampled, but also its input signal $g(\tau)$ and transfer function. In this case, equality (2) is replaced by the system of linear equations

$$\sum_{n=-\infty}^{\infty} h[(q-n)\Delta t]\,g_n = f_q ,$$

where $g_n = g(n\Delta t)$ - sampled input signal $g(\tau)$ EAU.

The solution to this system of equations has the form

$$g(q\Delta t)=g_q=\frac{1}{2\pi}\int_{-\pi/\Delta t}^{\pi/\Delta t}\frac{F(\omega)e^{jq\omega\Delta t}}{H(\omega)}d\omega \quad (20)$$

where $H(\omega)=H(\omega,0)$.

At the same discrete points $\tau=q\Delta t$, skeletal solution (19) gives

$$g_L(q\Delta t)=\frac{1}{2\pi}\int_{-\pi/\Delta t}^{\pi/\Delta t}F(\omega)e^{jq\omega\Delta t}H^*(\omega)\,d\omega\times \times\left[\int_0^1\left|H(\omega,\gamma)\right|^2 d\gamma\right]^{-1} \quad (21)$$

Comparison of expressions (20) and (21) shows that the isolated zeros of the frequency response of the EAU are eliminated in the solution $g_L(q\Delta t)$ compared to the solution $g(q\Delta t)$. Therefore, if, for example, the noise contains components whose frequencies fall on isolated zeros of the function $H(\omega)$, then solution (20) will be unstable, while solution (21) remains stable, i.e. a sharp increase in noise in the skeletal signal $g_L(q\Delta t)$ does not occur. In addition, averaging over the initial discretization phase also leads to a decrease in the skeletal solution fallibility.

Formula (19) makes it possible to establish the form of the solution not only at discrete points $\tau=q\Delta t$, but also at intermediate points, depending on the input signal $g(\tau)$. To do this, substitute equality (8) for $F(\omega)$ and, using expressions (2) and (15), we obtain

$$g_L(\tau)=\int_{-\pi/\Delta t}^{\pi/\Delta t}L(\tau,\tau')\,g(\tau)\,d\tau \quad (22)$$

where

$$L(\tau,\tau')=\frac{1}{2\pi}\int_{-\pi/\Delta t}^{\pi/\Delta t}H(\omega,\gamma)H^*(\omega,\gamma')e^{(q'-q)\omega\Delta t}d\omega\times \left[\int_0^1\left|H(\omega,\gamma)\right|^2 d\gamma\right]^{-1}; \quad (23)$$

$$q=\left[\frac{\tau}{\Delta t}\right];\quad q'=\left[\frac{\tau'}{\Delta t}\right];\quad \gamma=\left\{\frac{\tau}{\Delta t}\right\};\quad \gamma'=\left\{\frac{\tau'}{\Delta t}\right\}.$$

An operator $\hat{L}$ with matrix elements $L(\tau,\tau')$ is a projection operator from a space $L+\overline{L}$ onto a subspace L.

Let us obtain the dependence of the approximating (skeletal) signal $g_L(\tau)$ on the input signal $g(\tau)$ of the EAU. Let us first consider a special case when the impulse response $h(\tau)$ of the EAU changes little within the discretization interval Δt. Under this condition, the frequency response of the $H(\omega,\gamma)$ EAU is practically independent of γ and from (23) we have

$$L(\tau,\tau')=\frac{1}{2\pi}\int_{-\pi/\Delta t}^{\pi/\Delta t}e^{(q'-q)\omega\Delta t}d\omega=$$
$$=\frac{1}{\Delta t}\delta_{q'q}=\frac{1}{\Delta t}\begin{cases}1 \text{ at } q'=q;\\ 0 \text{ at } q'\neq q.\end{cases} \quad (24)$$

Then for the skeletal solution $g_L(\tau)$ from (22) taking into account (24) on the interval $q\Delta t<\tau<(q+1)\Delta t$ we find

$$g_L(\tau)=\frac{1}{\Delta t}\int_{q\Delta t}^{(q+1)\Delta t}g(\tau')d\tau';\ q\Delta t<\tau<(q+1)\Delta t . \quad (25)$$

Thus, for the particular case under consideration, the skeletal signal $g_L(\tau)$ represents a stepwise approximation of the input signal $g(\tau)$, and the value of the function $g_L(\tau)$ at each discretization interval Δt is the average value of the signal $g(\tau)$ in this interval.

