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Resource Allocation in Wireless Networks

Theory and Algorithms

Springer

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To our families

Der König hat viele Gnade für meine geringen Dienste, und das Publikum viel Nachsicht für die unbedeutenden Versuche meiner Feder; ich wünschte, dass ich einigermaßen etwas zu der Verbesserung des Geschmackes in meinem Lande, zur Ausbreitung der Wissenschaften beitragen könnte. Denn sie sind's allein, die uns mit anderen Nationen verbinden, sie sind's, die aus den entferntsten Geistern Freunde machen, und die angenehmste Vereinigung unter denen selbst erhalten, die leider durch Staatsverhältnisse öfters getrennt werden.

Clavigo, Johann Wolfgang von Goethe

Nader wiele łaski raczy mi okazywać król za moje skromne służby, publiczność zaś zbyt jest wyrozumiała dla niepozornych płodów mego pióra, byłbym wszelako szczęśliwy mogąc się nieco przyczynić do urobienia literackiego smaku w moim kraju, do rozprzestrzenienia nauk. Jedynie to bowiem może nas zbliżyć do innych nacji, tylko dzięki temu zdobywamy w najdalszych stronach przyjaciół pośród przodujących umysłów, przyczyniając się do utrwalenia najcenniejszych więzów, które niestety jakże często zrywają interesa państwowe.

Clavigo, Johann Wolfgang von Goethe
(Tłumaczenie: Wanda Markowska)

Preface

The wireless industry is in the midst of a fundamental shift from providing voice-only services to offering customers an array of multimedia services, including a wide variety of audio, video and data communications capabilities. Future wireless networks will be integrated into every aspect of daily life, and therefore could affect our life in a magnitude similar to that of the Internet and cellular phones. However, the emerging applications and directions require fundamental understanding on how to design and control wireless networks that lies far beyond what the currently existing theory can provide. We are deeply convinced that mathematics is the key technology to cope with central technical problems in the design of wireless networks since the complexity of the problem simply precludes the use of engineering common sense alone to identify good solutions.

The main objective of this book is to provide tools for better understanding the fundamental tradeoffs and interdependencies in wireless networks, with the goal of designing resource allocation strategies that exploit these interdependencies to achieve significant performance gains. The book consists of three largely independent parts: theory, applications and appendices. The first part ends with some bibliographical comments and the second part starts with a short introduction to the problem of resource allocation in wireless networks. Below we briefly summarize the content of each part.

Theory: Chapters 1 and 2 deal with some fundamental problems in the theory of nonnegative matrices and provide a theoretical basis for the resource allocation problem addressed in the second part of the book. It should be emphasized that our intent is *not* to provide a thorough treatment of this wide subject. Instead, we focus on problems that naturally appear in the design of resource allocation strategies for wireless networks. When developing such strategies, different characterizations of the Perron root of nonnegative irreducible matrices turn out to be vital to better understanding of fundamental tradeoffs between diverse optimization objectives. Our main attention will be directed to the Perron root of nonnegative irreducible matrices whose entries continuously depend on some parameter vector. In this case, the Perron root can be viewed as a map from a convex parameter set into the set of positive reals. The book is concerned with the properties of this map and, in particular, with the question under which conditions it is a convex function of the parameter vector. With few exceptions, we focus on a special structure of matrix-valued functions that is particularly relevant to applications in wireless networks. We provide necessary and sufficient conditions for the Perron root to be a convex function of the parameter vector as well as address a closely related problem of convexity of the so-called feasibility set. Chapter 2 is devoted to some properties of a positive solution to a system of linear equations with nonnegative coefficients. Applications that involve systems of linear equations with nonnegative coefficients are numerous, ranging from the physical and engineering sciences to other mathematical areas like graph theory and optimization. Such systems also occur in the power control problem for power-constrained wireless networks.

Applications: The second part of the book (Chaps. 4-6) deals with the problem of resource allocation in wireless networks. Roughly speaking, the objective is to maximize the sum of utilities of link rates for best-effort (elastic) traffic. This is equivalent to the problem of joint power control and link scheduling, which has been extensively investigated in the literature and is known to be notoriously difficult to solve, even in a centralized manner. Although the book provides some interesting insights into this problem, the main focus will be on the power control part. In particular, a class of utility functions is identified for which the power control problem can be converted into an equivalent convex optimization problem. The convexity property is a key ingredient in the development of powerful and efficient power control algorithms.

