# Spontaneous coalition forming. Why some are stable? 

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#### Abstract

A model to describe the spontaneous formation of military and economic coalitions among a group of countries is proposed using spin glass theory. Between each couple of countries, there exists a bond exchange coupling which is either zero, cooperative or conflicting. It depends on their common history, specific nature, and cannot be varied. Then, given a frozen random bond distribution, coalitions are found to spontaneously form. However they are also unstable making the system very disordered. Countries shift coalitions all the time. Only the setting of macro extra national coalition are shown to stabilize alliances among countries. The model gives new light on the recent instabilities produced in Eastern Europe by the Warsow pact dissolution at odd to the previous communist stability. Current European stability is also discussed with respect to the European Union construction.


## 1 Introduction

Twenty years ago, using physics to describe political or social behavior was a very odd approach. Among very scarce attempts, one paper was calling on

[^0]to the creation of a new field under the name of "Sociophysics" []]. It stayed without real continuation. Only in the last years did physicists start to get involved along this line of research [2]. Among various subjects [3, [4], we can cite voting process [5, [6], group decision making [7], competing opinion spreading [8, 9, 10], and very recently international terrorism [11].

In this paper we adress the question of spontaneous coalition forming within military alliances among a set of independant countries [12, 13, 14]. A model is built from the complexe physics of spin glasses [15. While coalitions are found to form spontaneously, they are unstable. It is only the construction of extra-territory macro organizations which are able to produce stable alliances.

The following of the paper is organised as follows. The second part contains the presentation of the model. Basic features of the dynamics of spontaneous froming bimodal coalitions are outlined. The building of extra-territory coaltions is described in Section 3. The cold war situation is then analysed in Section 4. Section 5 is devoted to the situation in which only one world coalition is active. A new explanation is given in Section 6 to Eastern European instabilities following the Warsaw pact dissolution as well as to Western European stability. Some hints are also obtained on how to stabilize these Eastern Europe instabilities. Last Section contains some concluding remarks.

## 2 Presentation of the model

We start from a group of $N$ independant countries [12]. From historical, cultural and economic experience, bilateral propensities $J_{i, j}$ have emerged between pairs of countries $i$ and $j$. They are either favoring cooperation $\left(J_{i, j}>0\right)$, conflict $\left(J_{i, j}<0\right)$ or ignorance ( $J_{i, j}=0$ ). Each propensity $J_{i, j}$ depends solely on the pair $(i, j)$ itself. Propensities $J_{i, j}$ are local and independant frozen bonds. Respective intensities may vary for each pair of countries but are always symmetric, i.e., $J_{i j}=J_{j i}$.

From the well known saying "the enemy of an enemy is a friend" we get the existence of only two competing coalitions. They are denoted respectively by A and B. Then each country has the choice to be in either one of two coalitions. A variable $\eta_{i}$ where index i runs from 1 to N , signals the $i$ actual belonging with $\eta_{i}=+1$ for alliance A and $\eta_{i}=-1$ for alliance B. From bimodal symmetry all A-members can turn to coalition B with a simultaneous flip of all B-members to coalition A.

Given a pair of countries $(i, j)$ their respective alignment is readily expressed through the product $\eta_{i} \eta_{j}$. The product is +1 when $i$ and $j$ belong to the same coalition and -1 otherwise. The associated "cost" between the countries is measured by the quantity $J_{i j} \eta_{i} \eta_{j}$ where $J_{i j}$ accounts for the amplitude of exchange which results from their respective geopolitical history and localization.