In the general case, when the impulse response h(τ) of the EAU can change noticeably within the discretization interval Δt, the value of the skeletal signal $g_L(\tau)$ is a weighted average of the signal $g(\tau)$ not only over this one, but also over neighboring discretization intervals. The type of the weighting function is determined by the impulse response $h(\tau)$.

The obtained solutions for the approximating (skeletal) signal $g_L(\tau)$ make it possible to find an expression for estimating the relative approximation fallibility ε_1 using the formula for the relative variance of this fallibility

$$\varepsilon_1^2=\frac{\|g-g_L\|^2}{\|g\|^2} \quad (26)$$

If the discretization rate is large enough so that the impulse response $h(\tau)$ of the EAU changes little over the discretization interval Δt, then the signal $g_L(\tau)$ is determined by formula (25), and the approximation fallibility represents the fallibility of the step approximation. If, moreover, within each discretization interval Δt the signal $g(\tau)$ changes smoothly, without sharp bursts and jumps, then it can be expanded in intervals Δt in a Taylor series and limited to the linear term

$$g(\tau)=g(q\Delta t)+g'(q\Delta t)(\tau-q\Delta t);\ q\Delta t<\tau<(q+1)\Delta t . \quad (27)$$

This decomposition is valid provided

$$\Delta t\cdot g''(\tau)<<g'(\tau)$$

Substituting (27) into (15) and performing calculations, we obtain

$$g_L(\tau)=g(q\Delta t)+g'(q\Delta t)\cdot\Delta t/2 \quad (28)$$

Relations (27) and (28) allow expressing the relative fallibility of approximation ε_1 as a function of the discretization interval Δt. For this we calculate

$$\|g-g_L\|^2=\sum_{q=-\infty}^{\infty}\int_{q\Delta t}^{(q+1)\Delta t}[g(\tau)-g_L(\tau)]^2d\tau=$$
$$=\frac{1}{12}(\Delta t)^2\sum_{q=-\infty}^{\infty}[g'(q\Delta t)]^2\Delta t\approx\frac{(\Delta t)^2}{12}\int_{-\infty}^{\infty}[g'(\tau)]^2d\tau.$$

After substituting this equality in (26), we find

$$\varepsilon_1\approx\Delta t/\theta . \quad (29)$$

where

$$\theta=\left\{\int_{-\infty}^{\infty}[g'(\tau)]^2d\tau\Big/\left[12\int_{-\infty}^{\infty}g^2(\tau)d\tau\right]\right\}^{-1/2} \quad (30)$$

Since formula (29) is obtained by expanding into a Taylor series up to the first term, the smaller the fallibility, the more accurate it is ε_1.

The value θ determined by equality (30), in the considered approximation, does not depend on the discretization interval Δt and is a parameter characterizing the temporal properties of the input signal $g(\tau)$. Let's call it the characteristic time, and the shorter it is, the faster and sharper the signal changes $g(\tau)$.

The presence of interference in the output discrete signal f_q of an ECB (or ADC) caused by noise and fallibilitys, in particular, quantization noise, makes it impossible to accurately reconstruct even an approximating signal $g_L(\tau)$. Since the frequency characteristics of real measuring transducers in the EAU decrease at sufficiently high frequencies, then when passing from a discrete signal f_q to the original signal $g(\tau)$, the signal-to-noise ratio decreases and for an unregulated solution it can reach zero, even with a zero signal-to-noise ratio in the analog signal $f(t)$. Its discretization regularizes the solution so that the signal-to-noise ratio in the reconstructed signal $g_L(\tau)$ remains finite, not equal to zero. It depends on the discretization frequency and the more, the lower this frequency. Let's prove it. Let us find the dependence of the signal-to-noise ratio in the signal $g_L(\tau)$on the discretization frequency (interval) for a given signal-to-noise ratio in a discrete signal f_q. Let us denote ξ_l by the random fallibility (noise) in this signal, then for the random fallibility of the signal spectrum f_q in accordance with equality (8) for $F(\omega)$ we have

$$\Delta F(\omega)=\sum_{l=-\infty}^{\infty}\xi_l e^{-jl\omega\Delta t} .$$

The fallibility $\Delta g_L(\tau)$ in the approximating signal $g_L(\tau)$, as follows from formula (11):

$$\Delta g_L(\tau)=\frac{\Delta t}{2\pi}\int_{-\pi/\Delta t}^{\pi/\Delta t}\frac{\Delta F(\omega)\cdot\psi(\omega,\tau)}{\lambda(\omega)}d\omega .$$

