# Nonlinear Adaptive Backstepping with Estimator Resetting using Multiple Observers

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**Abstract.** A multiple model based observer/estimator for the estimation of parameters is used to reset the parameter estimation in a conventional Lyapunov based nonlinear adaptive controller. The advantage of combining both approaches is that the performance of the controller with respect to disturbances can be considerably improved while a reduced controller gain will increase the robustness of the approach with respect to noise and unmodeled dynamics. Several alternative resetting criteria are developed based on a control Lyapunov function.

## 1 Introduction

The use of multiple models to switch or reset parameter estimators has been proposed in order to speed up the convergence rate of certainty equivalence adaptive control of linear systems [1–8].

In this paper we present a hybrid approach to speed up transients in continuous Lyapunov based nonlinear adaptive control systems. Hereby, a multiple model observer (MMO) is used to reset the parameter estimation in a nonlinear adaptive controller. The advantage of combining both approaches is that transients due to adaptation can be damped out while the performance of the controller with respect to disturbances can be improved. As a consequence the gain of the continuous adaptive controller can be considerably lowered thus, increasing the robustness of the approach with respect to noise and unmodeled dynamics. The parameter resetting is based on a Control Lyapunov function and can guarantee asymptotic stability. The main contributions of the paper are

- an extension of multiple model based adaptive control to the class of parametric strict feedback nonlinear systems,
- the formulation of a set of sufficient closed loop stability conditions for resetting tuning function based nonlinear adaptive controllers,
- the introduction of a fast multiple model observer, from which even under transient conditions an accurate parameter estimate can be obtained.

The paper is organised as follows: In Section 2 some results of constructive nonlinear adaptive control are briefly reviewed and a motivation for discontinuous parameter resetting is given. This is followed by an analysis of the closed loop stability implications of resetting parameter estimates (Section 3) where a first order and a second order example are used to illustrate the results. Section 4 describes the concept of multiple model observers and gives for a special plant structure sufficient conditions for stability of parameter resetting. At the end discussions of a first order system as an application of the method and some simulation results are given.

## 2 Nonlinear Adaptive Backstepping

Consider the adaptive tracking problem for a parametric strict-feedback system [9]

$$\begin{aligned}\dot{x}_1 &= x_2 + \varphi_1(x_1)^T\theta \\ &\vdots \\ \dot{x}_{n-1} &= x_n + \varphi_{n-1}(x_1, x_2, \dots, x_{n-1})^T\theta \\ \dot{x}_n &= \beta(x)u + \varphi_n(x)^T\theta \\ y &= x_1\end{aligned} \tag{1}$$

where $\theta \in \mathbb{R}^p$ is a vector of unknown constant parameters, $\beta$ and $F = [\varphi_1, \dots, \varphi_n]$ are smooth nonlinear functions taking arguments in $\mathbb{R}^n$. It has been shown that in a tuning function adaptive controller for such a system the adaptive control law and the parameter update law take the following form

$$u = \frac{1}{\beta(x)}\left[\alpha_n(x, \hat{\theta}, \bar{y}_r^{(n-1)}) + y_r^{(n)}\right] \tag{2}$$

$$\dot{\hat{\theta}} = \Gamma\tau_n(x, \hat{\theta}, \bar{y}_r^{(n-1)}) \tag{3}$$

where $y_r$ is the reference signal to be tracked by the output $y$

$$\bar{y}_r^{(i)} = (y_r, \dot{y}_r, \dots, y_r^{(i)}). \tag{4}$$

The control law and the tuning functions are given recursively by

$$z_i = x_i - y_r^{(i-1)} - \alpha_{i-1} \tag{5}$$

$$\begin{aligned}\alpha_i(\bar{x}_i, \hat{\theta}, \bar{y}_r^{(i-1)}) = &-z_{i-1} - c_i z_i - w_i^T\hat{\theta} + \sum_{k=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_k}x_{k+1} + \frac{\partial\alpha_{i-1}}{\partial y_r^{(k-1)}}y_r^{(k)}\right) \\ &-\kappa_i|w_i|^2 z_i + \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\Gamma\tau_i + \sum_{k=2}^{i-1}\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\Gamma w_i z_k\end{aligned} \tag{6}$$

$$\tau_i(\bar{x}_i, \hat{\theta}, \bar{y}_r^{(i-1)}) = \tau_{i-1} + w_i z_i \tag{7}$$

$$w_i(\bar{x}_i, \hat{\theta}, \bar{y}_r^{(i-1)}) = \varphi_i - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}\varphi_k \tag{8}$$

$$i = 1 \dots n \tag{9}$$

where $\bar{x}_i = (x_1, \dots, x_i)$, $\alpha_0 = 0$, $\tau_0 = 0$, $c_i > 0$. The control law together with the parameter update law render the time derivative of the Lyapunov function

$$V_n = \frac{1}{2} z^T z + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \quad \text{with} \quad \tilde{\theta} = \theta - \hat{\theta} \tag{10}$$

negative semidefinite along trajectories of the closed loop system:

$$\dot{V}_n = -\sum_{k=1}^{n} c_k z_k^2 - \sum_{k=i}^{n} \kappa_i |w_i|^2 z_i^2 \leq -c_0 \, |z|^2 \quad \text{where} \quad c_0 = \min_{1 \leq i \leq n} c_i \tag{11}$$

Our main objective is to improve the transient performance of the closed loop system, in particular with respect to the unknown parameter vector $\theta$ which is assumed to be constant with respect to time.

