

# Lecture Notes in Computer Science

885

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# Closed Object Boundaries from Scattered Points

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# Preface

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This Ph.D. dissertation presents the result of research carried out between 1985 and 1992, first at Leiden University as a scientific assistant researcher, and later at CWI (Centre for Mathematics and Computer Science), Amsterdam, as a researcher on the NFI IIICAD project, funded by NWO (Dutch Organization for Scientific Research) under Grant NF-51/62-514.

The research goal was the development of new methods and techniques for the construction of closed object boundaries from scattered points in both 2D and 3D. These points are either synthetic or measured from the boundary of an existing object.

New results are presented in Chapters 3, 5, 7, and 9. Chapter 3 introduces ‘the  $\gamma$ -neighborhood graph’, which provides a geometrical structure on the scattered points. Chapter 5 presents a method to construct a piecewise linear boundary through all given scattered points which is based on the  $\gamma$ -neighborhood graph. Chapter 7 introduces ‘the flintstones scheme’, a hierarchical approximation and localization scheme. Chapter 9 presents methods to construct a smooth piecewise cubic boundary from a piecewise linear one (e.g. resulted from methods of Chapter 5 or 7).

The material presented in this dissertation has partly appeared before in other publications. The correspondence between the chapters and publications is as follows: Chapters 2 and 3: [Veltkamp, 92c], Chapters 4 and 5: [Veltkamp, 89a] and [Veltkamp, 91], Chapters 6 and 7: [Veltkamp, 90] and [Veltkamp, 92b], Chapter 8: [Veltkamp, 92d], Chapter 9: [Veltkamp, 92a].

The text of this thesis could never have matured without the suggestions and constructive criticism of many colleagues, above all my Ph.D. supervisors Jan van den Bos and Mark Overmars. Arie de Bruin, Nies Huijsmans, Pia Pfluger,

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