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Finite Versus Infinite

Contributions to an Eternal Dilemma

Springer

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Preface

The finite – infinite interplay is central in human thinking, from ancient philosophers and mathematicians (Zeno, Pythagoras), to modern mathematics (Cantor, Hilbert) and computer science (Turing, Gödel). Recent developments in mathematics and computer science suggest a) radically new answers to classical questions (e.g., does infinity exist?, where does infinity come from?, how to reconcile the finiteness of the human brain with the infinity of ideas it produces?), b) new questions of debate (e.g., what is the role played by randomness?, are computers capable of handling the infinity through unconventional media of computation?, how can one approximate efficiently the finite by the infinite and, conversely, the infinite by finite?).

Distinguished authors from around the world, many of them architects of the mathematics and computer science for the new century, contribute to the volume. Papers are as varied as Professor Marcus' activity, to whom this volume is dedicated. They range from real analysis to DNA computing, from linguistics to logic, from combinatorics on words to symbolic dynamics, from automata theory to geography, and so on, plus an incursion into the old history of conceptions about infinity and a list of philosophical "open problems". They are mainly mathematical and theoretical computer science texts, but not all of them are purely mathematical. They deal directly with the finite – infinite interplay (one proves several times that paths from finite to infinite and conversely, from infinite to finite, are both inspiring and useful), or only implicitly (each grammar or automaton is a *finite* device meant to describe a possibly *infinite* object, a language; similarly, an axiomatic system is a finite construction aiming to cover an infinity of truths – whence the limits of such systems).

We emphasize the precaution taken in the title. The reader can find here only some *contributions* to an *eternal* debate. This is just a step in an infinite sequence of steps. We hope that this is a step forward...

*

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Cristian S. Calude
Gheorghe Păun
November 1999

Tribute to Professor Solomon Marcus

The present book is presented in honour of Professor Solomon Marcus, on the occasion of his 75th birthday (1st of March, 2000).

The co-editors of this volume have debated for a while what would be the most appropriate topic for such a work. The decision was, at the same time, difficult and easy.

Professor Marcus started by being a "pure mathematician", building for himself an international reputation in mathematical analysis and related areas; then, at the end of the fifties, he switched to an interesting career in mathematical linguistics (mainly analytical approaches to phonological, morphological, syntactic and semantic categories, but also automata and formal languages).[1] Although he was one of the most cited authors in this area, he has once again enlarged his interests – and definitely others' as well – by founding one more domain, mathematical poetics. Just *en passant*, in 1968, he introduced (Marcus) contextual grammars, a genuinely new generative device inspired from analytical models in the study of language.[2] This was not enough for Professor Marcus. In the same way as real analysis and mathematical linguistics provided to him tools for mathematical poetics, all these have been applied to semiotics, natural and social sciences.

Then, what to choose for a *Festscrift*? None of these alone would be enough, but all together are hard to be contained under the covers of a single book.[3]

A tempting idea was to look for major trends or meta-ideas in Professor Marcus' *oeuvre*. Again a difficult choice – at least for the present co-editors, although both of them were (and psychologically still are!) his students and had/have the privilege to be his close collaborators. Analogies/bridges between fields which, at the first sight, look remote? The finesse of study, no

[1] His book *Finite Grammars and Automata*, published unfortunately only in Romanian in 1964, was one of the earliest monographs on this topic.

[2] The domain has expanded beyond his expectations (as he confessed somewhere): a comprehensive book was published recently by Kluwer: *Marcus Contextual Grammars.*

[3] Another book in honour of Professor Marcus, including only papers in mathematical and computational linguistics, will be published by the Romanian Academy Publishing House.

matter what object of research? The attraction for counter-intuitive, even paradoxical facts, proven then to be common-frequent-beneficial? The visible passion for dichotomies? Which one? Artistic versus scientific? Finite physical existence versus infinite spiritual existence? Old versus young provocation? Discrete versus continuous? Analytical versus generative? Local versus global? Finite versus infinite?

That's it! It fits well with Professor Marcus' activity and personality, both of which leave the impression of infinity... About four hundred papers, more than forty authored or co-authored books (published in ten languages not including Romanian), twenty five edited books, hundreds of conferences attended, hundreds of lectures at universities, many domains of direct research, many more domains of general interest, an authoritative leader of schools, a respected cultural presence, a continuous promoter of the Romanian mathematical heritage and, at the same time, an energizer of the first steps of many young researchers, a perfect memory, a rigorous life style, active scientifically and physically at 75 (at a level which is tiring for many of his much younger colleagues) – all these have something to do with infinity...

Antonia, a 10 year old Romanian-Spanish girl from Tarragona, Spain, had a decisive influence in choosing the topic of the volume. Earlier than us, she called Professor Marcus *El infinito*, under circumstances pointing to his manner of approaching kids. In his opinion, a child should rediscover Zeno's paradoxes via a Socrates-type dialogue of the following kind: S.M.: "Look, Antonia, you have a bread, a usual one, and you eat today half of it. How much does it remain for tomorrow?" A.: "Half a bread, of course." S.M.: "Okay, but tomorrow you eat half of what you have got. Does some bread remain for after tomorrow?" A.: "Yes, sir, some bread still remains." S.M.: "Good, but after tomorrow you again eat half of the bread you have. Is it true that some bread still remains?" A. (already doubtful): "Yes, but"... After about five iterations, the two parts are always separated: A., or any other "victim", knowing for sure that (s)he can eat a *normal* bread in a day or so, hence any continuation of the dialogue is senseless (because it's breadless), S.M. looking desperately for the bright light of infinity in the others' eyes: eating every day half of the piece of bread you have got means securing bread for infinitely many days, isn't it?... In many cases, both sides are disappointed... In at least one case, S.M. got a surname: *El Infinito*...

Happy Birthday, *Profesore Infinito*!

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