It is a well known fact that for this adaptive control schemes the transient performance can be improved by increasing any of the design parameters $c_i$, $\kappa_i$ and $\Gamma$. The higher the gain the faster the transient response of the control systems. In practical applications however, high gain should be avoided as there are always unmodelled dynamics or even time delays (related to computer implementation) in the system which may lead to instability if the loop gain is too high. Thus, other strategies of counteracting uncertainties are highly desirable.

Such a strategy is provided by the multiple model switching and tuning approach, where the estimates are taken from a finite set

$$\theta_i, \quad i = 1, \dots, N.$$

The multiple model observer provides additional information on parameter uncertainies which can then be used to instanteneously reset the parameter estimate $\hat{\theta}$. Suppose the best estimate of the multiple model observer with respect to *prediction performance* is

$$\hat{\theta}^+ = \hat{\theta}_j.$$

Then a decision has to be made whether or not to use this additional information. In the case when the multiple model estimate is used the current continuous estimate $\hat{\theta}^-$ will be discarded and the continuous update law reset to the new value. This resetting decision should not be based on the modelling performance alone. It should also be guaranteed that the control performance and in particular the transient behaviour is improved via resetting.

In between the resetting events the parameter estimate will still be governed by the adaptation law and it will thus be piecewise continuous. This will result in discontinuous control and adaptation laws. Since the state transformation in Eq. (5) is parameterised by $\hat{\theta}$ the states $z_2, \dots, z_n$ will be discontinuous in time.

In the remainder of the paper the implications of such a resetting strategy will be studied.

## 3 Stability analysis of parameter resetting

### 3.1 Sufficient conditions for stability

Stability results for discontinuous Lyapunov functions exist, e.g. [10]. For stability it is sufficient that

1. $V(x)$ be continuous with respect to its arguments
2. $V(x)$ is non-increasing along trajectories in between switching events,
3. $V(x^+) \leq V(x^-)$ whenever there is a jump from $x^- = \lim_{t \downarrow t^*} x(t)$ to $x^+ = \lim_{t \uparrow t^*} x(t)$ at some time instant $t^*$.

Consider the Lyapunov function (10) of the tuning function approach

$$V_n(z, \theta, \hat{\theta}) = \frac{1}{2} z^T z + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \quad \text{with} \quad \tilde{\theta} = \theta - \hat{\theta}. \tag{12}$$

For the tuning function approach it can be easily shown that properties 1 and 2 hold due to the stability of the closed loop system when no resetting is applied. When the parameter estimate $\hat{\theta}$ is reset, the state variable $z$ depending on $\hat{\theta}$ changes discontinuously with time. Then, to obtain a sufficient condition for stability it remains to be analysed whether

$$\Delta V_n = V_n(z(\hat{\theta}^+), \theta, \hat{\theta}^+) - V_n(z(\hat{\theta}^-), \theta, \hat{\theta}^-) \leq 0 \tag{13}$$

holds. If this is the case then a resetting of $\hat{\theta}$ from $\hat{\theta}^-$ to $\hat{\theta}^+$ is admissible. In general the state vector $z$ will depend on $\hat{\theta}$ in a nonlinear way. In order to develop some stability criteria the following assumption may be made (it will be shown in later sections how this can be replaced by other assumptions):

**Assumption 3.1.** *Set the step change in parameter*

$$\Delta\hat{\theta} = \hat{\theta}^+ - \hat{\theta}^-. \tag{14}$$

*There exist a matrix-valued function $M(z^-, \hat{\theta}^-, \bar{y}_r^{(i-1)})$ such that*

$$\left(z^+\right)^T \left(z^+\right) \leq \left(z^- + M\Delta\hat{\theta}\right)^T \left(z^- + M\Delta\hat{\theta}\right) \tag{15}$$