Appendices: The main purpose of the appendices is to make the book more understandable to readers who are not familiar with some basic concepts and results from linear algebra and convex analysis. The treatment is very superficial and formal proofs are presented only for the most important results such as the Perron–Frobenius theorem. Moreover, the presentation is limited to results used somewhere in the book. However, we hope that this collection of basic results will help some readers to better understand the material covered by the book. Finally, the presentation introduces the notation and terminology used throughout the book.

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And finally, we would like to thank our families for their patience, support and understanding. This book is dedicated to you.

Berlin, June 2006

Sławomir Stańczak
Marcin Wiczanowski
Holger Boche

Contents

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List of Symbols

$a, b, c, \alpha, \beta, \mu, ...$	Scalars over $\mathbb{R}$ or $\mathbb{C}$
$\mathbf{A}, \mathbf{B}, \mathbf{X}, \mathbf{Y}...$	Matrices; Sect. A.2
$\mathbf{A} \leq \mathbf{B}$	Partial ordering; Sect. A.2
$\mathbf{A}^{-1}$	Matrix inverse; Sect. A.3
$\mathbf{A}^T$	Transpose matrix; Definition A.3
$\mathrm{A}_K(\mathbf{X})$	Eq. (1.8)
$\mathrm{A} \times \mathrm{B}$	Cartesian product
A	Sect. 5.2.1
$\mathbf{A} \circ \mathbf{B}$	Hadamard product; Sect. A.2
B_K	Sect. 1.6.2
$\overline{\mathrm{B}}_K$	Sect. 1.6.2
B	Sects. 4.3 and 5.2.1
$\mathbb{C}$	Sect. A.1
$\mathrm{cl}(\mathrm{A})$	Closure
C	Eq. (5.11)
$\tilde{\mathrm{C}}$	Eq. (5.15)
$\mathrm{diag}(\mathbf{u})$	Diagonal matrix; Sect. A.2
$\det(\mathbf{A})$	Matrix determinant; Sect. A.3
δ_l	The Kronecker delta
$\mathbf{e}_i$	Sect. A.1
$\mathrm{E}_K(\mathbf{X})$	Sect. 1.6.2
$\mathrm{E}_K^+(\mathbf{X})$	Sect. 1.6.2
$\boldsymbol{\eta}(\mathbf{p})$	Eq. (6.30)
F	Eq. (1.53) and Eq. (2.5)
$\partial\mathrm{F}$	Eq. (1.55)
F^c	Eq. (1.60)

$\mathrm{F}(P_{\mathrm{t}})$	Eq. (2.9)
$\mathrm{F}(P_1, \dots, P_K)$	Eq. (2.11)
$\mathrm{F}_k(\alpha)$	Eq. (2.12)
$\mathrm{F}(P_{\mathrm{t}}; P_1, \dots, P_K)$	Eq. (2.13)
$\partial\mathrm{F}(P_{\mathrm{t}})$	Definition 2.15
$\partial\mathrm{F}(P_1, \dots, P_K)$	Definition 2.15
$\mathrm{F}^c(P_{\mathrm{t}})$	Sect. 2.4
F_γ	Eq. (5.32)
$\mathrm{F}_\gamma(\mathrm{P})$	Eq. (5.31)
$\partial\mathrm{F}_\gamma(\mathrm{P})$	Eq. (5.34)
$f'(x), x \in \mathbb{R}$	The first derivative; Sect. B.1
$f''(x), x \in \mathbb{R}$	The second derivative; Sect. B.1
$F(\mathbf{p})$	Eq. (6.2)
$F_e(\mathbf{s})$	Eq. (6.12)
$\mathbf{\Gamma}(\boldsymbol{\omega})$	Eqs. (1.48) and (5.12)
$g_k(\mathbf{p})$	(6.21)
$h_k(\mathbf{s})$	Eq. (6.14)
$\mathbf{I}$	Identity matrix; Sect. A.2
$I_k(\mathbf{p})$	Eq. (6.5)
K	Sect. 4.1
$\mathsf{K}(n)$	Sect. 4.1
$\lambda_p(\boldsymbol{\omega})$	Sect. 1.3.1
$\mathrm{LC}_K(\Omega)$	Definition 1.34
$\mathrm{lc}(\Omega)$	Sect. 2.3
L	Sect. 4.1
$\mathbb{N}$	Natural numbers
$\mathbb{N}_0$	Nonnegative integers
N	Sect. 4.1
$\mathrm{N}_K = \mathbb{R}_+^{K\times K}$	Definition A.18
N_K^+	Sect. 1.6.2
$\mathrm{N}_K(\Omega)$	Definition 1.32
$\mathrm{N}_{K,\mathbf{r}}(\Omega)$	Eq. (2.8)
$\|\mathbf{u}\|$	Vector norms; Sect. A.1
$\|\mathbf{X}\|$	Matrix norms; Sect. A.2
$\nabla_k f(\mathbf{x})$	Definition B.9
$\nabla f(\mathbf{x})$	Eq. (B.3)
$\nabla^2 f(\mathbf{x})$	Definition B.12

