A New Topological Classification of Points in 3D Images

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Abstract.

We propose, in this paper, a new topological classification of points in 3D images. This classification is based on two connected components numbers computed on the neighborhood of the points. These numbers allow to classify a point as an interior or isolated, border, curve, surface point or as different kinds of junctions.

The main result is that the new border point type corresponds exactly to a simple point. This allows the detection of simple points in a 3D image by counting only connected components in a neighborhood. Furthermore other types of points are better characterized.

This classification allows to extract features in a 3D image. For example, the different kinds of junction points may be used for characterizing a 3D object. An example of such an approach for the analysis of medical images is presented.

1 Introduction

Image analysis deals more and more with three-dimensional (3D) images. They may come from several fields, the most popular one is the medical imagery. 3D images need specific tools for their processing and their interpretation. This interpretation task involves often a matching stage, between two 3D images or between a 3D image and a model.

Before this matching stage, it is necessary to extract useful information of the image and to organize it into a high-level structure. It can be done by extracting the 3D edges of the image (see [6]) and then by searching some particular qualitative features on these edges. These features are geometrical (see [5]) or topological (see [4]). In both cases, they are : intrinsic to the 3D object, stable to rigid transformations and locally defined.

In this paper, we propose a new topological classification which improves the one proposed in [4]. After recalling some basic definitions of 3D digital topology (section 2), we give the principle of the topological classification (section 3.1) and we present its advantages (section 3.3). It is defined by computing two connected components numbers. The main result is that we can characterize simple points with these numbers without any Euler number (genus) computation. An example of application in medical imagery is given (section 5).

2 Basic Definitions

We recall some basic definitions of digital topology (see [1] and [2]).

A 3D digital image is a subset of $\mathbb{Z}^3$. A point $x \in \mathbb{Z}^3$ is defined by (x_1, x_2, x_3) with $x_i \in \mathbb{Z}$. We can use the following distances defined in $\mathbb{R}^n$ with their associated neighborhoods :

- $D_1(x,y) = \sum_{i=1}^{n} |y_i - x_i|$ with $V_1^r(x) = \{y / D_1(x,y) \leq r\}$
- $D_\infty(x,y) = \text{MAX}_{\,i=1..n} |y_i - x_i|$ with $V_\infty^r(x) = \{y / D_\infty(x,y) \leq r\}$

We commonly use the following neighborhoods :

6-neighborhood : We note $N_6(x) = V_1^1(x)$ and $N_6^*(x) = N_6(x) \setminus \{x\}$
26-neighborhood : We note $N_{26}(x) = V_\infty^1(x)$ and $N_{26}^*(x) = N_{26}(x) \setminus \{x\}$
18-neighborhood : We note $N_{18}(x) = V_1^2(x) \cap V_\infty^1(x)$ and $N_{18}^*(x) = N_{18}(x) \setminus \{x\}$

A binary image consists of one object X and its complementary set $\overline{X}$ called the background. In order to avoid any connectivity paradox, we commonly use the 26-connectivity for the object X and the 6-connectivity for the background $\overline{X}$. These connectivities are the one's used in this paper.

3 The Topological Classification

3.1 Principle

Let us consider an object X in the real space $\mathbb{R}^3$, let $x \in X$, and let $V(X)$ be an arbitrarily small neighborhood of x. Let us consider the numbers $C_{\mathbb{R}^3}$ and $\overline{C_{\mathbb{R}^3}}$ which are respectively the numbers of connected components in $X \cap (V(x) \setminus \{x\})$ and in $\overline{X} \cap (V(x) \setminus \{x\})$ adjacent to x. These numbers may be used as topological descriptors of x. For example a point of a surface is such that we can choose a small neighborhood $V(X)$ such as $C_{\mathbb{R}^3} = 1$ and $\overline{C_{\mathbb{R}^3}} = 2$.

Such numbers are commonly used for thinning algorithms and for characterizing simple points in 3D. The acute point of their adaptation to a digital topology is the choice of the small neighborhood $V(X)$.