Here factorisation over $i$ and $j$ is not possible since we are dealing with competing bonds [15]. It makes teh problem very hard to solve analytically. Given a configuration $X$ of countries distributed among coaltions A and B, for each nation $i$ we can measure its overall degree of conflict and cooperation with all others $N-1$ countries via the quantity,

$$
\begin{equation*}
E_{i}=\sum_{j=1}^{N} J_{i j} \eta_{j} \tag{1}
\end{equation*}
$$

where the summation is taken over all other countries including $i$ itself with $J_{i i} \equiv 0$. The product $\eta_{i} E_{i}$ then evaluates the "cost" associated with country $i$ choice with respect to all other country choices. Summing up all country individual "cost" yields,

$$
\begin{equation*}
E(X)=\frac{1}{2} \sum_{i=1}^{N} \eta_{i} E_{i} \tag{2}
\end{equation*}
$$

where the $1 / 2$ accounts for the double counting of pairs. This "cost" measures indeed the level of global satisfaction from the whole country set. It can be recast as,

$$
\begin{equation*}
E(X)=\frac{1}{2} \sum_{<i, j>} J_{i j} \eta_{i} \eta_{j}, \tag{3}
\end{equation*}
$$

where the sum runs over the $N(N-1)$ pairs $(i, j)$. At this stage it sounds reasonable to assume each country chooses its coalition in order to minimize its indivual cost. Accordingly to make two cooperating countries $\left(J_{i, j}>0\right)$ in the same alliance, we put a minus sign in from of the expression of Eq. (3) to get,

$$
\begin{equation*}
H=-\frac{1}{2} \sum_{<i, j>} J_{i j} \eta_{i} \eta_{j} \tag{4}
\end{equation*}
$$

which is indeed the Hamiltonian of an Ising random bond magnetic system. There exist by symmetry $2^{N} / 2$ distinct sets of alliances each country having

2 choices for coalition. Starting from any initial configuration, the dynamics of the system is implemented by single country coalition flips. An country turns to the competing coalition only if the flip decreases its local cost. The system has reached its stable state once no more flip occurs. Given $\left\{J_{i j}\right\}$, the $\left\{\eta_{i}\right\}$ are thus obtained minimizing Eq. (4).

Since the system stable configuration minimizes the energy, we are from the physical viewpoint, at the temperature $T=0$. In practise for any finite system the theory can tell which coalitions are possible. However, if several coalitions have the same energy, the system is unstable and flips continuously from one coalition set to another one at random and with no end.

For instance, in the case of three conflicting nations like Israel, Syria and Iraq, any possible alliance configuration leaves always someone unsatisfied. Let us label them respectively by $1,2,3$ and consider equal and negative exchange interactions $J_{12}=J_{13}=J_{23}=-J$ with $J>0$ as shown in Fig. (1). The associated minimum of the energy is equal to $-J$. However this minimum value is realized for several possible and equivalent coalitions which are respectively $(A, B, A),(B, A, A),(A, A, B),(B, A, B),(A, B, B)$, and ( $\mathrm{B}, \mathrm{B}, \mathrm{A})$. First three are identical to last ones by symmetry since here what matters is which countries are together within the same coalition. This peculiar property of a degenerate ground state makes the system unstable. There exists no one single stable configuration which is stable. Some dynamics is shown in Fig. (1). The system jumps continuously and at random between $(\mathrm{A}, \mathrm{B}, \mathrm{A}),(\mathrm{B}, \mathrm{A}, \mathrm{A})$ and $(\mathrm{A}, \mathrm{A}, \mathrm{B})$.

To make the dynamics more explicit, consider a given site $i$. Interactions with all others sites can be represented by a field,

$$
\begin{equation*}
h_{i}=\sum_{j=1}^{N} J_{i j} \eta_{j} \tag{5}
\end{equation*}
$$

resulting in an energy contribution

$$
\begin{equation*}
E_{i}=-\eta_{i} h_{i} \tag{6}
\end{equation*}
$$

to the Hamiltonian $H=\frac{1}{2} \sum_{i=1}^{N} E_{i}$. Eq. (6) is minimum for $\eta_{i}$ and $h_{i}$ having the same sign. For a given $h_{i}$ there exists always a well defined coalition choice except for $h_{i}=0$. In this case site $i$ is unstable. Then both coalitions are identical with respect to its local energy which stays equal to zero. An unstable site flips continuously with probability $\frac{1}{2}$ (see Fig. (1)).


