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# Relational Matching

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# Preface

This book is the result of an elementary study on relational matching. Relational matching is a method for finding the best correspondences between structural descriptions. In computer vision it is widely used for the recognition and location of objects in digital images. For this purpose, the digital images and the object models are represented by structural descriptions. The matching algorithm then has to determine which image elements and object model parts correspond.

This study particularly focuses on the evaluation of the correspondences. In order to find the best match, one needs a measure to evaluate the quality of a match. This measure usually quantifies the similarity between the image and the model elements. This strategy is based on the assumption that corresponding elements will have similar characteristics (like size, shape, etc.). This study reviews the evaluation measures that have been suggested over the past few decades and presents a new measure that is based on information theory. This new measure is integrated into tree search methods that are utilized to find the best match.

The resulting relational matching theory hence combines matching strategies, information theory, and tree search methods. Because the reader may not be familiar with all aspects, comprehensive introductions are given to these topics.

I would like to thank my supervisor, Prof. Wolfgang Förstner, for the pleasant cooperation and the many interesting discussions we had. I would also like to thank the German Research Society, which financed the Special Research Program "High Precision Navigation" (SFB 228) at the University of Stuttgart. The research for this thesis was performed within the image processing project of this research program.

Stuttgart, June 1992

George Vosselman

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