*for all $\Delta\hat{\theta} \in D \subseteq \mathbb{R}^p$.*

Under assumption 3.1 the following bound on the step change of the Lyapunov function (10) can be given:

$$\begin{aligned} \Delta V_n = {} & \left(z^+\right)^T \left(z^+\right) + \left(\theta - \hat{\theta}^+\right)^T \Gamma^{-1} \left(\theta - \hat{\theta}^+\right) \\ & - \left(z^-\right)^T \left(z^-\right) - \left(\theta - \hat{\theta}^-\right)^T \Gamma^{-1} \left(\theta - \hat{\theta}^-\right) \end{aligned} \tag{16}$$

$$\Delta V_n \leq 2\left[M^T z^- - \Gamma^{-1}\tilde{\theta}^-\right]^T \Delta\hat{\theta} + \Delta\hat{\theta}^T \left[M^T M + \Gamma^{-1}\right] \Delta\hat{\theta} \tag{17}$$

$$\tilde{\theta}^- = \theta - \hat{\theta}^-$$

For positive definite $M^T M + \Gamma^{-1} > 0$ the sufficient condition for stability $\Delta V_n \leq 0$ is satisfied inside the hyper-ellipse

$$2\left[M^T z^- - \Gamma^{-1}\tilde{\theta}^-\right]^T \Delta\hat{\theta} + \Delta\hat{\theta}^T \left[M^T M + \Gamma^{-1}\right] \Delta\hat{\theta} = 0 \tag{18}$$

It can be easily verified that even in the case when $\hat{\theta}$ steps from $\hat{\theta}^-$ to the *correct parameter value* $\hat{\theta}^+ = \theta$ the condition for stability is not necessarily satisfied because in this case the requirement would be:

$$2(z^-)^T M\tilde{\theta}^- + (\tilde{\theta}^-)^T(M^T M - \Gamma^{-1})\tilde{\theta}^- \leq 0. \tag{19}$$

It has been shown above that the set of admissible parameter changes $\Delta\theta$ depends on the state $z$ and on the parameter error $\tilde{\theta}$. While $z^-$ and $z^+$ can be computed, additional information on the estimation error is necessary to check the admissibility of $\Delta\theta$. In the remainder of the paper two ways of obtaining the required knowledge of $\theta$ will be presented. The first approach is by exploiting properties of the closed loop system while the second approach uses additional information supplied by an multiple observer.

### 3.2 Reference trajectory resetting

The condition (13) on $\Delta V$ can be considerably simplified when resetting of the reference trajectory $y_r$ is used in combination with the parameter resetting.

Reference trajectory resetting can be applied most easily in the case where $y_r$ and its derivative are generated by a linear reference model which is driven by some external reference input signal $r(t)$. For the following calculations we assume the existence of a reference model since the states of such a system can be reset directly. In the other case where $y_r$ and its derivatives are generated externally the reset is accomplished by modification of the reference signal using the output $\delta, \delta^{(1)}, \ldots, \delta^{(n-1)}$ of an additional linear asymptotically stable autonomous system $y_{rmod}^{(i)} = y_r^{(i)} + \delta^{(i)}$.

Reference trajectory initialisation is originally a tool for improving the transients in adaptive tuning function control systems [9]. In fact, by resetting the $n$ values $y_r(t^+), \dot{y}_r(t^+), \ldots, y_r^{(n-1)}(t^+)$ an additional degree of freedom is obtained which enables us to set $z^+ = 0$. From Eq. (5) it can be seen that $z^+ = 0$ requires the solution of set of equations

$$y_r^{(i-1)}(t^+) = x_i - \alpha_{i-1}(\bar{x}_1, \ldots, x_{i-1}, \hat{\theta}^+, y_r(t^+), \ldots, y_r^{(i-2)}(t^+)), \qquad i = 1, \ldots, n \tag{20}$$

It can be shown [9] that the solution to these equations does not depend on the controller parameters.

The step change in the Lyapunov function with reference trajectory resetting is

$$\begin{aligned}\Delta V_n &= \left(\theta - \hat{\theta}^+\right)^T \Gamma^{-1} \left(\theta - \hat{\theta}^+\right) - \left(z^-\right)^T \left(z^-\right) - \left(\theta - \hat{\theta}^-\right)^T \Gamma^{-1} \left(\theta - \hat{\theta}^-\right) \\ &= \Delta\hat{\theta}^T \Gamma^{-1} \Delta\hat{\theta} - 2\left(\tilde{\theta}^-\right)^T \Gamma^{-1} \Delta\hat{\theta} - \left(z^-\right)^T \left(z^-\right) \end{aligned} \tag{21}$$

for which we can obtain a controller independent upper bound

$$\Delta V_n \leq \Delta\hat{\theta}^T \Gamma^{-1} \Delta\hat{\theta} - 2\left(\tilde{\theta}^-\right)^T \Gamma^{-1} \Delta\hat{\theta} \tag{22}$$

When trajectory resetting is used, the Lipschitz assumption 3.1 (where $M$ might be difficult to compute) is no longer required because $z^+ = 0$ in Eq. (16).