We obtain an expression for the relative variance of the random fallibility ε_2^2, i.e. for the ratio of the interference power $\|\Delta g_L\|^2$ to the signal power $\|g_L(\tau)\|^2$. Taking into account the orthogonality of the functions $\psi(\omega,\tau)$ according to (14), we find

$$\varepsilon_2^2=\frac{\|\Delta g_L(\tau)\|^2}{\|g_L(\tau)\|^2}=\int_{-\pi/\Delta t}^{\pi/\Delta t}\frac{S(\omega)}{\lambda(\omega)}d\omega\left[\int_{-\pi/\Delta t}^{\pi/\Delta t}\frac{S_0(\omega)}{\lambda(\omega)}d\omega\right]^{-1}, \quad (31)$$

where $S(\omega)$ is the spectral power density of the interference ξ_l in the discrete signal f_q; $S_0(\omega)$ - spectral power density of the signal f_q.

In the most unfavorable case, when the interference spectrum in the signal f_q is concentrated near the smallest value $\lambda(\omega)$ equal to λ_{min}, and the signal spectrum f_q is near the largest value $S(\omega)$ equal to λ_{max}, from (31) we have

$$\varepsilon_2' = \sqrt{\lambda_{max}/\lambda_{min}}\sqrt{P/P_0}\ , \tag{32}$$

where $P = \int_{-\pi/\Delta t}^{\pi/\Delta t} S(\omega)d\omega$ - the power of the interference ξ_l in the signal f_q; $P_0 = \int_{-\pi/\Delta t}^{\pi/\Delta t} S_0(\omega)d\omega$ - signal strength f_q.

If the interference ξ_l is uniformly distributed over the spectrum of the discrete signal f_q within the limits $(-\pi/\Delta t, \pi/\Delta t)$, then a smaller value of the fallibility is obtained

$$\varepsilon_2'' = \left[\lambda_{max}\left(\frac{1}{\lambda}\right)_m\right]^{1/2}\sqrt{P/P_0} \tag{33}$$

where

$$\left(\frac{1}{\lambda}\right)_m = \frac{\Delta t}{2\pi}\int_{-\pi/\Delta t}^{\pi/\Delta t}\frac{d\omega}{\lambda(\omega)}$$

The quantity P_0/P is the signal-to-noise ratio at the output of the ECB (or ADC), and the quantity $(\varepsilon_2')^2$, $(\varepsilon_2'')^2$ is the signal-to-noise ratio in the reconstructed skeletal signal $g_L(\tau)$. Therefore, the noise generation factor k_n, which is equal to the signal-to-noise ratio of the skeletal signal $g_L(\tau)$ divided by the signal-to-noise ratio of the discrete signal f_q, in the most unfavorable case is determined, as follows from (32), by the expression

$$k_n' = \lambda_{max}/\lambda_{min} \tag{34}$$

and with a uniform distribution of interference over the signal f_q spectrum according to (33):

$$k_n'' = \lambda_{max}(1/\lambda)_m \tag{35}$$

From formulas (32) and (33), taking into account equalities (34) and (35), it can be seen that the fallibilitys ε_2' and ε_2'' are related to the noise generation coefficients k_n' and k_n'' the relations

$$\varepsilon_2' = \sqrt{k_n'}\cdot\eta \quad \varepsilon_2'' = \sqrt{k_n''}\cdot\eta \tag{36}$$

where $\eta^2 = P/P_0$ is the relative power of the interference in the discrete signal f_q, i.e. interference / signal ratio at the output of the ECB (or ADC).

For a fixed input signal $g(\tau)$, the relative approximation fallibility ε_1 is systematic, and the value ε_2^2, i.e. $(\varepsilon_2')^2$ or $(\varepsilon_2'')^2$, - the relative variance of the random fallibility.

Relationships (34), (35), (36) determine the relative fallibility in the reconstructed input signal $\hat{g}(\tau)$ caused by a random fallibility (interference) in the output discrete signal f_q of the ECB, depending on the value of the discretization interval and the type of impulse response of the EAU. Together with formulas (26), (29) for the relative approximation fallibility, they make it possible to reasonably approach the determination of the optimal discretization interval (frequency). Since the relative fallibility increases with an increase in the discretization interval Δt, such an optimum exists. In the simplest case, the optimal value of the discretization interval Δt can be found from the condition $\varepsilon_1 = \varepsilon_2$ or from the condition of the minimum total fallibility.

