Ingo Wegener

Complexity Theory

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Exploring the Limits of Efficient Algorithms

Translated from the German by Randall Pruim

With 31 Figures

Springer

Ingo Wegener
Universität Dortmund
Fachbereich Informatik
Lehrstuhl Informatik II
Baroper Str. 301
44221 Dortmund
Germany
wegener@ls2.cs.uni-dortmund.de

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Preface to the Original German Edition

At least since the development of the theory of NP-completeness, complexity theory has become a central area of instruction and research within computer science. The $\mathsf{NP} \neq \mathsf{P}$-problem represents one of the great intellectual challenges of the present. In contrast to other areas within computer science, where it is often suggested that nearly all problems are solvable with the aid of computers, the goals of complexity theory include showing what computers cannot do. Delineating the boundary between problems that can be efficiently solved and those that can only be solved with an unreasonable amount of resources is a practically relevant question, but so is the structural question of what determines the complexity or "complicatedness" of problems.

The development of complexity theory is presented in this book as essentially a reaction to algorithmic development. For this reason, the investigation of practically important optimization problems plays a predominant role. From this algorithmic perspective, reduction concepts can be thought of as methods to solve problems with the help of algorithms for other problems. From this it follows conversely that we can derive the difficulty of a problem from the difficulty of other problems.

In this book we choose an unusual approach to the central concept of nondeterminism. The usual description, based on computers that guess a correct computation path or for which a suitable computation path exists, is often confusing to students encountering nondeterminism for the first time. Here this description is replaced with an introduction to randomized algorithms. Nondeterminism is then simply the special case of one-sided error with an error-rate that may be larger than is tolerable in applications. In this presentation, nondeterministic algorithms can be run on normal computers, but do not provide a satisfactory solution to problems. Based on experience, we are hopeful that this algorithmic approach will make it simpler for students to grasp the concept of nondeterminism.

Since this is not intended to be a research monograph, the content has been limited to results that are important and useful for students of computer science. In particular, this text is aimed at students who want an introduction to complexity theory but do not necessarily plan to specialize in this area. For this reason, an emphasis has been placed on informal descriptions of the proof ideas, which are, of course, followed by complete proofs. The emphasis is on modern themes like the PCP-theorem, approximation problems, randomization, and communication complexity at the expense of structural and abstract complexity theory.

The first nine chapters describe the foundation of complexity theory. Beyond that, instructors can choose various emphases:

- Chapters 10, 13, and 14 describe a more classically oriented introduction to complexity theory,
- Chapters 11 and 12 treat the complexity of approximation problems, and
- Chapters 14, 15, and 16 treat the complexity of Boolean functions.

Many ideas have come together in this text that arose in conversations. Since it is often no longer possible to recall where, when, and with whom these conversations were held, I would like to thank all those who have discussed with me science in general and complexity theory in particular. Many thanks to Beate Bollig, Stefan Droste, Oliver Giel, Thomas Hofmeister, Martin Sauerhoff, and Carsten Witt, who read the [original German] manuscript and contributed to improvements through their critical comments, and to Alice Czerniejewski, Danny Rozynski, Marion Scheel, Nicole Skaradzinski, and Dirk Sudholt for their careful typesetting.

Finally, I want to thank Christa for not setting any limits on the time I could spend on this book.

Dortmund/Bielefeld, January 2003 *Ingo Wegener*

Preface to the English Edition

This book is the second translation project I have undertaken for Springer. My goal each time has been to produce a text that will serve its new audience as well as the original book served its audience. Thus I have tried to mimic as far as possible the style and "flavor" of the original text while making the necessary adaptations. At the same time, a translation affords an opportunity to make some improvements, which I have done in consultation with the original author. And so, in some sense, the result is a translation of a second edition that was never written.

Most of the revisions to the book are quite minor. Some bibliography items have been added or updated; a number of German sources have been deleted. Occasionally I have added or rearranged a paragraph, or included some additional detail, but for the most part I have followed the original quite closely. Where I found errors in the original, I have tried to fix them; I hope I have corrected more than I have introduced.

It is always a good feeling to come to the end of a large project like this one, and in looking back on the project there are always a number of people to thank. Much of the work done to prepare the English edition of this book was done while visiting the University of Ulm in the first half of 2004. The last revisions and final touches were completed during my subsequent visit at the University of Michigan. I would like to thank all my colleagues at both institutions for their hospitality during these visits.

A writer is always the worst editor of his own writing, so for reading portions of the text, identifying errors, and providing various suggestions for improvement, I want to thank Beate Bollig, Stefan Droste, Jeremy Frens, Judy Goldsmith, André Gronemeier, Jens Jägersküpper, Thomas Jansen, Marcus Schaefer, Tobias Storch, and Dieter van Melkebeek, each of whom read one or more chapters. In addition, my wife, Pennylyn, read nearly the entire manuscript. Their volunteered efforts have helped to ensure a more accurate and stylistically consistent text. A list of those (I hope few) errors that have escaped detection until after the printing of the book will be available at

`ls2-www.cs.uni-dortmund.de/monographs/ct`

Finally, a special thanks goes to Ingo Wegener, who not only wrote the original text but also responded to my comments and questions, and read the English translation with a careful eye for details; and to Hermann Engesser and Dorothea Glaunsinger at Springer for their encouragement, assistance, and patience, and for a fine *Kaffeestunde* on a sunny afternoon in Heidelberg.

Ann Arbor, January 2005 *Randall Pruim*

It is possible to write a research monograph in a non-native language. In fact, I have done this before. But a textbook with a pictorial language needs a native speaker as translator. Moreover, the translator should have a good feeling for the formulations and a background to understand and even to shape and direct the text. Such a person is hard to find, and it is Randall Pruim who made this project possible and, as I am convinced, in a perfect way. Indeed, he did more than a translation. He found some mistakes and corrected them, and he improved many arguments. Also thanks to Dorothea Glaunsinger and Hermann Engesser from Springer for their enthusiastic encouragement and for their suggestion to engage Randall Pruim as translator.

Bielefeld/Dortmund, January 2005 *Ingo Wegener*

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