Figure 1: Top left shows one possible configuration of alliances with countries 1 and 2 in A and country 3 in B. From it, countries 1 and 2 being in a mixed situation with respect to optimzing their respective bilateral interations, three possible and equiprobable distributions are possible. In the first possible following configuration (top right), country 1 has shifted alliance from A to B. However its move keeps it in its mixed situation while making country 2 happy and country 3 mixed. Instead it could have been country 2 which had shifted alliance (low right) making 1 happy and 3 mixed. Last possibility (low left) is both 1 and 2 shifting simultaneously. It is the worse since each country is unhappy.

## 3 Setting up extra territory coalitions

In parallel to the spontaneous emergence of unstable coalitions, some extra territory organizations have been set in the past to create alliances at a global world level. Among the recent more powerfull ones stand Nato and the former Warsow pact. These alliances were set above the country level and produce economic and military exchanges. Each country is then adjusting to its best interest with respect to these organisations. A variable $\epsilon_{i}$ accounts for each country $i$ natural belonging. For coalition A it is $\epsilon_{i}=+1$ and $\epsilon_{i}=-1$ for B . The value $\epsilon_{i}=0$ marks no apriori. These natural belongings are also induced by cultural and political history.

Exchanges generated by these coalitions produce additional pairwise propensities with amplitudes $\left\{C_{i, j}\right\}$. Sharing resources, informations, weapons is basically profitable when both countries are in the same alliance. However, being in opposite coalitions produces an equivalent loss. Therefore a pair $(i, j)$ propensity is $\epsilon_{i} \epsilon_{j} C_{i, j}$ which can be positive, negative or zero to mark respective cooperation, conflict or ignorance. It is a site induced bond [?]. Adding it to the former bond propensity yields an overall pair propensity,

$$
\begin{equation*}
J_{i, j}+\epsilon_{i} \epsilon_{j} C_{i, j} \tag{7}
\end{equation*}
$$

between two countries $i$ and $j$.
At this stage an additional variable $\beta_{i}= \pm 1$ is introduced to account for benefit from economic and military pressure attached to a given alignment. It is still $\beta_{i}=+1$ for A and $\beta_{i}=-1$ for B with $\beta_{i}=0$ in case of no pressure. The amplitude of this economical and military interest is measured by a local positive field $b_{i}$ which also accounts for the country size and its importance. At this stage, the sets $\left\{\epsilon_{i}\right\}$ and $\left\{\beta_{i}\right\}$ are independent.

Actual country choices to cooperate or to conflict result from the given set of above quantites. The associated total cost becomes,

$$
\begin{equation*}
H=-\frac{1}{2} \sum_{<i, j>}\left\{J_{i, j}+\epsilon_{i} \epsilon_{j} C_{i j}\right\} \eta_{i} \eta_{j}-\sum_{i=1}^{N} \beta_{i} b_{i} \eta_{i} \tag{8}
\end{equation*}
$$

An illustration is given in Fig. (2) with above exemple of Israel, Syria and Iraq labeled respectively by $1,2,3$ with $J_{12}=J_{13}=J_{23}=-J$ and $b_{1}=b_{2}=b_{3}=0$. Suppose an arab coalition is set against Israel with $\epsilon_{1}=+1$ and $\epsilon_{2}=\epsilon_{3}=-1$. The new propensities become respectively $-J+C,-J-C,-J-C$. They are now minimized by $\eta_{1}=-\eta_{2}=\eta_{3}$ to gives an energy of $-J-C$ to all three couplings.


Initial configuration Unstable


New stable configuration

Figure 2: Starting from one possible unstable configuration of alliances (left) with countries 1 and 2 in $A$ and country 3 in $B$, the stabilization is shown to result from the existence of the various $\epsilon$ with $\epsilon_{1}=\epsilon_{2}=-\epsilon_{3}$.

## 4 Cold war scenario

The cold war scenario means that the two existing world level coalitions generate much stonger couplings than purely bilateral ones, i.e., $\left|J_{i, j}\right|<C_{i, j}$ since to belong to a world level coalition produces more advantages than purely local unproper relationship. Local bond propensities are neutralized since overwhelmed by the two block site exchanges. The overall system is very stable. There exists one stable distribution between both competing alliances.