### 3.3 Application to a first order system

Consider the tracking control of the first order system

$$\dot{x}_1 = \varphi_1(x_1)\theta + u \tag{23}$$

An adaptive tuning function controller is simply

$$\begin{aligned} u &= -\varphi_1(x_1)\hat{\theta} - c_1 z_1 - \dot{y}_r && (24) \\ \dot{\hat{\theta}} &= \gamma z_1 \varphi_1(x_1) = \gamma \tau_1 && (25) \\ z_1 &= x_1 - y_r \end{aligned}$$

This controller based on the control Lyapunov function

$$V = \frac{1}{2} z_1^2 + \frac{1}{2\gamma}\left(\theta - \hat{\theta}\right)^2 \tag{26}$$

renders the derivative of the Lyapunov function negative semi-definite

$$\dot{V} = -c_1 z_1^2 \leq 0.$$

The closed loop system is given by

$$\dot{z}_1 = -c_1 z_1 + \varphi_1(x_1)\tilde{\theta} \tag{27}$$

The time derivative of the squared error along the solution of (27) is

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2} z_1^2\right) = z_1 \dot{z}_1 = -c_1 z_1^2 + z_1 \varphi_1(x_1)\tilde{\theta} \tag{28}$$

For the rest of the discussion of the first order case we assume that $\varphi_1(x_1) > 0$. This assumption is not necessary for the approach in general but it simplifies the switching law considerably.

For the first order system (23) and the Lyapunov function (26) we obtain by use of Eq. (16) the following sufficient stability condition:

$$\Delta V = V^{+} - V^{-} = \frac{1}{2\gamma}\left(\hat{\theta}^{+} - \hat{\theta}^{-}\right)^{2} - \frac{1}{\gamma}\left(\left(\theta - \hat{\theta}^{-}\right)\left(\hat{\theta}^{+} - \hat{\theta}^{-}\right)\right) \leq 0 \qquad (29)$$

This gives the following bounds on the step change in the parameter estimate:

$$\operatorname{sgn}\left(\Delta\hat{\theta}\right) = \operatorname{sgn}\left(\tilde{\theta}^{-}\right) \qquad (30)$$

$$\left|\Delta\hat{\theta}\right| \leq 2\left|\tilde{\theta}^{-}\right| \qquad (31)$$

In general, condition (31) cannot be verified without additional information on the parameter estimate. However a switching law $S(z_1, \Delta\hat{\theta})$ can be designed such that condition (30) holds.

Using this switching law the parameter resetting law is constructed in the following way

$$\hat{\theta} = \hat{\theta}^{-} + \left(\hat{\theta}^{+} - \hat{\theta}^{-}\right) S(z_1, \Delta\hat{\theta}) = \hat{\theta}^{-} + \Delta\hat{\theta}\; S(z_1, \Delta\hat{\theta}) \qquad (32)$$

where $S$ assumes the values 1 or 0 according to the following set of inequalities

$$\begin{aligned} &S = 1 \quad \text{whenever} \quad \begin{cases} z_1 > \varepsilon_1 \;\wedge\; \Delta\hat{\theta} > \varepsilon_2 \\ \qquad\quad \vee \\ z_1 < -\varepsilon_1 \wedge \Delta\hat{\theta} < -\varepsilon_2 \end{cases} \\ &S = 0 \quad \text{elsewhere} \end{aligned} \qquad (33)$$

Condition (32) states that resetting occurs whenever the magnitude of the control error $z_1$ exceeds some threshold and at the same time there is a significant discrepancy between continuous parameter estimate and multiple model parameter estimate having the same sign as the control error.

Note that due to the assumption that $\varphi$ is always positive we obtain from the closed loop error equation (27):

$$\dot{z}_1 z_1 > 0 \quad \text{implies} \quad \operatorname{sgn}(\dot{z}_1) = \operatorname{sgn}(\tilde{\theta}) \qquad (34)$$

Thus, provided that $|z_1|$ is increasing while it crosses the threshold $\varepsilon_1$ the sign of $\dot{z}_1$ is a direct indicator of the sign of the parameter error $\tilde{\theta}$. In the general case, the sign of $\varphi$ will be known and the resetting law can be modified accordingly.

This leads us to the following theorem

**Theorem 3.2.** *1. Consider the first order system (23) together with the continuous control law (24) and the update law (25). Assuming $\varphi_1(x_1) > 0$, $\gamma > 0$ and $c_1 > 0$. If the parameter $\hat{\theta}$ is reset under the condition*

$$z_1 \operatorname{sgn}(\dot{z}_1) = \varepsilon_1 \bigwedge z_1 \Delta\hat{\theta} > \varepsilon_1\varepsilon_2, \quad \varepsilon_1 > 0, \varepsilon_2 > 0 \qquad (35)$$

*then, the sign condition (30) is satisfied.*

2. *Provided the sign condition is satisfied, then a decrease of $V$ in Equation (26) at the switching instant is obtained provided that*

$$\left|\Delta\hat{\theta}\right| < 2\left|\tilde{\theta}^-\right| \tag{36}$$

*holds. Thus a sufficient condition for stability is satisfied.*

3. *If to the contrary*

$$\left|\Delta\hat{\theta}\right| \geq 2\left|\tilde{\theta}^-\right| \tag{37}$$

*holds then the control error $z_1$ is driven towards zero as long as $|z_1| > \varepsilon_1$ despite of the increase in value of $V$.*

*Proof* The first and second part of the Theorem has been proven above.

