Lecture Notes in Artificial Intelligence 681

Subseries of Lecture Notes in Computer Science
Edited by J. Siekmann

Lecture Notes in Computer Science
Edited by G. Goos and J. Hartmanis

Heinrich Wansing

The Logic of Information Structures

Springer-Verlag
Berlin Heidelberg New York
London Paris Tokyo
Hong Kong Barcelona
Budapest

Series Editor

Jörg Siekmann
University of Saarland
German Research Center for Artificial Intelligence (DFKI)
Stuhlsatzenhausweg 3,
D-66123 Saarbrücken, Germany

Author

Heinrich Wansing
FB Informatik, Universität Hamburg
Bodenstedtstraße 16, D-22765 Hamburg, Germany

CR Subject Classification (1991): I.2

ISBN 3-540-56734-8 Springer-Verlag Berlin Heidelberg New York
ISBN 0-387-56734-8 Springer-Verlag New York Berlin Heidelberg

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, re-use of illustrations, recitation, broadcasting, reproduction on microfilms or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer-Verlag. Violations are liable for prosecution under the German Copyright Law.

© Springer-Verlag Berlin Heidelberg 1993
Printed in Germany

Typesetting: Camera ready by author
45/3140-543210 - Printed on acid-free paper

Preface

The present monograph is a revised version of my doctoral thesis at the Fachbereich Philosophie und Sozialwissenschaften I of the Free University of Berlin. It contains my contribution to the interdisciplinary research project "Systeme der Logik als theoretische Grundlage der Wissens- und Informationsverarbeitung" at the Institute for Philosophy of the Free University of Berlin.

I am very glad that this preface gives me the opportunity to express my gratitude to various people and institutions. First of all, I would like to thank my thesis advisers, David Pearce and Johan van Benthem. With each of them I associate one of the basic ideas which in combination led me to writing this book, viz. (i) to regard negative and positive information as equally relevant and (ii) to vary in a systematic way structural rules of inference. Without their inspiration this thesis would not have come into existence, and it would not have been completed without Johan van Benthem's critical comments and encouragement. Moreover, I gratefully acknowledge scholarships from the Senat von Berlin and the Studienstiftung des deutschen Volkes. To the latter I am indebted for support over many years. A number of colleagues have, in one way or another, contributed to the realization of this book. I would like to thank Kosta Došen and Dirk Roorda for their critical remarks on an early version of Chapter 5 and several stimulating discussions. To Dirk in addition I return thanks for practical help concerning 'huisvesting'. I would also like to thank Peter Schroeder-Heister. His insistance on elimination rules enormously helped me to understand the proof-theoretic approach towards the problem of functional completeness. André Fuhrmann was so kind to comment on an early version of parts of Chapter 9. I also wish to thank Gerd Wagner for numerous conversations and three referees of the *Journal of Logic, Language and Information* for their reports. Finally, I am grateful to Jörg Siekmann for recommending this book for Springer Lecture Notes in AI. From the very beginning I could not have carried out this work without the support from Petra, Kasimir and my parents. They deserve my special, whole-hearted thanks.

As indicated in the text, Chapter 5 is based on 'Formulas-as-types for a hierarchy of sublogics of intuitionistic propositional logic' in D. Pearce & H. Wansing (eds.), *Nonclassical Logics and Information Processing*, Lecture Notes in AI, Vol. 619, Springer-Verlag, Berlin, 1992, and Chapter 4 (the part consisting of Sections 4.1 to 4.4) is (based on) my paper 'Functional completeness for subsystems of intuitionistic propositional logic', forthcoming in the *Journal of Philosophical Logic*. Chapter 4, in particular Section 4.5, is also used in 'On the expressiveness of Categorial Grammar', to appear in J. Wolenski (ed.), *The Legacy of Ajdukiewicz*, Rodopi, Amsterdam. Material from, essentially, Chapter 9 will appear as 'Informational Interpretation of Substructural Propositional Logics' in the *Journal of Logic, Language and Information*.