The distance associated to the 26-connectivity is D_∞, it is then natural to choose $V_\infty^1(x) = N_{26}(x)$ which is the smallest neighborhood associated to D_∞. Usually, the same neighborhood is chosen when using other connectivities. But the distance associated to the 6-connectivity is D_1, then $V_1^1(x) = N_6(x)$ is the smallest neighborhood associated to D_1. The trouble is that $N_6^*(x)$ is not 6-connected. Then $V_1^2(x)$ seems to be the good choice. In this neighborhood, some points have only one neighbor and have no topological interest, by removing them we obtain the 18-neighborhood $N_{18}(x)$.

3.2 Application to Digital Topology

We propose the same methodology of classification than in [4] :

1. Each point is labeled with a topological type using the computation of two connected components numbers in a small neighborhood.
2. Because some points (junctions points) are not detected with the two numbers, a less local approach is used for extracting them.

We are using the two following numbers of connected components :

- $C = NC_a[X \cap N_{26}^*(x)]$ which is the number of 26-connected components of $X \cap N_{26}^*(x)$ 26-adjacent to x. All points in the 26-neighborhood are 26-adjacent to x, therefore C is the number of 26-connected components of $X \cap N_{26}^*(x)$. It is not necessary to check the adjacency to x.

Type A - interior point :	$\overline{C} = 0$	
Type B - isolated point :		$C = 0$
Type C - border point :	$\overline{C} = 1$	$C = 1$
Type D - curve point :	$\overline{C} = 1$	$C = 2$
Type E - curves junction :	$\overline{C} = 1$	$C > 2$
Type F - surface point :	$\overline{C} = 2$	$C = 1$
Type G - surface - curve junction :	$\overline{C} = 2$	$C \geq 2$
Type H - surfaces junction :	$\overline{C} > 2$	$C = 1$
Type I - surfaces - curve junction :	$\overline{C} > 2$	$C \geq 2$

Table 1. Topological classification of 3D points according to the values of $\overline{C}$ and C

- $\overline{C} = NC_a[\overline{X} \cap N^*_{18}(x)]$ which is the number of 6-connected components of $\overline{X} \cap N^*_{18}(x)$ 6-adjacent to x.

We obtain then a first local topological classification of each point of the object using these two numbers (see Table 1).

However, this classification depends only on the 26-neighborhood of each point and some junction points belonging to a set of junction points which is not of unit-width are not detected. We propose the following procedures for extracting such points :

For curves : we only need to count the number of neighbors of each curve point (type **D**), if this number is greater than two, the point is a missed curves junction point (type **E**).

For surfaces : we use the notion of simple surface introduced in [4]. If a point of type **F** or **G** is adjacent to more than one simple surface in a 5x5x5 neighborhood, it is considered as a missed point of type **H** or **I**.

3.3 Advantages

The main difference of our new classification is that we count the connected components of the background $\overline{X}$ in a 18-neighborhood $N^*_{18}(x)$ instead of in a 26-neighborhood $N^*_{26}(x)$ as in [4]. By using a smaller neighborhood, we are able to see finer details of the object.

The main result due to this difference is that the border point type corresponds exactly to the characterization of simple points (see [7] and [1]).

Proposition 1. *A point $x \in X$ is simple if and only it verifies :*

$$C = NC_a[X \cap N^*_{26}(x)] = 1 \tag{1}$$

$$\overline{C} = NC_a[\overline{X} \cap N^*_{18}(x)] = 1 \tag{2}$$

Proof. The complete proof of this proposition can not be written here by lack of space (see [3] for details).

This new characterization of simple points needs only two conditions (instead of three as usual, see [1]), and these two conditions only need the computation of numbers of connected components. The computation of the genus, which requires quite a lot of computational effort, is no more necessary.