We consider first the coherent case in which cultural and economical trends go along the same coalition, i.e., $\beta_{i}=\epsilon_{i}$. Then from Eq. (8) the minimum of $H$ is unique with all country propensities satisfied. Each country chooses its coalition according to its natural belonging, i.e., $\eta_{i}=\epsilon_{i}$. This result is readily proven via the variable change $\tau \equiv \epsilon_{i} \eta_{i}$ which turns the energy to,

$$
\begin{equation*}
H_{1}=-\frac{1}{2} \sum_{<i, j>} C_{i j} \tau_{i} \tau_{j}-\sum_{i=1}^{N} b_{i} \tau_{i} . \tag{9}
\end{equation*}
$$

Above Hamiltonian representd a ferromagnetic Ising Hamiltonian in positive symmetry breaking fields $b_{i}$. Indeed it has one unique minimum with all
$\tau_{i}=+1$.
The remarkable result here is that the existence of two apriori world level coalitions is identical to the case of a unique coalition with every country in it. It shed light on the stability of the Cold War situation where each country satisfies its proper relationship. Differences and conflicts appear to be part of an overall cooperation within this scenario.

The dynamics for one unique coalition including every country, or two competing alliances, is the same since what matters is the existence of a well defined stable configuration. However there exists a difference which is not relevant at this stage of the model since we assumed $J_{i, j}=0$. In reality $J_{i, j} \neq 0$ makes the existence of two coalitions to produce a lower "energy" than a unique coalition since then, more $J_{i, j}$ can also be satisfied.

It worth to notice that field terms $b_{i} \epsilon_{i} \eta_{i}$ account for the difference in energy cost in breaking a pair proper relationship for respectively a large and a small country. Consider for instance two countries $i$ and $j$ with $b_{i}=$ $2 b_{j}=2 b_{0}$. Associated pair energy is

$$
\begin{equation*}
H_{i j} \equiv-C_{i j} \epsilon_{i} \eta_{i} \epsilon_{j} \eta_{j}-2 b_{0} \epsilon_{i} \eta_{i}-b_{0} \epsilon_{j} \eta_{j} \tag{10}
\end{equation*}
$$

Conditions $\eta_{i}=\epsilon_{i}$ and $\eta_{j}=\epsilon_{j}$ give the minimum energy,

$$
\begin{equation*}
H_{i j}^{m}=-J_{i j}-2 b_{0}-b_{0} . \tag{11}
\end{equation*}
$$

From Eq. (11) it is easily seen that in case $j$ breaks proper alignment shifting to $\eta_{j}=-\epsilon_{j}$ the cost in energy is $2 J_{i j}+2 b_{0}$. In parallel when $i$ shifts to $\eta_{i}=-\epsilon_{i}$ the cost is higher with $2 J_{i j}+4 b_{0}$. Therfore the cost in energy is lower for a breaking from proper alignment by the small country $\left(b_{j}=b_{0}\right)$ than by the large country $\left(b_{j}=2 b_{0}\right)$. In the real world, it is clearly not the same for instance for the US to be against Argentina than to Argentina to be against the US.

We now consider the uncoherent case in which cultural and economical trends may go along opposite coalitions, i.e., $\beta_{i} \neq \epsilon_{i}$. Using above variable change $\tau \equiv \epsilon_{i} \eta_{i}$, the Hamiltonian becomes,

$$
\begin{equation*}
H_{2}=-\frac{1}{2} \sum_{<i, j>} J_{i j} \tau_{i} \tau_{j}-\sum_{i=1}^{N} \delta_{i} b_{i} \tau_{i}, \tag{12}
\end{equation*}
$$

where $\delta_{i} \equiv \beta_{i} \epsilon_{i}$ is given and equal to $\pm 1 . H_{2}$ is formally identical to the ferromagnetic Ising Hamiltonian in random fields $\pm b_{i}$.