If the assumptions of the third part of the theorem hold then, outside $|z_1| > \varepsilon_1$ we have along the solutions of the closed loop equation:

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}z_1^2\right) &= z_1\dot{z}_1 \\ &= -c_1 z_1^2 + z_1\varphi_1(x_1)\left[\tilde{\theta}^- - \Delta\hat{\theta}S(y,\Delta\hat{\theta})\right] \\ &\leq -c_1 z_1 + |z_1\varphi_1(x_1)|\left[\left|\tilde{\theta}^-\right| - \left|\Delta\hat{\theta}\right|\right] < 0 \end{aligned} \tag{38}$$

due to (37) which implies that $z_1$ is driven towards the origin. ■

As a remark, one might note, that case 3 of Theorem 3.2 implies stability but possibly with reduced transient performance and chattering.

The negative jump in the Lyapunov function could be interpreted as improved transient performance. This follows from the dependency of transient performance of the tuning function approach on the initial conditions which has been analysed in [9].

### 3.4 Application to a second order system

Consider the second order system with one parameter

$$\begin{aligned} \dot{x}_1 &= x_2 + \varphi(x_1)\theta \\ \dot{x}_2 &= u. \end{aligned} \tag{39}$$

Designing the tuning function controller (2) for such a system requires one backstep. Assuming that the parameter estimate $\hat{\theta}$ can vary discontinuously with time we will thus have also discontinuous changes with time in $\alpha_1$ and $z_2$ and in the corresponding Lyapunov function $V = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 + \frac{1}{2\gamma}\tilde{\theta}^2$. The step change in the Lyapunov function can be expressed as

$$\begin{aligned} \Delta V = V^+ - V^- &= z_2^-\varphi_1(x_1)\Delta\hat{\theta} + \frac{1}{2}\varphi_1^2(x_1)\Delta\hat{\theta}^2 - \frac{1}{\gamma}\tilde{\theta}^-\Delta\hat{\theta} + \frac{1}{2\gamma}\Delta\hat{\theta}^2 \\ &= \frac{1}{2}\left(\frac{1}{\gamma} + \varphi_1^2(x_1)\right)\Delta\hat{\theta}^2 - \left(\frac{1}{\gamma}\tilde{\theta}^- - z_2^-\varphi(x_1)\right)\Delta\hat{\theta} \end{aligned} \tag{40}$$

This corresponds with Assumption 3.1 and Eq. (15) where

$$
\begin{aligned}
z^+ &= z^- + M\Delta\hat{\theta} \\
M &= \begin{pmatrix} 0 \\ \varphi_1(x_1) \end{pmatrix}, \qquad M^T M = \varphi_1^2(x_1).
\end{aligned}
$$

The reset conditions discussed in sections 3.3 and 3.4 require the information whether the states of $z_1$ and $z_2$ cross some threshold from above or below. No explicit knowledge of the derivatives of the states is required. In case of noisy state measurement multiple crossing of the threshold may occur, however, by imposing an additional threshold on $\Delta\hat{\theta}$ a hysteresis is introduced and chattering cannot occur.

## 4 Multiple Model Observer (MMO)

As explained above a multiple observer approach can be used to avoid large transient errors in continuous adaptive control. Quite similar to the multiple model estimation described in [2–4, 7], the idea is to construct a finite set of parallel observers each of which is designed for a fixed parameter value. In its simplest form the MMO constists of a set $O$ of $N$ individual observers $o_i$ each parameterised with a fixed parameter value $\theta_i$. All $N$ observer cover the range of admissible parameter values. Figure (1) shows the structure of a multi-observer parameter estimation. Each of the $N$ observer estimates the states of the system and is driven by the residual $e_{1i} = x_1 - \hat{x}_{1i}$. Since any mismatch between a single observer and the physical system will in general lead to a steady-state estimation error, this error can be used to determine the best observer for the actual system.

Using discontinuous output injection functions is common in sliding mode observers [11]. A hybrid observer using convergence information to switch between several discontinuous output injection functions for nonlinear systems has been reported in [12]. Here, we propose instead to use a set of observers with fixed output injection functions which can have considerably faster transients.

A performance index $Q_i(\hat{\boldsymbol{x}}_i, y)$ is defined for each observer of the set $O$. The performance index weighs the output error of the observer, thus quantifies the mismatch between the plant and the individual observer. A switching logic $L$ is used to determine the estimate $\theta_i$ of the multi-observer $O$. $L$ satisfies two purposes:

1. selecting the coefficient $\theta_i$ corresponding to the observer $o_i$ with the best performance.
2. providing a mechanism that ensures a convergence of the estimator after a finite number of switches.