4. Discussion of Experimental Results

To illustrate the proposed method for determining the optimal discretization interval (frequency), let us consider the simplest example in which calculations can be carried out analytically.

Example. Let the EAU be a simple aperiodic link with an impulse response

$$h(\tau) = \begin{cases}\alpha e^{-\alpha\tau}, & \tau > 0;\\ 0 & , \tau < 0,\end{cases} \tag{37}$$

where α^{-1} - link time constant.

Let's calculate the frequency response $H(\omega,\gamma)$ of the EAU. Substituting (37) into (16), we obtain

$$H(\omega,\gamma) = \alpha\sum_{i=1}^{\infty} e^{-\alpha[(i-\gamma)\Delta t] - ji\omega\Delta t} = e^{\alpha\gamma\Delta t}H(\omega) \tag{38}$$

where

$$H(\omega) = \alpha\sum_{i=1}^{\infty} e^{-(\alpha+j\omega)i\Delta t} = \alpha\left[e^{(\alpha+j\omega)\Delta t} - 1\right]^{-1}.$$

The function $H(\omega)$ does not depend on the fractional part $\gamma = \{\tau/\Delta t\}$, therefore

$$\int_0^1 \left|H(\omega,\gamma)\right|^2 d\gamma = b(\alpha)\left|H(\omega)\right|^2 , \tag{39}$$

where

$$b(\alpha) = \int_0^1 e^{2\alpha\gamma\Delta t}d\gamma = \frac{1}{2\alpha\Delta t}\left(e^{2\alpha\Delta t} - 1\right)$$

Is a numerical coefficient.

Substituting (38), (39) into (19), we find the approximating (skeletal) signal

$$g_L(\tau) = \frac{e^{\alpha\gamma\Delta t}}{2\pi\alpha\, b(\alpha)}\int_{-\pi/\Delta t}^{\pi/\Delta t} F(\omega)e^{jq\omega\Delta t}\left[e^{(\alpha+j\omega)\Delta t} - 1\right]d\omega.$$

After substituting expressions (8) for $F(\omega)$ this formula, we finally obtain

$$g_L(\tau) = \frac{2e^{\alpha\gamma\Delta t}}{e^{\alpha\Delta t} - 1}\left(e^{\alpha\Delta t} f_{q+1} - f_q\right)$$

The signal $g_L(\tau)$ is a multiplier modulated step curve $exp[\alpha\{\tau/\Delta t\}\Delta t]$. If $\alpha\Delta t << 1$, then the modulation disappears and the signal approaches the stepped curve

$$g_L(\alpha) = \frac{1}{2\alpha\Delta t}\left(f_{q+1} - f_q\right).$$

From (18) we find

$$\lambda(\omega) = b(\alpha)\Delta t\left|H(\omega)\right|^2 =$$
$$= \alpha^2 \Delta t b(\alpha)\left(e^{2\alpha\Delta t} + 1 - 2e^{\alpha\Delta t}\cos\omega\Delta t\right)^{-1}.$$

From this we get:

$$\lambda_{max} = \alpha^2 \Delta t b(\alpha)\left(e^{\alpha\Delta t} - 1\right)^{-2}$$
$$\lambda_{min} = \alpha^2 \Delta t b(\alpha)\left(e^{\alpha\Delta t} + 1\right)^{-2}$$
$$(1/\lambda)_m = \frac{1}{\alpha^2 \Delta t b(\alpha)}\left(e^{2\alpha\Delta t} + 1\right)$$

From (34), (35), we calculate the noise generation coefficients:

$$k_n' = \left(e^{\alpha\Delta t} + 1\right)^2\left(e^{\alpha\Delta t} - 1\right)^{-2} \quad k_m'' = \left(e^{2\alpha\Delta t} + 1\right)\left(e^{\alpha\Delta t} - 1\right)^{-2}.$$

Determination of the optimal discretization interval Δt_o for the simplest condition $\varepsilon_1 = \varepsilon_2$ and the most unfavorable, concentrated, interference leads to the equation

$$\frac{\Delta t_o}{\theta} = \frac{e^{\alpha\Delta t_o} + 1}{e^{\alpha\Delta t_o} - 1}\sigma,$$

where σ is the relative mean value of the interference.