The local field term $\delta_{i} b_{i} \tau_{i}$ modifies the country field $h_{i}$ in Eq. (9) to $h_{i}+\delta_{i} b_{i}$ which now can happen to be zero. This change is qualitative since now there exists the possibility to have "unstability", i.e., zero local effective field coupled to the individual choice. Moreover countries which have opposite cultural and economical trends may now follow their economical interest against their cultural interest or vice versa. Two qualitatively different situations may occur.

- Unbalanced economical power: in this case we have $\sum_{i}^{N} \delta_{i} b_{i} \neq 0$.

The symmetry is now broken in favor of one of the coalition. But still there exists only one minimum.

- Balanced economical power: in this case we have $\sum_{i}^{N} \delta_{i} b_{i}=0$.

Symmetry is preserved and $H_{2}$ is identical to the ferromagnetic Ising Hamiltonian in random fields which has one unique minimum.

## 5 Unique world leader

Very recently the Eastern block has disappeared. However it the Western block is still active as before. In this model, within our notations, denoting A the Western alignment, we have still $\epsilon_{i}=+1$ for countries which had $\epsilon_{i}=+1$. On the opposite, countries which had $\epsilon_{i}=-1$ have now turned to either $\epsilon_{i}=+1$ if joining Nato or to $\epsilon_{i}=0$ otherwise.

Therefore above $J_{i, j}=0$ assumption based on the inequality $\left|J_{i, j}\right|<$ $\left|\epsilon_{i} \epsilon_{j}\right| C_{i, j}$ no longer holds for each pair of countries. In particular propensity $p_{i, j}$ become equal to $J_{i, j}$ in all cases where $\epsilon_{i}=0, \epsilon_{j}=0$ and $\epsilon_{i}=\epsilon_{j}=0$.

A new distribution of countries results from the collapse of one block. On the one hand A coalition countries still determine their actual choices between themselves according to $C_{i, j}$. On the other hand former B coalition countries are now determining their choices according to competing links $J_{i, j}$ which did not automatically agree with former $C_{i, j}$.

This subset of countries has turned from a random site spin glasses without frustration into a random bond spin glasses with frustration. The former B coalition subset has jumped from one stable minimum to a highly degenerated unstable landscape with many local minima. This property could be related to the fragmentation process where ethnic minorities and states are
shifting rapidly allegiances back and forth while they were formerly part of a stable structure just few years ago.

While the B coalition world organization has disappeared, the A coalition world organization did not change and is still active. The condition $\left|J_{i, j}\right|<$ $C_{i, j}$ is still valid for A pair of countries with $\epsilon_{i} \epsilon_{j}=+1$. Associated countries thus maintain a stable relationship and avoid a fragmentation process. This result supports a posteriori argument against the dissolution of Nato once Warsaw Pact was disolved. It also favors the viewpoint that former Warsaw Pact countries should now join Nato.

Above situation could also shed some light on the current European debate. It would mean European stability is mainly the result of the existence of European structures with economical reality and not the outcome of a new friendship among former ennemies. These structures produce associated propensities $C_{i, j}$ much stronger than local competing propensities $J_{i, j}$ which are still there. European stability would indeed result from $C_{i, j}>\left|J_{i, j}\right|$ and not from all having $J_{i, j}>0$. An eventual setback in the European construction $\left(\epsilon_{i} \epsilon_{j} C_{i, j}=0\right)$ would then automatically produce a fragmentation process analogous of what happened in former Yugoslavia with the activation of ancestral bilateral local conflicts.

## 6 Conclusion

In this paper we have proposed a new way to describe alliance forming phenomena among a set of countries. It was shown that within our model the cold war stabilty was not the result of two opposite alliances but rather the existence of alliances which neutralize the conflicting interactions within allies. It means also that having two alliances or just one is qualitatively the same with respect to stability.

From this viewpoint the strong instabilies which resulted from the Warsow pact dissolution are given a simple explanation. Simultaneously some hints are obtained about possible policies to stabilize world nation relationships. Along this line, the importance of European construction was also underlined. At this stage, our model remains rather basic. However it opens some new road to explore and to forecast international policies.

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