
EVOLUTIONARY ALGORITHMS FOR SOLVING MULTI-OBJECTIVE PROBLEMS

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Evolutionary Computation

EVOLUTIONARY ALGORITHMS FOR SOLVING MULTI-OBJECTIVE PROBLEMS

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To our beloved wives

Foreword

Researchers and practitioners alike are increasingly turning to search, optimization, and machine-learning procedures based on natural selection and natural genetics to solve problems across the spectrum of human endeavor. These genetic algorithms and techniques of evolutionary computation are solving problems and inventing new hardware and software that rival human designs. The Kluwer Series on Genetic Algorithms and Evolutionary Computation publishes research monographs, edited collections, and graduate-level texts in this rapidly growing field. Primary areas of coverage include the theory, implementation, and application of genetic algorithms (GAs), evolution strategies (ESs), evolutionary programming (EP), learning classifier systems (LCSs) and other variants of genetic and evolutionary computation (GEC). The series also publishes texts in related fields such as artificial life, adaptive behavior, artificial immune systems, agent-based systems, neural computing, fuzzy systems, and quantum computing as long as GEC techniques are part of or inspiration for the system being described.

This encyclopedic volume on the use of the algorithms of genetic and evolutionary computation for the solution of multi-objective problems is a landmark addition to the literature that comes just in the nick of time. Multi-objective evolutionary algorithms (MOEAs) are receiving increasing and unprecedented attention. Researchers and practitioners are finding an irresistible match between the population available in most genetic and evolutionary algorithms and the need in multi-objective problems to approximate the Pareto trade-off curve or surface.

The authors have done a remarkable job in collecting, organizing, and interpreting the burgeoning literature of MOEAs in a form that should be welcomed by novices and old hands alike. The volume starts with an extraordinarily thorough introduction, including short vignettes and photographs of many of the pioneers of multi-objective optimization. It continues with as complete a discussion of the many varieties of MOEA as appears anywhere in the literature.

A discussion of MOEA test suites surveys the important landscape of test landscapes and is followed with important chapters on empirical testing and MOEA theory. Practitioners will especially welcome the thorough survey of real-world MOEA applications, the ample discussion of parallelization, and the discussion of MOEA in multi-criteria decision making. The final chapter of special topics discusses the relation of MOEA techniques to other methods in soft computation such as simulated annealing, ant colony optimization, and memetic algorithms. The researcher will especially appreciate the large appendices that help classify the existing literature as an aid to subsequent work.

I urge those interested in the growing field of multi-objective genetic and evolutionary algorithms to run—don't walk—to your nearest on-line or off-line book purveyor and click, signal, or otherwise buy this important addition to our literature.

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Preface

The solving of multi-objective problems (MOPs) has been a continuing effort by humans in many diverse areas including computer science, engineering, economics, finance, industry, physics, chemistry, and ecology, among others. Many powerful deterministic and stochastic techniques for solving these large dimensional optimization problems have risen out of operations research, decision science, engineering, computer science and other related disciplines. The explosion in computing power continues to arouse extraordinary interest in stochastic search algorithms that require high computational speed and very large memories. A generic stochastic approach is that of evolutionary algorithms (EAs). Such algorithms have been demonstrated to be very powerful and generally applicable for solving difficult single objective problems. Their fundamental algorithmic structures can also be applied to solving many multi-objective problems. In this book, the various features of multi-objective evolutionary algorithms (MOEAs) are presented in an innovative and unique fashion, with detailed customized forms suggested for a variety of applications. Also, extensive MOEA discussion questions and possible research directions are presented at the end of each chapter.

Based upon the original contributions of Darwin and Mendel, evolution occurs through natural selection and adaptation. Using this basic biological model, various evolutionary algorithm structures have been developed. Single objective EAs and in particular genetic algorithms (GAs), evolutionary programming (EP) and evolution strategies (ES) have been shown to find if not the optimal solution something that is satisfactory; i.e. “satisfices” the user. The goal of course is to search the associated objective/fitness function landscape (phenotype space) through exploration and exploitation for the “optimal” solution. Such activity is controlled through the use of biologically inspired “mating”, “mutation” and “selection” operators. Specific evolutionary algorithm development involves the encoding of the independent variables (genotype) and the structuring of specific parametric mating, mutation, and selection

operators. These operators manipulate each genotype individual appropriately as the search proceeds through the phenotype landscape.

The design of innovative evolutionary algorithmic approaches for multi-objective problems is built upon the research and development for single objective functions. Understanding this body of knowledge lends insight to the design and implementation of MOEAs. The use of MOEAs requires insight not only of the algorithmic domain, but also knowledge of the application problem domain. This monograph addresses such variations in the development of multi-objective evolutionary algorithms (MOEA), associated theory, appropriate multi-objective problems (MOPs) for MOEA testing, and experience with real-world applications. Many references are included and suggested for further reading.