We introduce dimensionless variables in this equation $\zeta = \alpha\Delta t_o$ and $\beta = \alpha\theta\sigma$, after which we transform to the form

$$\zeta = \beta\frac{e^{\zeta} + 1}{e^{\zeta} - 1}. \tag{40}$$

Similarly, with a uniform distribution of interference in the output signal f_q of the EDU over its spectrum for the optimal discretization interval Δt_o, we obtain the equation

$$\frac{\Delta t_o}{\theta} = \frac{\sqrt{e^{2\alpha\Delta t_o} + 1}}{e^{\alpha\Delta t_o} - 1}\sigma \;.$$

In the same dimensionless variables, this equation takes the form

$$\zeta = \beta\frac{\sqrt{e^{2\zeta} + 1}}{e^{\zeta} - 1}. \tag{41}$$

In figure 2 shows the graphs of the dependence of solutions $\zeta = \alpha\Delta t_o$ on the parameter $\beta = \alpha\theta\sigma$ provided $\varepsilon_1 = \varepsilon_2$.

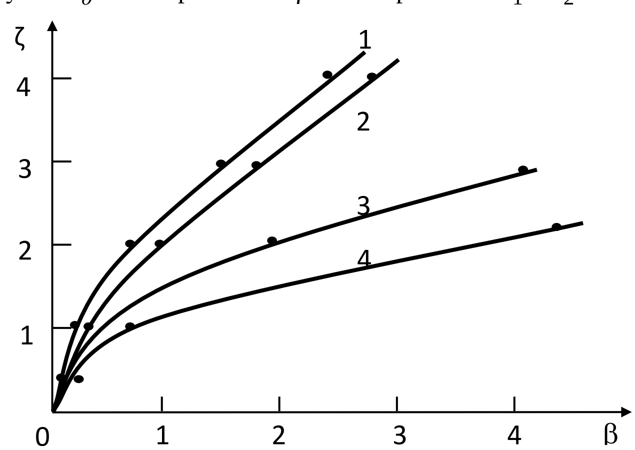

Figure 2. Graphs of the dependence of the dimensionless variable ζ on the parameter β for various types of noise

Curve 1 for equation (40) is frequency-concentrated interference and curve 2 for equation (41) is uniformly distributed interference over the spectrum of the EDU output signal f_q.

The same figure shows the dependences of solutions $\zeta = \alpha\Delta t_o$ on the parameter $\beta = \alpha\theta\sigma$, determined by the minimum of the total relative recovery fallibility $\varepsilon = \varepsilon_1 + \varepsilon_2$, for concentrated interference (curve 3) and uniformly distributed interference over the spectrum of the EDU output signal f_q (curve 4).

In figure 3 shows the dependences of the normalized fallibility ε/σ on the value ζ, and the numbering of the curves corresponds to the same four options for which the curves in Fig. 2.

In the limiting cases, the solution to equations (40) and (41) is easily found analytically. So, for $\alpha\Delta t_o << 1$, using the expansion in a power series up to the first term, we have

$e^{\alpha\Delta t_o} + 1 \approx 2$; $e^{\alpha\Delta t_o} - 1 \approx \alpha\Delta t_o$; $e^{2\alpha\Delta t_o} + 1 \approx 2$,

therefore, the solution to equation (40)

$$\Delta t_o \approx \sqrt{2\sigma\theta/\alpha}, \ \sigma\theta\alpha << 1,$$

and the solution to equation (41)

$$\Delta t_o \approx \sqrt{\sqrt{2}\sigma\theta/\alpha} = \sqrt{1{,}41\sigma\theta/\alpha}$$

For relative fallibilitys, we get:

- for interference concentrated in frequency in the spectrum of the EDU output signal f_q:

$$\varepsilon_1 = \varepsilon_2' = 2\sigma/(\alpha\Delta t_o);$$

for interference uniformly distributed over the spectrum of the EDU output signal f_q:

$$\varepsilon_1 = \varepsilon_2'' = \sqrt{2}\sigma/(\alpha\Delta t_o).$$

Since $\alpha\Delta t_o << 1$, when the input signal $g(\tau)$ is restored, the interference is amplified and restoration is possible only with a sufficiently low level of interference ξ in the output signal f_q of the EDU. In this case, the signal $g(\tau)$ details are restored at discretization intervals Δt_o that are much smaller than the averaging interval α of the EAU impulse response, i.e. "super resolution" occurs

.