In order to prevent chattering, two different approaches have been suggested in literature

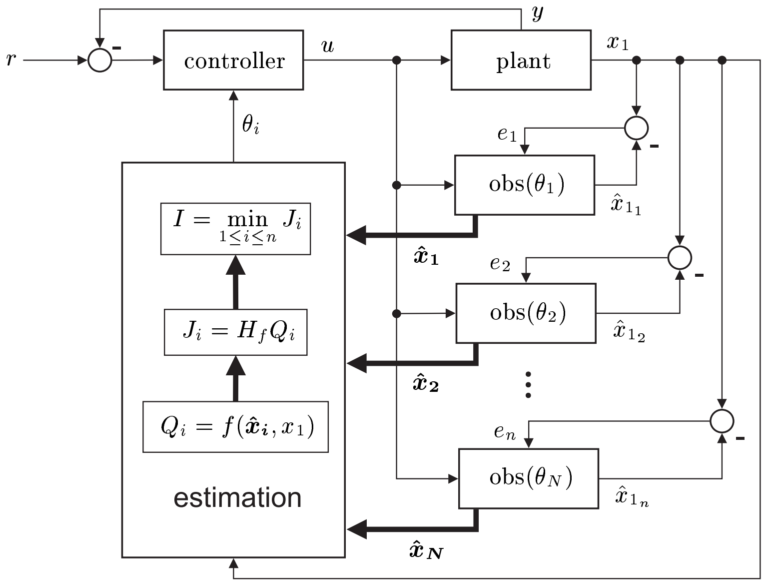


**Fig. 1.** Multiple model observer parameter estimation

- Dwell time switching [5] where after each switch for a certain period of time switching is prohibited.
- Hysteresis switching [1, 13]: Let $o_p$ be the valid observer at time $t^-$ then a switch to a new observer $o_i$ occurs only if $Q_i(t^+)\,(1+h) < Q_p(t^+)$ where $Q_p(t^+)$ is the current performance of the observer $o_p$ and $h > 0$ is the hysteresis. Otherwise no switching will occur and $o_p$ will remain valid.

### 4.1 Construction of the individual observers in the first order case

Consider the system (23) where the parameter $\theta$ is treated as an augmented state

$$\begin{aligned} \dot{x}_1 &= \varphi_1(y)\theta + u \\ \dot{\theta} &= 0 \\ y &= x_1 \end{aligned} \tag{41}$$

It is assumed that $\varphi_1(y) > 0$ and that the parameter $\theta$ is contained in a closed interval $[\theta_{min}, \theta_{max}]$. The interval is discretised using a set of $N$ parameter values $\theta_{min} < \theta_1 < \theta_2 < \ldots < \theta_N < \theta_{max}$. Each of the $N$ individual observers of the multiple model observer will be centered around one of the discrete parameter values $\theta_i$. For this purpose Eq. (41) is rewritten into

$$\begin{aligned} \dot{x}_1 &= \varphi_1(y)\theta_i + \varphi_1(y)x_{2i} + u \\ \dot{x}_{2i} &= 0 \end{aligned} \tag{42}$$

where $x_{2i} = \theta - \theta_i$. Following the Lyapunov based observer design in [14] we propose to use the following individual nonlinear observer

$$\begin{aligned} \dot{\hat{x}}_{1i} &= \varphi_1(y)\theta_i + 2\omega\varphi_1(y)(y - \hat{x}_{1i}) + u + \varphi_1(y)\hat{x}_{2i} \\ \dot{\hat{x}}_{2i} &= \omega^2\varphi_1(y)(y - \hat{x}_{1i}), \qquad \omega > 0. \end{aligned} \tag{43}$$

Defining the error $e_i = [e_{1i}, e_{2i}]^T = [y - \hat{x}_{1i}, x_{2i} - \hat{x}_{2i}]^T$ the observer will result in the bilinear error dynamics

$$\dot{e}_i = \varphi(y) \underbrace{\begin{pmatrix} -2\omega & 1 \\ -\omega^2 & 0 \end{pmatrix}}_{A} e_i. \tag{44}$$

where the matrix $A$ is Hurwitz and $\varphi(y)$ represents the nonlinearity in the system output. The observer design renders the derivative of the Lyapunov function

$$V_i(e_i) = \frac{1}{2} e_i^T \begin{pmatrix} 1 & 0 \\ 0 & \omega^{-2} \end{pmatrix} e_i \tag{45}$$

negative definite $\dot{V}_i = -2\omega\varphi(y)e_{1i}^2 < 0$.