Applying the fundamental concepts of MOEAs to real-world problems was initially a curiosity, but today is a common trend. By using the concepts and techniques presented in this book one can obtain insight into the selection of an MOEA software platform and associated tuning of the various operator parameters for complex applications. Moreover, most complex real-world applications have side constraints which requires MOEA tailoring in searching the fitness landscape. This book attempts to address all these issues through the following features:

- It has been conceived to be a self-contained reference. This book provides all the necessary elements to guide a newcomer in the design, implementation, validation and application of MOEAs.
- Researchers in the field will benefit from the book's comprehensive review of state-of-the-art concepts and discussions of open research topics.
- The book is also written for graduate students in computer science, computer engineering, operations research, management science, and other scientific and engineering disciplines, who are interested in multi-objective optimization using evolutionary algorithms.
- The book has also been conceived for professionals interested in developing practical applications of evolutionary algorithms to real-world multi-objective optimization problems.
- Each chapter is complemented by discussion questions and several ideas that attempt to trigger novel research paths. Supplementary reading is strongly suggested for deepen the understanding of MOEAs.
- Key features include MOEA classifications and explanations, MOEA applications and techniques, MOEA test function suites, and MOEA performance measurements.

The flow of material in each chapter is intended to present a natural and comprehensive development of MOEAs from basic concepts to complex applications. As previously stated, at the end of each chapter a list of possible research topics is given along with a number of pertinent discussion questions. Chapter 1 presents and motivates MOP and MOEA terminology and nomenclature that is used in the following chapters.

In Chapter 2, the developmental history of MOEAs is presented, noting that it has proceeded in number of ways from aggregated forms of single objective EAs to true multi-objective approaches such as MOGA, MOMGA, NPGA, NSGA, PAES, SPEA and their extensions. Additionally, each MOEA is presented with historical and algorithmic insight. Being aware of the many facets of historical multiobjective problem solving provides a foundational understanding of the discipline. The various MOEA techniques and constructs are compared leading to a generic algorithm incorporating general MOEA operators and parameters. In addition, the variety of techniques for incorporating MOEA constructs for constrained MOPs are delineated. A comprehensive comparison of contemporary MOEAs provides insight to their individual advantages and disadvantages. Software implementation issues of structure, user friendly interface and graphics presentations are also addressed.

Chapter 3 presents a detailed development of MOP test suites from numerical functions (unconstrained and with side constraints) and generated functions to discrete *NP*-Complete problems and real-world applications. Associated appendices include extensive tables and the Pareto optimal set and the associated Pareto front for many of the proposed test functions. The objective is to provide a comprehensive listing and a classification of contemporary MOEA test functions. This knowledge leads to an understanding and an ability to select appropriate MOEA test suites based upon a set of desired comparative characteristics.

MOEA performance comparisons are presented in Chapter 4 using many of the test function suites discussed in Chapter 3. Also, an extensive discussion of possible comparison metrics and presentation techniques are discussed. The selection of key algorithmic parameter values (population size, termination, etc.) is emphasized. A limited set of MOEA results are related to the design and analysis of efficient and effective MOEAs employing these various MOP test suites and appropriate metrics. Thus, a wide spectrum of empirical testing and analysis techniques are provided to the MOEA user.

Although MOEA theory is relatively limited, Chapter 5 presents a summary of known results related to MOEA convergence to the Pareto front. Also, theoretical and practical issues ranging from Pareto ranking and niching to mating restriction, stability, and complexity are discussed.

Although is unrealistic to present every MOP application, Chapter 6 attempts to group and classify the contemporary multitude of various MOEA applications

via representative examples. Such a compendium provides the reader with a starting point for their own application based upon a similar problem domain. Genetic operators as well as encodings adopted in many MOEA applications are also briefly discussed.

In Chapter 7, research and development of parallel MOEAs is classified and analyzed. The three foundational paradigms (master-slave, island, diffusion) are defined. Using these three structures, many contemporary MOEA parallel developments are algorithmically compared and analyzed in terms of advantages and disadvantages for different computational architectures. Some general observations about the current state of parallel and distributed MOEAs are also stated.

Chapter 8 discusses and compares the two main schools of thought regarding multi-criteria decision making (MCDM): Outranking approaches and Multi-Attribute Utility Theory (MAUT). Aspects such as the operational attitude of the Decision Maker (DM), the different stages at which preferences can be incorporated, scalability, transitivity and group decision making are also discussed. However, the main emphasis is to describe the most representative research regarding preference articulation into MOEAs. This comprehensive review includes brief descriptions of the approaches reported in the literature as well as an analysis of their advantages and disadvantages.

Chapter 9 discusses multiobjective extensions of other search heuristics that can be integrated into MOEAs. The main techniques covered include tabu search, scatter search, simulated annealing, ant colony, distributed reinforcement learning, and memetic algorithms. Such techniques are being incorporated into MOEAs for efficiency purposes.

To profit from the book, one should have at least single objective EA knowledge and experience. Also, some mathematical knowledge is appropriate to understand symbolic functions as well as theoretical aspects of MOEAs. This includes basic linear algebra, calculus, probability and statistics. The use of this text in a graduate course on MOEAs is recommended using these prerequisites.

In support of this text, one can find up-to-date MOEA reference listings of journal papers, conference papers, MOP software, and MOEA software at the Evolutionary Multi-Objective Optimization (EMOO) Repository internet web site <http://www.lania.mx/~ccoello/EMOO> or the USA mirror repository at <http://www.jeo.org/emo/>. These sites will continually be updated in support of this text. If you have a contribution, please send it to ccoello@cs.cinvestav.mx.

The genesis of this book resides in the PhD dissertation of David A. Van Veldhuizen (Graduate school of Engineering, Air Force Institute of Technology, 1999) as well as the research papers of the authors.

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CARLOS A. COELLO COELLO
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