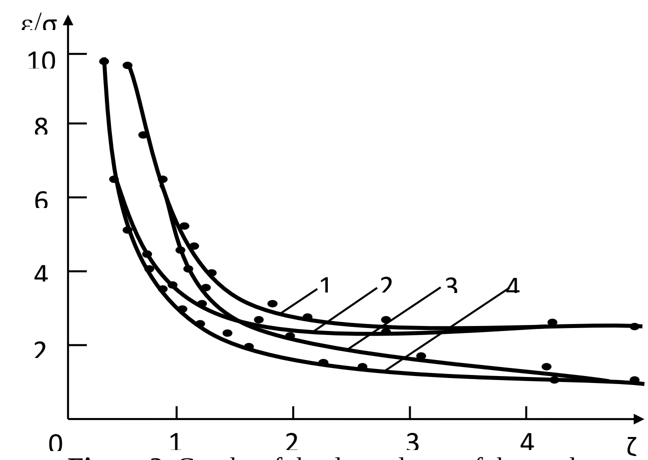

Figure 3. Graphs of the dependence of the random discretization error ε/σ on ζ for various types of noise

For $\alpha\Delta t_o >> 1$ both from equation (40) and from equation (41) we have $\Delta t_o \approx \sigma\theta$; $\sigma\theta\alpha >> 1$

In this case $\varepsilon_1 = \varepsilon_2' = \varepsilon_2'' = \sigma$, no amplification of the interference occurs during signal $g(\tau)$ recovery.

For interference, concentrated and evenly distributed over the spectrum of the EDU output signal f_q, from equations (40) and (41) we obtain the relationship between the interference component of the fallibility and the approximation fallibility:

$$\varepsilon_2' = \sigma \frac{e^{\alpha\theta\varepsilon_1} + 1}{e^{\alpha\theta\varepsilon_1} - 1} \tag{42}$$

$$\varepsilon_2'' = \sigma \frac{\sqrt{e^{2\alpha\theta\varepsilon_1} + 1}}{e^{\alpha\theta\varepsilon_1} - 1} \tag{43}$$

It can be seen from formulas (42), (43) that the desire to reduce one of the components of the reconstruction fallibility leads to an increase in its other component. So, in the region of “superresolution” (at $\alpha\theta\varepsilon_1 << 1$) we have:

$$\varepsilon_2' \approx 2\sigma/(\alpha\theta\varepsilon_1);\ \varepsilon_2'' \approx \sqrt{2}\sigma/(\alpha\theta\varepsilon_1).$$

5. Conclusions

The analysis revealed that the optimal sampling rate (or interval) for analog-to-digital signal processing is directly related, first, to the time characteristic of the input signal. Secondly, with pulse or frequency response. And, thirdly, with the level of interference in the output signal, ie with all the parameters and characteristics of the actual measuring channel.

It is proved that to determine the optimal sampling interval there is no need to carry out the actual reconstruction of the input signal, although it is possible to do so by the obtained expressions. If there is a priori information about the input signal, the approximating signal may be supplemented by a signal orthogonal to it, to take into account such information. Just keep in mind that this increases the power of the input signal. In practice, it is sufficient to know the characteristic time of the input signal, the relative dispersion of the interference in the output signal, and the frequency spectrum determined, respectively, in the most unfavorable case, knowledge of the ratio of frequency-focused interference. This makes it possible to determine both components of the error that affect the choice of sampling frequency - the approximation error and the noise component of the error as a function of the sampling interval. Using one of the criteria of optimality finds the optimal sampling interval (or frequency). In this example, the criterion of equality of the components of infallibility and the criterion of a minimum total error lead to close values of the optimal sampling interval, this also occurs in the General case.

Thus, reducing the sampling rate below the optimum leads to an increase in the approximation error and the loss of some information about the input signal. At the same time, unjustified oversampling, which complicates the technical implementation of the device, is not useful because it not only does not increase the information about the input signal but if necessary restores it leads to its reduction by increasing the noise effect of the output signal on recovery accuracy. input signal. The proposed method of signal sampling, based on the minimum error of information recovery to determine the optimal sampling rate, allows us to resolve this contradiction.

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