An important property of the error differential equation (44) is that its solution can be explicitly given. Knowing the measurable output error $e_{1i}(t - T)$ and $e_{1i}(t)$ at some time instant $t$ the parameter estimation error

$$e_{2i}(t) = \frac{1}{y^*} \left[ (1 + \omega y^*)\, e_{1i}(t) - \mathrm{e}^{-\omega y^*} e_{1i}(t - T) \right] \tag{46}$$

can be determined, where $y^*(t - T, t) = \int_{t-T}^{t} \varphi_1(y(\tau))\mathrm{d}\tau > 0$. Thus, even under observer transients a parameter estimate

$$\hat{\theta}_i = \theta_i + \hat{x}_{2i}(t) + e_{2i}(t) \tag{47}$$

can be computed.

Anti-windup is introduced for the observer state $\hat{x}_{2i}$ by defining the local bounds $\bar{\theta}_i$. The state equation $\dot{\hat{x}}_{2i}$ is set to zero if $\hat{x}_{2i} + \theta_i \notin [\bar{\theta}_{i-1}, \bar{\theta}_i]$ and $(y - \hat{x}_{1i})\hat{x}_{2i} > 0$. Hence, only one individual observer will have an output error converging to zero and consequently a cost index $Q_i$ converging to zero independently of the particular cost index that is used.

The properties of the MMO can be used to derive the following resetting law:

**Theorem 4.1.** *Consider the control system (23) together with the control law (24), the parameter update law (25) and the MMO (43). Suppose that $o_i$ is the observer that has been selected according to the cost index. Then, setting $\hat{\theta}^+ = \theta_i$ will result in a negative step of the Lyapunov function (26) if*

1. *$x_{2i}(\tau)$ does not saturate within the time intervall $\tau \in [t - T, t]$.*
2. *$\bar{\theta}_{i-1} < \hat{\theta}_i < \bar{\theta}_i$.*
3. *either (a) $\hat{\theta}^- - \bar{\theta}_i > \bar{\theta}_i - \theta_i$ or (b) $\bar{\theta}_{i-1} - \hat{\theta}^- > \theta_i - \bar{\theta}_{i-1}$.*

*Proof* If condition 1 of the theorem holds, according to Eqs. (46) and (47) we have

$$\hat{\theta}_i = \theta_i + \hat{x}_{2i}(t) + e_{2i}(y^*(t, t-T), e_{1i}(t), e_{1i}(t-T)). \tag{48}$$

If in addition to this, condition 2 is satisfied, then it can be implied that the real parameter is contained in

$$\bar{\theta}_{i-1} < \theta < \bar{\theta}_i. \tag{49}$$

From condition 3 it follows that either 3a is satisfied in which case we obtain by adding $\hat{\theta}$ to both sides, rearranging and employing (49)

$$-\Delta\hat{\theta} = \hat{\theta}^- - \theta_i < 2(\hat{\theta} - \bar{\theta}_i) \leq 2(\hat{\theta}^- - \theta) = -2\tilde{\theta}^- \tag{50}$$

If on the other hand 3b is satisfied then by subtracting $\hat{\theta}$ from both sides and employing (49)

$$\Delta\hat{\theta} = \theta_i - \hat{\theta}^- < 2(\bar{\theta}_{i-1} - \hat{\theta}^-) \leq 2(\theta - \hat{\theta}^-) = 2\tilde{\theta}^-. \tag{51}$$

Consequently, conditions (30) and (31) are satisfied which is sufficient for stability. ∎

Note that the MMO approach does not rely on assumption 3.1.

## 5 First order system

Consider the first order system (41) where $\varphi_1(x_1) = x_1^2$ together with the control law (24) and the update law (25). The design of the MMO (43) is done by using five parameter hypotheses $\theta_i \in \{-10, -5, 0, 5, 10\}$. The parameter estimate $\hat{\theta}$ is reset if the Theorem 4.1 together with (32) hold. The simulation results with and without parameter resetting are depicted in Figure (2). Consider the simulation scenario where the system should follow a ramp signal with the slope 0.1sec$^{-1}$. The parameter $\theta$ jumps at time $t$ = 4sec from $\theta = 9$ to $\theta = -8$ and at time $t$ = 7sec to $\theta = 4$. White noise is distributed to the system's output. Note that the scenario differs slightly from the above theoretical considerations where the parameter $\theta$ is assumed to be time invariant. The upper left picture in Figure (2) shows the control error for both cases with (fat black line) and without (gray line) using the MMO. The upper right picture shows the control signal respectively. The lower left picture depicts the real parameter value $\theta$ (dotted), the estimate of the MMO $\theta_i$ (dashed gray), the estimate $\hat{\theta}$ with parameter resetting (solid fat)and $\hat{\theta}$ without resetting (dashed fat line). Using the MMO estimation, $\hat{\theta}$ converges faster to the real parameter value and the control error is removed faster. The lower right picture of Figure (2) shows the faster decrease of the Lyapunov function (26) and the performance enhancement. The simulation shows an improved performance even for step disturbances in the parameter.