Hamburg, March 1993 Heinrich Wansing

Contents

1 Introduction 1

1.1 Intuitionistic propositional logic IPL . . . 2
1.2 Kripke's interpretation of IPL . . . 4
1.3 Grzegorczyk's interpretation of IPL . . . 8
1.4 The BHK interpretation of IPL . . . 10
1.5 Appendix: Derivations in sequent calculi . . . 11

2 Generalizations 13

2.1 Positive and negative information . . . 13
2.2 The fine-structure of information processing . . . 16
2.3 Critical remarks on the BHK interpretation of IPL . . . 18
2.3.1 Ambiguity of the BHK interpretation . . . 20
2.3.2 The proper treatment of negation . . . 22
2.4 Examples . . . 24
2.4.1 Nelson's constructive systems $\mathbf{N}^-$ and $\mathbf{N}$ . . . 24
2.4.2 Relevant implicational logic $R_{\supset}$ . . . 26
2.4.3 Categorial logics . . . 28
2.5 Appendix: Possible constraints on 'informational interpretation' . . . 29

3 Intuitionistic minimal and intuitionistic information processing 31

3.1 Substructural subsystems of MPL and IPL . . . 31
3.2 Some standard properties . . . 34
3.2.1 Cut-elimination . . . 34
3.2.2 Decidability . . . 38
3.2.3 Interpolation . . . 42

4 Functional completeness for substructural subsystems of IPL 45

4.1 The higher-level Gentzen calculus $\mathbf{G}$ . . . 46
4.2 Gentzen semantics . . . 48
4.3 Functional completeness for $ISPL$ . . . 50
4.4 Functional completeness for $ISPL_{\Delta}$ and $MSPL_{\Delta}$ . . . 54
4.5 Digression: On the expressiveness of Categorial Grammar . . . 54
4.5.1 Extended Lambek Calculus . . . 55
4.5.2 Limitations . . . 58

5 Formulas-as-types for substructural subsystems of IPL **59**
5.1 The typed lambda calculus $\lambda_{/,\backslash}$ . . . 59
5.2 Encoding proofs in $ISPL_{/,\backslash}$ . . . 62
5.3 Cut-elimination in $PROOF_{ISPL_{/,\backslash}}$ and β-reduction in $\Lambda_{ISPL_{/,\backslash}}$ as homomorphic images of each other . . . 63
5.4 $\beta\eta$-reduction in $\Lambda_{ISPL_{/,\backslash}}$ as a monomorphic image of normalizing (cut)-free proofs . . . 66
5.5 Encoding proofs in $ISPL_{/,\backslash,\circ}$ . . . 68
5.6 Encoding proofs in structural extensions of $ISPL_{/,\backslash}$ . . . 72
5.7 Cut-elimination in $PROOF_{ISPL_{/,\backslash\ominus}}$ and β-reduction in $\Lambda_{ISPL_{/,\backslash\ominus}}$ as homomorphic images of each other . . . 74
5.8 Appendix: Proof of strong normalization for $\Lambda_{/,\backslash}$ wrt $\twoheadrightarrow_\beta$ and $\twoheadrightarrow_{\beta\eta}$ 77

6 Constructive minimal and constructive information processing **81**
6.1 Substructural subsystems of $\mathbf{N}^-$ and $\mathbf{N}$ . . . 81
6.2 Cut-elimination, decidability, and interpolation . . . 86
6.3 The BHK interpretation reconsidered . . . 89
6.3.1 Ambiguity as providing degrees of freedom . . . 89
6.3.2 The proof/disproof interpretation and its soundness wrt $COSPL^-_\ominus$ resp. $COSPL_\ominus$ . . . 90

7 Functional completeness for substructural subsystems of N **93**
7.1 Disproofs as mirror-images of proofs . . . 93
7.2 The higher-level Gentzen calculus $\mathbf{GN}^-$ . . . 94
7.3 Generalized Gentzen semantics . . . 97
7.4 Functional completeness for $COSPL^-_\Delta$ . . . 100
7.5 Functional completeness for $COSPL_\Delta$ . . . 105
7.6 Digression: Negation in Categorial Grammar . . . 105

8 The constructive typed λ-calculus λ^c and formulas-as-types for $\mathbf{N}^-$ **107**
8.1 The typed λ-calculus λ^c . . . 107
8.1.1 The syntax of λ^c . . . 107
8.1.2 Models for λ^c . . . 110
8.1.3 Intended models for λ^c . . . 112
8.1.4 The canonical model and completeness . . . 113
8.1.5 Completeness wrt intended models . . . 115
8.2 Formulas-as-types for $\mathbf{N}^-$ . . . 118

9 Monoid models and the informational interpretation of substructural propositional logics **123**
9.1 Monoid models . . . 123
9.2 Informational interpretation . . . 128
9.3 Extension with (linear) modalities . . . 134
9.4 Dynamics of interpretation . . . 136
9.5 *Slomos* as an exhaustive format of abstract information structures . . . 138
9.6 Appendix: Applications . . . 140
9.6.1 Monotonicity and paraconsistency . . . 140

9.6.2 Negation in a formal system 141
9.6.3 Beth's Theorem 144

Bibliography **149**

Summary **156**

Index **158**