$\mathbf{1}$	Sect. A.1
$\Omega \subset \mathbb{R}^K$	Eq. (1.45)
Π_K	Sect. 1.1
Π_K^+	Sect. 1.1
$\mathrm{P}_K = \mathbb{R}_{++}^{K\times K}$	Definition A.18
$\mathrm{P}_K(\Omega)$	Sect.
$\boldsymbol{\omega}$	Sect. 1.3.1
$\mathbf{p}(\mathbf{X})$	Eq. (1.2)
$\mathbf{p}(\boldsymbol{\omega})$	Eq. (2.4)
$\varPhi$	Eq. (4.7)
$\varPsi$	Definition 5.5
$\varPsi_e$	Definition 5.5
ψ	Eq. (5.36)
ψ_e	Definition 6.1
$\varPi_{\mathrm{S}}$	Eq. (B.19)
$\mathrm{P} \subset \mathbb{R}_+^K$	Eq. (4.6)
P_n	Eq. (4.6)
P_+	Eq. (6.10)
$\mathbf{q}(\mathbf{X})$	Eq. (1.2)
$\mathrm{Q} \subset \mathbb{R}$	Sect. 1.3.1
$\mathbb{R}$	Real numbers
$\mathbb{R}_+ \subset \mathbb{R}$	Sect. A.4
$\mathbb{R}_{++} \subset \mathbb{R}_+$	Sect. A.4
$\mathbb{R}^K$	Sect. A.1
$\mathbb{R}^{K\times K}$	Sect. A.2
$\mathbb{R}_{++}^K(\Omega)$	Sect. 2.1
$\sigma(\mathrm{A})$	Matrix spectrum; Definition A.7
$\rho(\mathbf{X})$	Spectral radius; Definition A.7
S_K	Sect. 1.2
$\mathrm{S}_K(\mathbf{X})$	Eq. (1.3)
$\mathrm{SIR}_k(\mathbf{p})$	Eq. (4.2)
$\mathrm{S} \subset \mathbb{R}^K$	Eq. (6.11)
S	Sect. 4.1
$\mathrm{trace}(\mathbf{X})$	Matrix trace; Sect. A.2
$\boldsymbol{\theta}(\mathbf{p})$	Eq. (6.30)
$\mathbf{u} \le \mathbf{v}$	Partial ordering; Sect. A.1
$\mathbf{p}, \mathbf{q}, \mathbf{s}, \mathbf{u}, \mathbf{v}, \mathbf{z}, \ldots$	Vectors; Sect. A.1

$\mathbf{V}$	Eq. (4.4)
$\mathrm{W}_K(\mathbf{X})$	Eq. (1.14)
$\mathrm{X}_K \subset \mathrm{N}_K$	Definition A.21
$\mathrm{X}_K(\Omega)$	Definition 1.32
$\mathrm{X}_{K,\mathbf{\Gamma}}(\Omega)$	Eq. (1.49)
$\mathrm{X}^s_{K,\mathbf{\Gamma}}(\Omega)$	Sect. 1.4.1
$\mathrm{X}^p_{K,\mathbf{\Gamma}}(\Omega)$	Sect. 1.4.2
$\mathrm{X}^0_{K,\mathbf{\Gamma}}(\Omega)$	Sect. 1.5
$\mathbf{0}$	Zero vector; Sect. A.1