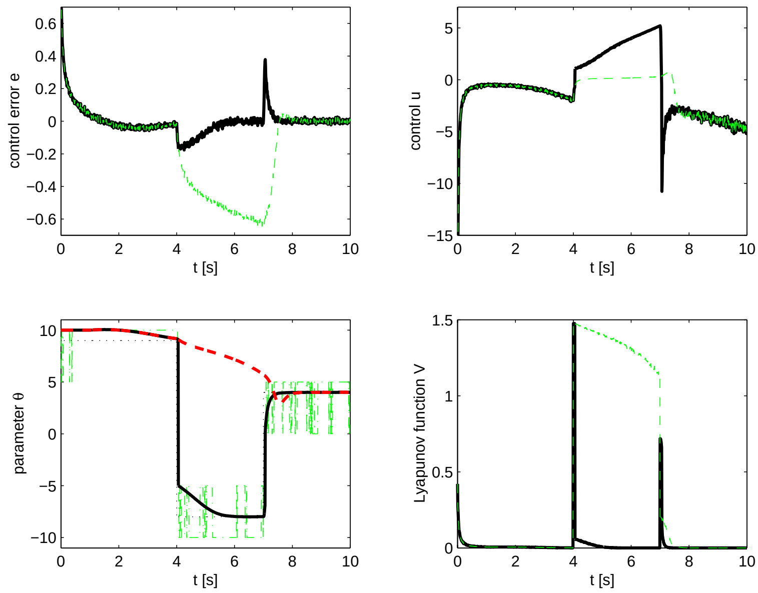


**Fig. 2.** First order example

# 6 Conclusions

The presented paper provided an extension of multiple model based adaptive control to the class of parametric strict feedback nonlinear systems. As a main contribution a set of sufficient closed loop stability conditions for resetting tuning function based nonlinear adaptive controllers was given. Also, a fast multiple model observer was introduced, from which even under transient conditions a parameter estimate can be obtained. A first order control example showed that recovering of the control error can be improved after instantaneous changes of the parameter.

Future work will be dedicated to the application of multiple observers in automotive wheel slip control where a fast recovery of wheel slip after instantaneous changes of the tyre/road friction coefficient is required.

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## References


1. R. H. Middleton, G. C. Goodwin, D. J. Hill, and D. Q. Mayne, "Design issues in adaptive control," *IEEE Transactions on Automatic Control*, vol. 33, pp. 50–58, 1 1988.
2. K. S. Narendra and J. Balakrishnan, "Improving transient response of adaptive control systems using multiple models and switching," *IEEE Transactions on Automatic Control*, vol. 39, no. 9, pp. 1861–1866, 1994.
3. K. S. Narendra and J. Balakrishnan, "Intelligent control using fixed and adaptive models," in *Proceedings on the 33rd CDC, Lake Buena vista, Florida*, pp. 1680–1685, December 1994.
4. K. S. Narendra, J. Balakrishnan, and M. K. Ciliz, "Adaption and learning using multiple models, switching and tuning," *IEEE Control Systems Magazine*, vol. 15, no. 3, pp. 37–51, 1995.
5. A. S. Morse, "Supervisory control of families of linear set-point controllers–part 1: Exact matching," *IEEE Transactions on Automatic Control*, vol. 41, pp. 1413–1431, October 1996.
6. A. S. Morse, "Supervisory control of families of linear set-point controllers–part 2: Robustness," *IEEE Transactions on Automatic Control*, vol. 42, pp. 1500–1515, 10 1997.
7. K. S. Narendra and C. Xiang, "Adaptive control of discrete–time systems using multiple models," in *Proceedings on the 37th CDC Tampa, Florida*, pp. 3978–3983, December 1998.
8. P. V. Zhivoglyadov, R. H. Middleton, and M. Fun, "Localisation based switching adaptive control for time-varying discrete time systems," *IEEE Transactions on Automatic Control*, vol. 45, no. 4, pp. 752–755, 2000.
9. M. Krstić, I. Kanellakopoulos, and P. Kokotović, *Nonlinear Adaptive Control Design*. John Wiley & Sons Inc., 1995.
10. A. van der Schaft and H. Schumacher, *An Introduction to Hybrid Dynamical Systems*. London: Springer-Verlag, 2000.
11. S. D. V. Utkin, "Sliding mode observers. Tutorial," in *Proceedings of the 34th IEEE CDC, New Orleans*, pp. 3376–3378, 1995.
12. Y. Liu, "Switching observer design for uncertain nonlinear system," in *Proceedings of the 34th IEEE CDC, New Orleans*, pp. 1756–1761, 1995.
13. J. Hespanha, *Logic-Based Switching Algorithms in Control*. Dissertation, Graduate School, Yale University, December 1998.
14. C. de Wit, R. Horowitz, and P.Tsiotras, "Model-based observers for tire/road contact friction prediction," in *New Directions in Nonlinear Observer Design, Springer Lecture Notes on Control and Information Science No. 244*.