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### Heinrich Wansing

# The Logic of Information Structures

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#### **Preface**

The present monograph is a revised version of my doctoral thesis at the Fachbereich Philosophie und Sozialwissenschaften I of the Free University of Berlin. It contains my contribution to the interdisciplinary research project "Systeme der Logik als theoretische Grundlage der Wissens- und Informationsverarbeitung" at the Institute for Philosophy of the Free University of Berlin.

I am very glad that this preface gives me the opportunity to express my gratitude to various people and institutions. First of all, I would like to thank my thesis advisers, David Pearce and Johan van Benthem. With each of them I associate one of the basic ideas which in combination led me to writing this book, viz. (i) to regard negative and positive information as equally relevant and (ii) to vary in a systematic way structural rules of inference. Without their inspiration this thesis would not have come into existence, and it would not have been completed without Johan van Benthem's critical comments and encouragement. Moreover, I gratefully acknowledge scholarships from the Senat von Berlin and the Studienstiftung des deutschen Volkes. To the latter I am indebted for support over many years. A number of colleagues have, in one way or another, contributed to the realization of this book. I would like to thank Kosta Došen and Dirk Roorda for their critical remarks on an early version of Chapter 5 and several stimulating discussions. To Dirk in addition I return thanks for practical help concerning 'huisvesting'. I would also like to thank Peter Schroeder-Heister. His insistance on elimination rules enormously helped me to understand the proof-theoretic approach towards the problem of functional completeness. André Fuhrmann was so kind to comment on an early version of parts of Chapter 9. I also wish to thank Gerd Wagner for numerous conversations and three referees of the Journal of Logic, Language and Information for their reports. Finally, I am grateful to Jörg Siekmann for recommending this book for Springer Lecture Notes in AI. From the very beginning I could not have carried out this work without the support from Petra, Kasimir and my parents. They deserve my special, whole-hearted thanks.

As indicated in the text, Chapter 5 is based on 'Formulas-as-types for a hierarchy of sublogics of intuitionistic propositional logic' in D. Pearce & H. Wansing (eds.), Nonclassical Logics and Information Processing, Lecture Notes in AI, Vol. 619, Springer-Verlag, Berlin, 1992, and Chapter 4 (the part consisting of Sections 4.1 to 4.4) is (based on) my paper 'Functional completeness for subsystems of intuitionistic propositional logic', forthcoming in the Journal of Philosophical Logic. Chapter 4, in particular Section 4.5, is also used in 'On the expressiveness of Categorial Grammar', to appear in J. Wolenski (ed.), The Legacy of Ajdukiewicz, Rodopi, Amsterdam. Material from, essentially, Chapter 9 will appear as 'Informational Interpretation of Substructural Propositional Logics' in the Journal of Logic, Language and Information.

Hamburg, March 1993

Heinrich Wansing

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