4 Counting the Connected Components

There exists some optimal algorithms for searching and labeling the k-connected components in a binary image (see [8]). These algorithms need only one scan of the picture by one half of the k-neighborhood, and use a table of labels for managing the conflicts when a point owns to several connected components already labeled.

We can use the same algorithm in our little neighborhoods, but it has a high computational cost. In these neighborhoods, we have an a priori knowledge about the possible adjacencies. We can store this knowledge in a table and use it in a propagation algorithm. For that, we scan the neighborhood, if we find an object's point which is not labeled, we assign a new label to it and we propagate this new label to the whole connected component which contains the point. Using this knowledge, the propagation algorithm is faster than the classical one.

5 Results

We consider two NMR 3D images of a skull scanned in two different positions (see Figure 1).

We apply a thinning algorithm (derived from our characterization of simple point) to the 3D image containing a skull. The 3D image contains 256*256*151 quasi-isotropic voxels of $0.8*0.8*1 \text{ mm}^3$.

We obtain then the skeleton of the skull. We apply our classification algorithm to label each point. Projections of the labeled skeleton are shown in Figure 2. It is easy to check the astonishing likeness between both results, in spite of the noise due to the scan and the skeletonization. This will be used with profit in a forthcoming 3D matching algorithm.

6 Conclusion

A new topological classification of points in a 3D image has been proposed. This classification allows the characterization of a point as an interior, isolated, border, curve, surface point or as different kinds of junctions. This classification allows also the detection of simple points (and applications like thinning or shrinking). This is done by computing two connected components numbers. The Euler number which leads to a lot of computational effort does not need to be evaluated. Furthermore the method for computing connected components in a small neighborhood enables fast computations of the two numbers.

References

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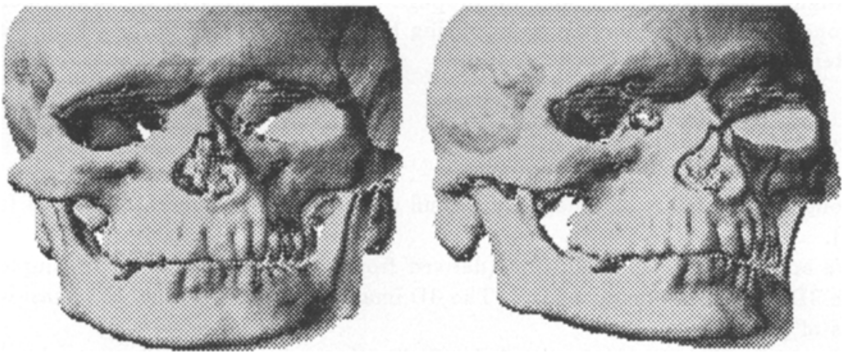

Fig. 1. 3D representations of a skull scanned in two positions

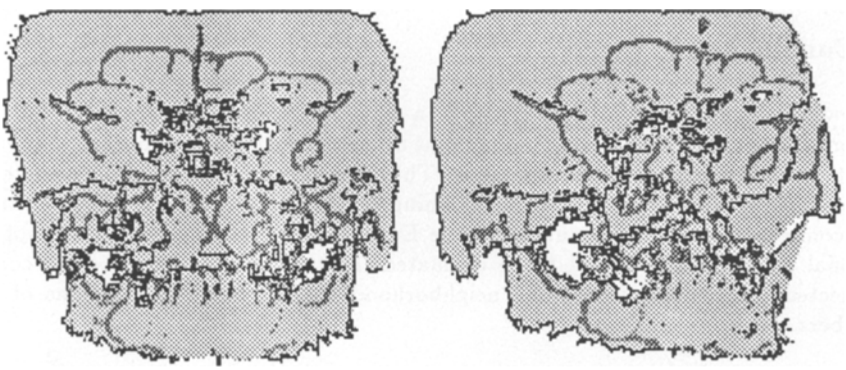

Fig. 2. Projection of the topological characterization of the skeleton of the skull : border are in black, surfaces in light grey and surfaces junctions in grey